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THE COUPLING CONSTANT VALUE AT THE GRAND UNIFICATION SCALE

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Using the analogy between Bogomolny's inequality in the field theory and the harmonic oscillator Hamiltonian in quantum mechanics, a possible explanation of the coupling constant numerical value at the Grand Unification scale is proposed which proves to be equal to $1/8\pi$.

Precise measurements of fundamental interaction constants [1] show that, in the framework of the Standard Model, the running coupling constants do not converge to a single point, while in the Minimal Supersymmetric Standard Model (MSSM) [2] this convergence does exist (within the error of modern experimental data) (see, e.g. [3]). As a rule, the latter fact has been used as an indirect evidence of supersymmetry existence. However, it can be regarded somewhat differently, i.e., as an indirect measurement of the coupling constant at the Grand Unification scale (α_{GUT}). Its numerical value turns out to be around 1/25.

At present there is no any theoretical explanation of the fact that the coupling constant takes this particular value. Apparently, the unified theory of all fundamental interactions should predict this value. It may be just a combination of some mathematical constants. We note that the numerical value of α_{GUT}^{-1} indirectly predicted by the MSSM is quite close to $8\pi \cong 25.13$. This value is certainly dependent on the renormalization scheme already in the two-loop approximation. It is known, however, that there exists, for example, in quantum chromodynamics, a certain physical renormalization scheme where calculations result in measurable quantities. Therefore, in what follows, we assume possible experimental measurement of α_{GUT} as well as existence of a preferred (physical) renormalization scheme, by analogy with the known case.

In the present paper we also try to explain why the coupling constant takes a certain chosen value. To this end we shall apply the analogy between the harmonic oscillator Hamiltonian in mechanics and Bogomolny's equation.

Actually, the harmonic oscillator is the simplest example of a self-dual model [4] since its Hamiltonian

$$H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2} \tag{1}$$

is invariant under the transformations

$$p \to -m\omega x; \qquad x \to p/m\omega. \tag{2}$$

Alternatively, first hypotheses of duality [5] were based on the Bogomolny inequality symmetry property [4, 6],

$$M \geq v(Q_e^2 + Q_m^2)^{1/2}, \tag{3}$$

where Q_e and Q_m are the electric and magnetic charges, and M is the state mass.

We note that the expression $Q_e^2 + Q_m^2$ closely resembles (1). Moreover, when standard fields are used, magnetic monopoles behave like classic particles. In the strong coupling region, however, the Lagrangian should be written in terms of dual fields [7, 8] interacting with the field of a magnetic monopole which is quantized under these conditions. This situation is in a way similar to the coordinate and momentum representations in quantum mechanics.

Furthermore, Dirac's quantization condition, $eQ_m = 2\pi N$, is somewhat similar in form to the Bohr–Sommerfeld quantization rule,

$$\int p\,dx = 2\pi N. \tag{4}$$

Now we make another step forward to this analogy development. We assume the electric and magnetic charges to be canonical conjugate operators,

$$\hat{Q}_e \to e, \qquad \hat{Q}_m \to \frac{1}{i}\frac{d}{de}. \tag{5}$$

Partly, these considerations are induced by the fact that the commutator of electric and magnetic fluxes across closed loops α and β in the *quantum* field theory is [9]

$$[\hat{B}(\alpha), \hat{E}(\beta)] = iGL(\alpha, \beta), \tag{6}$$

where $GL(\alpha,\beta)$ is an integer determining the number of turns of one loop around the other.

So, when the loops are constricted to a point, i.e., when a flux through a closed surface is considered, the commutator turns out to be ill-defined and the statement of simultaneous measurement of electric and magnetic charges can be doubted.

Now, by analogy with quantum mechanics, we write the Schrödinger-type equation,

$$\left(-\frac{d^2}{de^2} + e^2\right)\psi(e) = \lambda\psi(e). \tag{7}$$

Using the same analogy, $\lambda/4\pi$ can be interpreted as the coupling constant value, and ψ as its distribution probability amplitude.

