Moscow University Physics Bulletin
Vol. 54, No. 1, pp. 8–16, 1999

Vestnik Moskovskogo Universiteta. Fizika
UDC 537.874

ANALYSIS OF LIGHT SCATTERING BY A PARTICLE INSIDE A LAYER BY THE METHOD OF DISCRETE SOURCES

E. Yu. Eremina and A. G. Sveshnikov

A mathematical model of the problem of light scattering by a particle residing inside a layer on a substrate is analyzed on the basis of the method of discrete sources. A computational algorithm has been constructed. Computation results have been presented for light scattering by particles of various materials.

INTRODUCTION

The improvement of the hardware components of personal computers becomes more and more dynamic. The further miniaturization of memory and storage disk elements requires more stringent standards for the deviations of their configuration from the preset structure. One of the most important problems is the detection of contaminating particles striking the surface of silicon wafers at various stages of the manufacturing process. The detection of particles hidden beneath the film used in the course of wafer lithography is the most difficult-to-solve problem involved. Experimental investigations are difficult to perform because of the absence of the method of particles calibration, except for polystyrene ones, and take much money. Based on the method of discrete sources, the authors of [1] have analyzed the scattering of *P*- and *S*-polarized light by particles on a silicon substrate. In the present work, the method used in [1] is generalized to the studies of light scattering by particles residing inside an arbitrary dielectric layer on a substrate.

1. MATHEMATICAL MODEL OF A PERMEABLE PARTICLE IN A LAYER

We begin with the mathematical formulation of the problem in hand. Let $\{\mathbf{E}^0, \mathbf{H}^0\}$ be the field of a plane electromagnetic wave of linear polarization incident at an angle γ to the normal on the plane air-layer (D_0, D_f) interface Ξ_f (Fig. 1) and let the particle D_i with a smooth boundary ∂D be completely inside the layer of thickness d bounded by the planes Ξ_f and Ξ_1. The plane Ξ_1 separates the layer and the substrate D_1. Let the particle be axisymmetric so that its symmetry axis coincides with the direction of the external normal to the plane Ξ_1. We introduce a rectangular coordinate system with the origin on the plane Ξ_1 and with the Oz-axis directed along the symmetry axis of the scatterer. The mathematical formulation of the problem then has the form

$$\nabla \times \mathbf{H}_t = ik\varepsilon_t \mathbf{E}_t; \quad \nabla \times \mathbf{E}_t = -ik\mu_t \mathbf{H}_t \ \text{ in } \ D_t, \quad t = 0, f, 1, i, \tag{1}$$

$$\left.\begin{aligned} \mathbf{n}_p \times (\mathbf{E}_i(p) - \mathbf{E}_f(p)) = 0, \\ \mathbf{n}_p \times (\mathbf{H}_i(p) - \mathbf{H}_f(p)) = 0, \end{aligned}\right. \quad p \in \partial D, \tag{2}$$

$$\left.\begin{aligned} \mathbf{e}_z \times (\mathbf{E}_0(p) - \mathbf{E}_f(p)) = 0, \\ \mathbf{e}_z \times (\mathbf{H}_0(p) - \mathbf{H}_f(p)) = 0, \end{aligned}\right. \quad p \in \Xi_f, \tag{3}$$

$$\begin{aligned} \mathbf{e}_z \times (\mathbf{E}_f(p) - \mathbf{E}_1(p)) &= 0, \\ \mathbf{e}_z \times (\mathbf{H}_f(p) - \mathbf{H}_1(p)) &= 0, \end{aligned} \qquad p \in \Xi_1. \tag{4}$$

The above expressions should be supplemented with the emission (damping) conditions for scattered fields at infinity ($z \neq 0, d$).

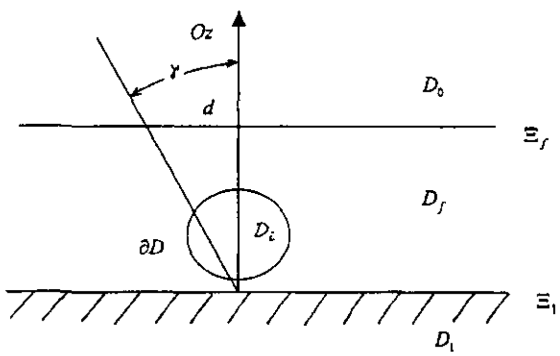

Fig. 1

Geometry of the problem.

Here $\mathbf{n}_p$ is the normal to the surface ∂D and $(\mathbf{E}_t, \mathbf{H}_t)$ is the total field in the corresponding region; specifically, the field $(\mathbf{E}_0, \mathbf{H}_0)$ is the sum of the incident field $(\mathbf{E}^0, \mathbf{H}^0)$, the field reflected from the surface Ξ_f, and the scattered field in D_0. We assume that the surface of the particle is smooth, i. e., $\partial D \subset C^{(1,\alpha)}$, and that the media parameters satisfy the conditions $\operatorname{Im} \varepsilon_t, \mu_t \leq 0$ (the time dependence $\exp\{i\omega t\}$). Then, boundary value problem (1)–(4) has the only solution.

Before proceeding to the construction of the approximate solution for the scattered fields, we solve the problem of diffraction of the plane wave field $(\mathbf{E}^0, \mathbf{H}^0)$ by the air-layer-substrate sandwich structure.

It has been known [2] that this solution can be written in an explicit form. We denote the field obtained as $(\mathbf{E}_t^0, \mathbf{H}_t^0)$, $t = 0, f, 1$. We will construct the approximate solution of boundary value problem (1)–(4) on the basis of the method of discrete sources [3], which essentially consists in representing the scattered field in the form of a finite linear combination of multipole fields satisfying the system of Maxwell equations in the domains $D_{0,f,1,i}$, the conditions at infinity for the scattered field in $D_{0,f,1}$, and also the adjointness conditions for the tangential field components at $\Xi_{f,1}$. In this case, the solution of boundary value scattering problem (1)–(4) is reduced to the problem of approximation of the external excitation field on the surface of the particle by the multipole fields. Thus, the unknown amplitudes of the discrete sources are only determined to satisfy the adjointness conditions on the surface ∂D of the particle. A characteristic feature of this approach is the possibility of the *a posteriori* estimation of the error of the solution obtained by calculating the discrepancy between the boundary values on ∂D.

The representation of the external field will be based on multipole sources satisfying the adjointness conditions on the planes $\Xi_{f,1}$. In this case, the structure of the fields will be governed by the Green tensor of the sandwich structure, which has the form

$$\mathbf{G}(Q,P) = \begin{bmatrix} g & 0 & 0 \\ 0 & g & 0 \\ \partial f/\partial x_Q & \partial f/\partial y_Q & \sigma \end{bmatrix}.$$

The fields of the multipole sources of electric (e) and magnetic (h) type are constructed on the basis of vector potentials having the following structure:

$$\mathbf{A}_{mx}^{e,h} = \mathbf{e}_x g_m^{e,h} - \mathbf{e}_z f_{m+1} \cos\phi,$$

$$\mathbf{A}_{my}^{e,h} = \mathbf{e}_y g_m^{e,h} - \mathbf{e}_z f_{m+1} \sin\phi,$$

where the corresponding azimuthal harmonics of the Green tensor components have the form

$$g_m^{e,h}(q, w_n) = \int_0^\infty J_m(\lambda\rho)\nu_{11}^{e,h}(z, w_n, \lambda)\lambda^{m+1}d\lambda, \tag{5}$$

$$f_m(q, w_n) = \int_0^\infty J_m(\lambda\rho)\nu_{31}^{e,h}(z, w_n, \lambda)\lambda^{m+1}d\lambda. \tag{6}$$

Here J_m is the cylindrical Bessel function, the point $q = (\rho, z)$ is located in the half-plane $\phi =$ const, and the multipoles are distributed along the symmetry axis $w_n \in Oz$ inside the particle D_i. In the given case, the following representations hold true for the spectral functions $\nu_{11}^{e,h}(\lambda, z)$ and $\nu_{31}^{e,h}(\lambda, z)$:

$$\nu_{11}^{e,h} = \begin{cases} A_{11}^{e,h}(\lambda, w_n, d)\exp\{-\eta_0(z-d)\}, & z > d, \\ \dfrac{\exp\{-n_f|z-z_0|\}}{\eta_f} + B_{11}^{e,h}\exp\{-\eta_f(d-z)\} + C_{11}^{e,h}\exp\{-\eta_f z\}, & d > z > 0, \\ D_{11}^{e,h}\exp\{\eta_1 z\}, & z < 0, \end{cases}$$

$$\nu_{31}^{e,h} = \begin{cases} A_{11}^{e,h}(\lambda, w_n, d)\exp\{-\eta_0(z-d)\}, & z > d, \\ B_{31}^{e,h}\exp\{-\eta_f(d-z)\} + C_{31}^{e,h}\exp\{-\eta_f z\}, & d > z > 0, \\ D_{31}^{e,h}\exp\{\eta_f z\}, & z < 0. \end{cases}$$

Moreover, the spectral functions satisfy the adjointness conditions at the boundaries $z = 0, d$ [4]

$$[\nu_{11}^e] = \left[\frac{d\nu_{11}^e/dz}{\mu}\right] = 0, \quad [\nu_{11}^h] = \left[\frac{d\nu_{11}^h/dz}{\varepsilon}\right] = 0,$$

$$\left[\frac{\nu_{33}^e}{\mu}\right] = \left[\frac{d\nu_{33}^e/dz}{\varepsilon\mu}\right] = 0, \quad \left[\frac{\nu_{33}^h}{\varepsilon}\right] = \left[\frac{d\nu_{33}^h/dz}{\varepsilon\mu}\right] = 0,$$

$$\left[\frac{d\nu_{31}^e/dz}{\varepsilon\mu}\right] = -\left[\frac{1}{\varepsilon\mu}\right]\nu_{11}^e, \quad \left[\frac{\nu_{31}^h}{\varepsilon\mu}\right] = -\left[\frac{1}{\varepsilon\mu}\right]\nu_{11}^h,$$

$$\left[\frac{d\nu_{31}^e/dz}{\mu}\right] = 0, \quad \left[\frac{\nu_{31}^h}{\varepsilon}\right] = 0.$$

Here $\eta_t^2 = \lambda^2 - k_t^2,\ k_t^2 = k^2\varepsilon_t\mu_t,\ t = 0, 1, f$.

In constructing the approximate solution, we follow the scheme of the method of discrete sources presented in [1]. The basis for the representation for the total field inside the particle are the regular functions, whose singularities are at infinity [3]

$$\mathbf{A}_{mx}^i = g_m^i(q, w_n)\mathbf{e}_x, \quad \mathbf{A}_{my}^i = g_m^i(q, w_n)\mathbf{e}_y, \tag{7}$$

$$g_m^i(q, w) = j_m(k_i R_{qw})\left(\frac{k_i\rho}{R_{qw}}\right)^m, \tag{8}$$

where j_m is the spherical Bessel function and $w_n \in Oz$.

One can easily see that $\rho/R_{qw} = \sin\theta_{qw}$, where θ_{qw} is the angle the spherical coordinate system makes with the origin at the point w. For this reason, the functions g_m^i may be rearranged in the form

$$g_m^i(q, w) = j_m(k_i R_{qw})P_m^m(\cos\theta_{qw})\frac{k_i^m}{(2m-1)!!}.$$

Here P_m^m are the adjoint Legendre polynomials. It is easy to see now that the functions

$$G_m^i(q,w) = g_m^i(q,w)\left\{\begin{array}{c}\sin m\phi\\ \cos m\phi\end{array}\right\}$$

satisfy the homogeneous Helmholtz equation $\Delta G_m^i + k_i^2 G_m^i = 0$ everywhere in the finite domain R^3 and thus are regular functions.

Now we can formally restrict ourselves to the construction of a representation for electromagnetic fields in the layer outside and inside of the particle only. We also construct an approximate solution, which considers not only the axial symmetry of the scatterer, but also the polarization of the external excitation [5].

We consider a P-polarized plane wave (whose $\mathbf{E}^0$ vector lies in the plane of incidence) propagating at an angle θ_0 to the Oz-axis. Having solved the problem of diffraction of the plane wave by the sandwich structure in the absence of the particle, we obtain the external excitation field inside the layer. It is represented in the form of a set of two plane waves propagating at some angle to each other in the opposite directions.

Based on the idea of the method of discrete sources [5] to allow for the polarization of the external field, we introduce the following combinations of potentials:

$$\begin{cases}\mathbf{A}_{\nu f}^e := \mathbf{A}_{mx}^e \cos m\phi - \mathbf{A}_{my}^e \sin m\phi,\\ \mathbf{A}_{\nu f}^h := \mathbf{A}_{mx}^h \sin m\phi + \mathbf{A}_{my}^h \cos m\phi,\end{cases}$$
$$\begin{cases}\mathbf{A}_{\nu i}^e := \mathbf{A}_{mx}^i \cos m\phi - \mathbf{A}_{my}^i \sin m\phi,\\ \mathbf{A}_{\nu i}^h := \mathbf{A}_{mx}^i \sin m\phi + \mathbf{A}_{my}^i \cos m\phi,\end{cases} \tag{9}$$

Here $\nu = \{m,n\}$ is the multiindex varying over the range $0 \le m \le M$, $1 \le n \le N$, where M is the maximum number of the Fourier harmonics and N is the number of multipoles, the same for all values of m. We write ν_0, Y ($Y = \{M,N\}$) for the inferior and upper indices of the multiindex. Moreover, we introduce vector potentials for the vertical dipoles. In the Green tensor, σ is responsible for the vertical dipoles, and so

$$\mathbf{A}_{n0}^e = \sigma^e(q,w_n)\mathbf{e}_z, \quad \mathbf{A}_{ni}^e = g_0^i(q,w_n)\mathbf{e}_z,$$
$$\sigma^e = g_0^h(q,w_n). \tag{10}$$

To render the exposition more convenient, we introduce abridged designations for the fields, which will be used in constructing the approximate solution

$$\begin{pmatrix}\mathbf{E}_{\nu t}^e\\ \mathbf{H}_{\nu t}^e\end{pmatrix} = \begin{pmatrix}\dfrac{i}{k\varepsilon_t\mu_t}\nabla\times\nabla\times\\ -\dfrac{1}{\mu_t}\nabla\times\end{pmatrix}\mathbf{A}_{\nu t}^e,$$
$$\begin{pmatrix}\mathbf{E}_{\nu t}^h\\ \mathbf{H}_{\nu t}^h\end{pmatrix} = \begin{pmatrix}\dfrac{1}{k\varepsilon_t}\nabla\times\\ \dfrac{i}{k\varepsilon_t\mu_t}\nabla\times\nabla\times\end{pmatrix}\mathbf{A}_{\nu t}^h. \tag{11}$$

Considering (11), we construct the representation for the approximate solution of boundary value problem (1)–(4) in the case of P-polarized field in the form

$$\begin{pmatrix}\mathbf{E}_t^Y\\ \mathbf{H}_t^Y\end{pmatrix} = \sum_{\nu=\nu_0}^{Y}\left\{p_\nu^t\begin{pmatrix}\mathbf{E}_{\nu t}^e\\ \mathbf{H}_{\nu t}^e\end{pmatrix} + q_\nu^t\begin{pmatrix}\mathbf{E}_{\nu t}^h\\ \mathbf{H}_{\nu t}^h\end{pmatrix}\right\} + \sum_{n=1}^{N} r_n^t\begin{pmatrix}\mathbf{E}_{nt}^e\\ \mathbf{H}_{nt}^e\end{pmatrix}, \tag{12}$$

where the last term corresponds to the vertical dipole sources.

As noted above, representation (12) meets all the conditions of boundary value problem (1)–(4), except for the adjointness conditions on the surface of the particle.

In the case of S-polarization of the incident wave, when the wave vector is perpendicular to the plane of incidence, the approximate solution of problem (1)–(4) will, as with the scheme [5], have the form of (12), where, instead of potentials (9), use is made of the following combinations:

$$\begin{cases} \mathbf{A}^e_{\nu f} := \mathbf{A}^e_{mx} \sin m\phi + \mathbf{A}^e_{my} \cos m\phi, \\ \mathbf{A}^h_{\nu f} := \mathbf{A}^h_{mx} \cos m\phi - \mathbf{A}^h_{my} \sin m\phi, \end{cases}$$
$$\begin{cases} \mathbf{A}^e_{\nu i} := \mathbf{A}^i_{mx} \sin m\phi + \mathbf{A}^i_{my} \cos m\phi, \\ \mathbf{A}^h_{\nu i} := \mathbf{A}^i_{mx} \cos m\phi - \mathbf{A}^i_{my} \sin m\phi. \end{cases} \tag{13}$$

Besides, the term corresponding to the vertical dipole sources in the right-hand side of (12) contains, instead of the field of vertical electric dipoles, $(\mathbf{E}^e_{nt}, \mathbf{H}^e_{nt})$, the field of vertical magnetic dipoles, $(\mathbf{E}^h_{nt}, \mathbf{H}^h_{nt})$, whose vector potentials have the form

$$\mathbf{A}^h_{n0} = \sigma^h(q, w_n)\mathbf{e}_z, \quad \mathbf{A}^h_{ni} = g^i_0(q, w_n)\mathbf{e}_z, \quad \sigma^h = g^e_0(q, w_n). \tag{14}$$

This is due to the fact that the vector $\mathbf{H}^0$ in the case of S-polarization lies in the plane of incidence.

So, the approximate solution of boundary value problem (1)–(4) for the scattered field $(\mathbf{E}^S_t, \mathbf{H}^S_t)$ in D_t, $t = 0, f, 1$ and the total field in D_i, which allows for the polarization of the exciting plane wave, possesses the following properties:

(1) it satisfies the system of Maxwell equations in the domains D_α, $\alpha = 0, f, 1, i$,

(2) it automatically satisfies the adjointness conditions for the tangential field components at the air-layer and layer-substrate interfaces $\Xi_{f,1}$,

(3) it satisfies the conditions at infinity.

By virtue of the completeness of the system of multipoles [3] and the orthogonality of multipole systems (9)–(10) and (13)–(14) for both types of polarization, the following theorem holds true.

Theorem. *Let* $(\mathbf{E}^0, \mathbf{H}^0)$ *be a field of P- or S-polarization, then for any* $\delta > 0$ *there exist* $Y(\delta)$ *and the coefficients* $\{p_\nu, q_\nu\}_{\nu=\nu_0}^{Y}$ *such that*

$$\left\| \begin{bmatrix} \mathbf{n}, \mathbf{E}^Y_i - \mathbf{E}^Y_f - \mathbf{E}^0_f \\ \mathbf{n}, \mathbf{H}^Y_i - \mathbf{H}^Y_f - \mathbf{H}^0_f \end{bmatrix} \right\|_{L_2(\partial D)} < \delta,$$

where $(\mathbf{E}^Y, \mathbf{H}^Y)$ *is specified by representation* (12) *and the vector potentials have the form of* (9)–(10) *or* (13)–(14).

2. COMPUTATIONAL ALGORITHM

In considering the computational algorithm, we will follow the scheme presented in [1], only placing emphasis on the main specific features. As noted above, approximate solution (12) satisfies all the conditions of boundary value problem (1)–(4), except the adjointness conditions on the surface of the particle. Consequently, the unknown amplitudes of the discrete sources are determined exactly to fulfill the conditions on the surface ∂D.

It is convenient to break down the algorithm for determining the unknown amplitudes into several steps. Insofar as the sources are localized on the symmetry axis or in the corresponding part of the complex plane [5], approximate solution (12) of boundary value problem (1)–(4) is a finite linear combination of Fourier harmonics in the azimuthal variable ϕ. Therefore, at the first step we expand the electric and magnetic field tangential components of the exciting plane waves into a Fourier series in ϕ using the following plane wave representation

$$\exp\{\pm i\widetilde{\omega}\cos\phi\} = \sum_{m=0}^{\infty} (2 - \delta_{0m})(\pm i)^m J_m(\widetilde{\omega}) \cos m\phi,$$

where $\widetilde{\omega} = k_f \rho \sin\theta_0$. Since representation (12) for the fields $(\mathbf{E}^Y_t, \mathbf{H}^Y_t)$ has the form of a finite linear combination of Fourier harmonics, the determination of the amplitudes of the discrete sources reduces to the determination of the amplitude vector from the harmonics (i. e., at a fixed serial number m of the azimuthal harmonic). As a result, the surface approximation is reduced to the approximation of the fields on the generatrix $\mathfrak{J}$ of the body D_i.

To solve the last problem, we use the collocation method joining the azimuthal harmonics on the set of collocation points, $\{\chi_l\}_{l=1}^{L} \subset \mathfrak{J}$, on the generatrix of the particle and solving the redetermined system of linear equations obtained. In doing so, we have to use numerical algorithms for computing Sommerfeld integrals (5)–(6) when approaching the surface of the body D_i from without.

Despite the difference in the representations for the approximate solution dependent on the polarization of the external excitation, we can construct the computational algorithm, so that it proves sufficient to use the pseudoinversion of one and the same matrix [1] to compute the amplitudes of the discrete sources for each azimuthal harmonic both for P- and for S-polarizations. The systems for the ϕ-independent harmonic corresponding to the vertical dipoles (their dimensionality is half as much) have to be solved twice, for P- and S-polarization separately.

To compute the intensity of the scattered field at infinity, it is necessary to have the scattering diagram. It is defined in the well-known way

$$\frac{\mathbf{E}(\mathbf{r})}{|\mathbf{E}^0(\mathbf{r})|} = \frac{\exp\{-ik_0 r\}}{r}\mathbf{F}(\theta,\phi) + O\left(\frac{1}{r^2}\right), \quad r \to \infty.$$

To obtain a particular scattering diagram, it is sufficient to use the asymptotic representations for the Sommerfeld integrals [6]

$$\int_0^\infty J_m(\lambda\rho) f(\lambda)\exp\{-\eta_0(z-w_n)\}\lambda^{1+m}\,d\lambda = \psi_0 ik_0\cos\theta(ik_0\sin\theta)^m G_n f(k_0\sin\theta) + o\left(\frac{1}{r}\right),$$

where

$$\psi_0 = \frac{\exp\{-ik_0 r\}}{r}, \quad G_n = \exp\{-ik_0 w_n\cos\theta\}, \quad r^2 = \rho^2 + z^2.$$

For the θ- and ϕ-components of the scattering diagram in the case of P-polarization, we then have

$$F_\theta^P = \frac{ik_0}{\varepsilon_0}\sum_{m=0}^{M}\cos(m+1)\phi(ik_0\sin\theta)^m$$
$$\times\sum_{n=1}^{N}\left\{p_{nm}^0\left[G_{nm}^e\cos\theta + ik_0\sin^2\theta\, F_m\right] + q_{nm}^0 G_{nm}^h\right\} - \frac{ik_0}{\varepsilon_0}\sin\theta\sum_{n=1}^{N} r_n^0 G_{n0}^h\frac{\mu_0}{\mu_f},$$

$$F_\phi^P = -\frac{ik_0}{\varepsilon_0}\sum_{m=0}^{M}\sin(m+1)\phi(ik_0\sin\theta)^m\sum_{n=1}^{N}\left\{p_{nm}^0 G_{nm}^e + q_{nm}^0\left[G_{nm}^h\cos\theta + ik_0\sin^2\theta F_m\right]\right\},$$

and in the case of S-polarization,

$$F_\theta^S = \frac{ik_0}{\varepsilon_0}\sum_{m=0}^{M}\sin(m+1)\phi(ik_0\sin\theta)^m\sum_{n=1}^{N}\left\{p_{nm}^0\left[G_{nm}^e\cos\theta + ik_0\sin^2\theta\, F_m\right] - q_{nm}^0 G_{nm}^h\right\},$$

$$F_\phi^S = \frac{ik_0}{\varepsilon_0}\sum_{m=0}^{M}\cos(m+1)\phi(ik_0\sin\theta)^m$$
$$\times\sum_{n=1}^{N}\left\{p_{nm}^0 G_{nm}^e - q_{nm}^0\left[G_{nm}^h\cos\theta + ik_0\sin^2\theta F_m\right]\right\} + \frac{ik_0}{\varepsilon_0}\sin\theta\sum_{n=1}^{N} r_n^0 G_{n0}^e\frac{\varepsilon_0}{\varepsilon_f}.$$

The spectral functions $G_{nm}^{e,h}$, F_m here have the following form:

$$G_{nm}^{e,h} = ik_0\cos\theta\exp\{ik_0\cos\theta d\}A_{11}^{e,h}(k_0\sin\theta, w_n, d),$$

$$F_m = ik_0\cos\theta\exp\{ik_0\cos\theta d\}A_{31}(k_0\sin\theta, w_n, d).$$

Thus, the formulas for the diagrams contain no Sommerfeld integrals and are easy to compute when the unknown amplitudes of the discrete sources are found.

The *a posteriori* estimation of the error of the results obtained was made by computing the discrepancy of the boundary conditions on the surface of the particle in the norm l_2. In the results presented below, the relative error is no more than 2–3%.

3. NUMERICAL RESULTS

We consider a spherical particle whose diameter does not exceed the thickness of the layer. The results presented below correspond to the wavelength $\lambda = 0.488$ μm of a plane incident wave. Figures 2 through 4

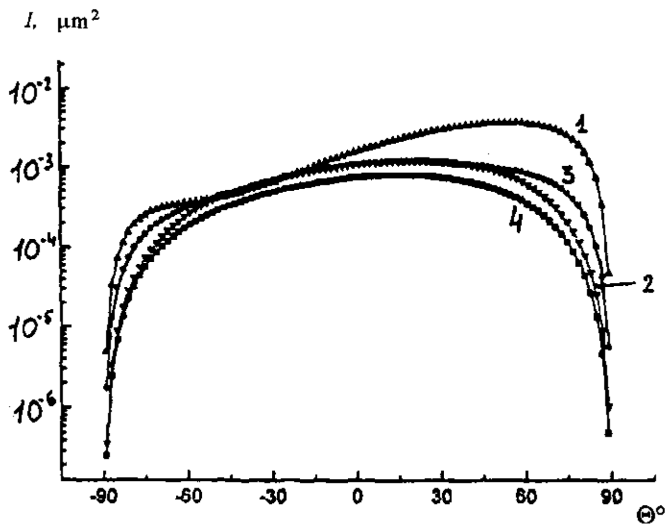

Fig. 2

Scattering of light by a silicon particle ($n = 4.5 - 0.4i$) with a diameter $D = 0.1$ μm in the absence of the layer for (*1*) *P*- and (*2*) *S*-polarization and in the presence of a layer $d = 0.2$ μm thick for (*3*) *P*- and (*4*) *S*-polarization.

show the intensity of the scattered light as a function of the observation angle θ in the plane formed by the half-planes $\phi = 0°$ and $\phi = 180°$ (the plane of incidence of the wave)

$$I^{P,S}(\theta,\phi) = \left|F_\theta^{P,S}(\theta,\phi)\right|^2 + \left|F_\phi^{P,S}(\theta,\phi)\right|^2 .$$

The layer is located on the flat surface of a silicon substrate ($n = 4.5 - 0.4i$). The most popular material — silica SiO_2 ($n = 1.44$) — has been considered as the layer.

The numerical experiments performed allow one to draw the following conclusions.

1. The presence of the layer causes a substantial change in the field scattered by the particle (Fig. 2).
2. The intensity of the scattered field in the case of *P*-polarization is higher than that in the case of *S*-polarization (Fig. 2).
3. Changing the position of the particle inside the layer causes a noticeable distortion of the diagram (Fig. 3).
4. The intensity of the scattered field basically reduces as the angle of incidence is increased (Fig. 4).

Using the algorithm suggested, we computed the ratio (in percentage terms) between the scattered energy and the energy flux through the surface of the particle for the cases of *P*- and *S*-polarization (Table 1).

This work was performed with a partial support of the Russian Foundation for Basic Research (Grant 97-01-01081)

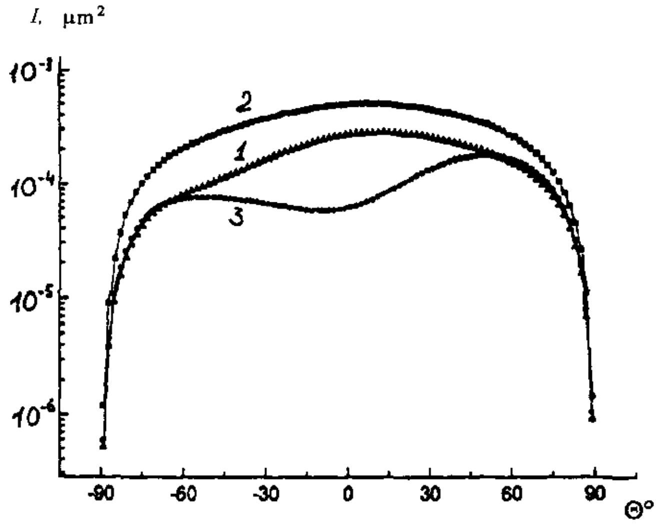

Fig. 3

Scattering of light by an iron particle ($n = 1.35 - 1.97i$) with a diameter $D = 0.1$ μm in a layer in the case of P-polarization, with the particle (*1*) lying on a substrate, (*2*) lifted to a height $h = 0.05$ μm and (*3*) 0.1 μm above the surface of the substrate.

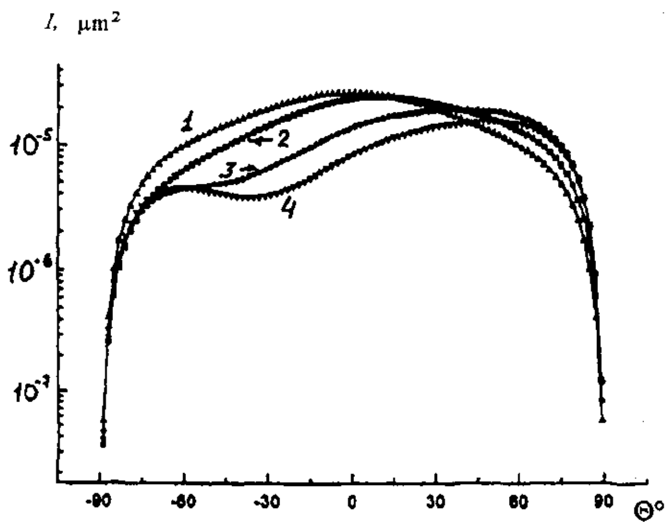

Fig. 4

Scattering of light by a silicon particle with a diameter $D = 0.06$ μm in a layer for various angles of incidence of a plane wave in the case of P-polarization: (*1*) normal incidence; (*2*) $\gamma = -15°$; (*3*) $\gamma = -45°$; (*4*) $\gamma = -65°$.

Table 1

Angle of incidence, γ, deg	Si		Fe		Al	
	P	S	P	S	P	S
0.0	24.7	24.7	23.4	23.4	20.2	20.2
5.0	24.6	24.6	23.4	23.4	20.2	20.2
10.0	24.5	24.6	23.2	23.4	20.1	20.2
15.0	24.2	24.5	23.0	23.5	20.1	20.2
20.0	23.8	24.5	22.7	23.5	20.0	20.2
25.0	23.3	24.3	22.2	23.6	19.8	20.3
30.0	22.6	24.2	21.6	23.6	19.7	20.3
35.0	21.8	24.1	20.9	23.7	19.4	20.3
40.0	21.0	23.9	20.1	23.7	19.2	20.4
45.0	20.0	23.7	19.1	23.8	18.8	20.5
50.0	19.2	23.5	18.1	23.9	18.5	20.5
55.0	18.1	23.4	17.1	23.9	18.0	20.5
60.0	17.3	23.2	16.1	24.0	17.6	20.6
65.0	16.6	23.1	15.1	24.0	17.2	20.6
70.0	15.7	23.0	14.3	24.1	16.8	20.6
75.0	15.3	22.9	13.7	24.1	16.5	20.7
80.0	14.8	22.9	13.2	24.1	16.2	20.7
85.0	14.6	22.7	12.9	24.2	16.1	20.7

Angle of incidence is reckoned from the normal to the substrate.

REFERENCES

1. Yu.A. Eremin, N.V. Orlov, and A.G. Sveshnikov, *Matem. Modelirovanie,* no. 10, p. 13, 1996.
2. M. Born, E. Volf, *Fundamentals of Optics* (in Russian), Moscow, 1970.
3. Yu.A. Eremin and A.G. Sveshnikov, *Method of Discrete Sources in Electromagnetic Diffraction Problems* (in Russian), Moscow, 1992.
4. V.I. Dmitriev, *Fields in Layered Media* (in Russian), Moscow, 1963.
5. Yu.A. