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BRIEF COMMUNICATIONS

THEORETICAL AND MATHEMATICAL PHYSICS

RADIATIVE SHIFT OF NEUTRINO ENERGY IN A MAGNETIZED ELECTRON-POSITRON GAS

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A radiative energy shift and an induced magnetic moment of a relativistic massive Dirac neutrino in a hot electron-positron plasma and in a degenerate electron gas in an external unquantizing magnetic field have been calculated.

Investigation of radiative effects that accompany neutrino propagation at finite temperature and matter density in the presence of a constant external magnetic field is of indisputable astrophysical interest. It is sufficient to mention the so-called solar neutrino problem [1], whose solution according to the Okun'–Voloshin–Vysotsky hypothesis [2] is only possible if the neutrino has electromagnetic moments of order $\sim 10^{-10}\mu_B$, where $\mu_B = e/2m$ is the Bohr magneton (we use the system of units with $\hbar = c = 1$). The value required is nine orders greater than the known value of the anomalous magnetic moment of the massive Dirac neutrino in vacuum, which was obtained in the framework of the Standard Model of electroweak interaction of Salam and Weinberg (see [1]),

$$\mu_\nu^{(0)} = \frac{3}{8\pi^2}\frac{G_F}{\sqrt{2}}m_\nu e \cong 3\times 10^{-19}\frac{m_\nu}{1\ \mathrm{eV}}\mu_B, \tag{1}$$

where $G_F = \dfrac{\sqrt{2}}{8M_W^2}$ is the Fermi constant, M_W is the W-boson mass, m_ν is the mass of an electron neutrino whose value is subject to the stringent experimental upper bound: $m_\nu \leq 15$ eV [3].

The use of extended modifications of the electroweak model [4] yielded better results (the value of the neutrino magnetic moment was $\mu_\nu \sim 10^{-13}\mu_B$). However, before giving preference to the alternative theories, one should perform an intense study of the question in the framework of the Standard Model. In particular, it might be well to estimate the effective corrections to the vacuum magnetic moment of an electron neutrino at the sacrifice of its interaction with particles of the external medium together with a strong external magnetic field.

In the present work we determine the radiative shift of the energy ΔE_ν and the induced magnetic moment $\Delta\mu_\nu$ of a massive Dirac neutrino in a magnetized electron-positron gas with a chemical potential μ at finite temperature T. The calculation is performed in the 1-loop approximation (only the contribution of the charged current is accounted for). This problem has already been considered in paper [5], where the use was made of the real time representation for the electron propagator in an external magnetic field $\mathbf{H} \uparrow\uparrow Oz$ at finite temperature. In the real time representation the contribution of interest made by the effects of a hot and dense medium to the radiative shift of the neutrino energy is automatically separated from the pure vacuum part [6]. The expression for the temperature term $\Delta E_\nu(T,\mu,H)$ found in paper [5] with no restriction on the values of the parameters T, μ, and H was also used there for estimations in the case of a degenerate ($T \to 0$) electron gas. We have obtained the general formula for $\Delta E_\nu(T, \mu, H)$ which is equivalent to the main result of [5] but is more suitable for particular calculations at high values of the occupation numbers of the Landau levels n of an electron gas both in a completely degenerate case and at finite temperature. In the new expression integration is performed over the discrete numbers n, an integral over the variable u appeared instead of the sum which has the meaning of the square of the transverse momentum of an electron with the energy $E = \sqrt{u + p_3^2 + m_e^2}$, the mass m_e and the momentum component p_3 along the field direction. The result has the following form:

$$\Delta E_\nu = \frac{-g^2}{32\pi^3} \sum_{\varepsilon=\pm 1} \int_{-\infty}^{\infty} dp_3 \int_0^{\infty} ds \int_0^{\infty} du \int_{-\infty}^{\infty} d\lambda \, \frac{\varepsilon}{\exp\left[\dfrac{E - \varepsilon\mu}{T}\right] + 1}$$
$$\times \exp\left[i\lambda u + is\left((\varepsilon E - E_\nu)^2 - (p_3 - q_3)^2 - M_W^2 - q_\perp^2 \frac{\sin(eHs)\sin(eH\lambda)}{eHs\sin(eH(s+\lambda))}\right)\right] \frac{eH}{\sin(eH(s+\lambda))} \tag{2}$$
$$\times \left[A^2\left(1 + \frac{\varepsilon p_3}{E}\right)\exp[ieH(2s+\lambda)] + B^2\left(1 - \frac{\varepsilon p_3}{E}\right)\exp[-ieH(2s+\lambda)] + AB\frac{2\varepsilon q_\perp}{E}\frac{\sin(eHs)}{\sin(eH(s+\lambda))}\right],$$

where the following notation has been made

$$A = \sqrt{\left(1 - \frac{q_3}{E_\nu}\right)\left(1 + \zeta_\nu \frac{m_\nu}{E_\nu^\perp}\right)}, \quad B = \sqrt{\left(1 + \frac{q_3}{E_\nu}\right)\left(1 - \zeta_\nu \frac{m_\nu}{E_\nu^\perp}\right)},$$

$-e$ is the electron charge, $(E_\nu, \mathbf{q})$ is the neutrino 4-momentum, and $\zeta_\nu = \pm 1$ determines the neutrino spin orientation along and contrary to the field direction respectively.

Let an ultrarelativistic neutrino move in the direction perpendicular to the magnetic field with intensity $H \ll M_W^2/e \cong 10^{24}$ Gs. Let us consider two limiting cases that are interesting for astrophysical applications:

$$\mu = 0, \quad \sqrt{eH} \ll T \ll M_W \ll E_\nu \ll M_W^2/T \tag{3}$$

is the case of a hot electron-positron plasma in a weak magnetic field, which was not considered in [5], and

$$T \to 0, \quad \sqrt{eH} \ll \mu \ll M_W \ll E_\nu \ll M_W^2/\mu \tag{4}$$

is the case of a completely degenerate electron gas in an unquantizing magnetic field, then the chemical potential $\mu \cong \sqrt{m_e^2 + (3\pi^2 n_e)^{3/2}}$, where n_e is the electron concentration.

General expression (2) for the contribution to the radiative shift of the neutrino energy arising due to the effects of finite values of temperature and the medium density has the following asymptotics in the two mentioned cases:

$$\Delta E_\nu(T, H) = -\frac{7}{30}\frac{\pi^2}{\sqrt{2}} G_F \frac{T^4}{M_W^2} E_\nu - \zeta_\nu H \Delta\mu_\nu(T)$$

when conditions (3) are fulfilled and

$$\Delta E_\nu(n_e, H) = \sqrt{2} n_e G_F - \zeta_\nu H \Delta\mu_\nu(n_e)$$

when conditions (4) are fulfilled. Here $\Delta\mu_\nu(T)$ and $\Delta\mu_\nu(n_e)$ are the corrections to the magnitude of the neutrino magnetic moment, which are due to the medium contributions

$$\Delta\mu_\nu(T) = \frac{8}{9}\frac{T^2}{M_W^2}\mu_\nu^{(0)}, \tag{5}$$

$$\Delta\mu_\nu(n_e) = -\frac{16}{3}\frac{(3\pi^2 n_e)^{1/3}}{E_\nu}\mu_\nu^{(0)}. \tag{6}$$

We note that in papers [5, 7] the induced neutrino magnetic moment is determined for a neutrino at rest, i.e., according to the formula

$$\operatorname{Re} \Delta E_\nu(\zeta, H, \Delta\mu_\nu) = -\frac{m_\nu}{E_\nu}\zeta H \Delta\mu_\nu.$$

In this case one should assume $E_\nu = m_\nu$ in (6), then the corresponding contribution to the magnetic moment proves to be significant [5, 7]. For a relativistic neutrino numerical estimates (5) and (6) demonstrate that the correction to the magnitude of the neutrino magnetic moment which is due to its interaction with medium particles is small as compared to (1) (this follows from inequalities (3) and (4) respectively) and has different signs in the limiting cases considered.

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