The lowest operator eigenvalue in the right-hand side of (7) is equal to unity, and the corresponding eigenfunction is

$$\psi_0 = \frac{1}{\pi^{1/4}}\exp(-e^2/2). \tag{8}$$

Actually, equation (7) is a hypothesis of the coupling constant distribution with a certain probability. In this case, the monopole contribution to the effective action above the magnetic monopole production threshold is reduced to the replacement of the coupling constant by the above operator. The origin of the derivative with respect to the coupling constant is the magnetic charge which has the mass $\cong M_{GUT}$ [10] in Grand Unification models. In the low-energy theory such terms are absent. Hence, one may assume that below the monopole production threshold the squared magnetic charge operator can be omitted and the following mean value can be calculated:

$$\alpha_{GUT} = \frac{\langle e^2\rangle}{4\pi} = \frac{1}{4\pi}\int de\; e^2 \frac{1}{\sqrt{\pi}}\exp(-e^2) = \frac{1}{8\pi}. \tag{9}$$

It should be noted that this magnitude depends neither on the choice of the system of units (which is easily verified), nor on the choice of the wave function normalization because according to equation (7) we have

$$\left\langle -\frac{d^2}{de^2} + e^2\right\rangle = 2\langle e^2\rangle = 1, \tag{10}$$

and, therefore, α_{GUT} is exclusively determined by the operator eigenvalue in the right-hand side of (7).

(Interpreting $\lambda/4\pi$ as the coupling constant value specifies unique computation of mean values

$$\left\langle \hat{f}\right\rangle = \frac{\int de\,\bar{\psi}\hat{f}\psi}{\int de\,\bar{\psi}\psi}, \tag{11}$$

so that the normalization constant is canceled between the numerator and denominator.)

The numerical value of α_{GUT}^{-1} is around 25.13, which is in good agreement with the MSSM indirect predictions [11] in the light of recent experimental data [1].

Above the Grand Unification scale we have

$$\alpha_{GUT} = \frac{\lambda}{4\pi} = \frac{1}{4\pi}. \tag{12}$$

The interaction constant jump occurring as the production threshold for certain particles is crossed is known in the field theory. Particularly, in breaking the symmetry in the Standard Model, $SU(2) \times U(1) \to U(1)$, none of the running coupling constants is continuous. However, in our case we deal with a somewhat different phenomenon since the jump does not now relate to the normalization change. This is caused by vanishing of magnetic monopoles contribution to the coupling constant as the threshold is crossed. In some way, this resembles the phenomenon of coupling constants separation in crossing the gauge boson production threshold. The discontinuity of the function and not of the derivative is related to the fact that the magnetic contribution appears to be present in the total effective action, but not only in the loop corrections. The substitution $e \to \sqrt{\langle e^2 \rangle}$ in the effective action leads to the running coupling constants behavior, depicted in Fig. 1.

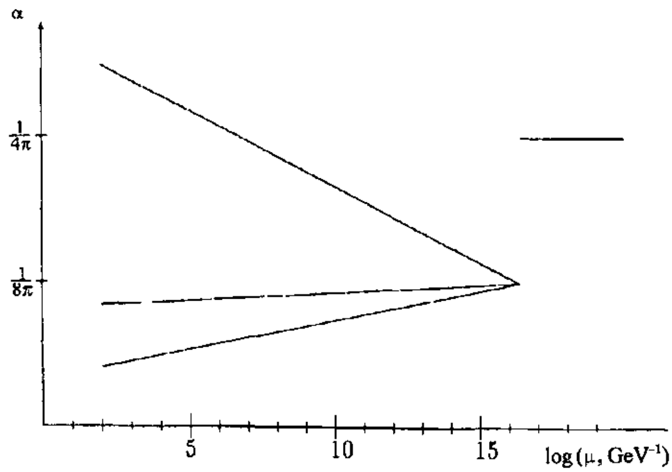

Fig. 1

Behavior of the running coupling constants.

As a concluding remark, we note that, although our basic assumptions are not rigorously justified and need further development, the present hypothesis may have the right to exist, the more so as other hypotheses are presently absent. It is also possible that some of the ideas suggested in the present paper may be of use for future investigations in this line. However, even now the exact numerical value of α_{GUT}, should the corresponding renormalization scheme be known, may be of interest to phenomenology studies on the Standard Model parameters forecast.

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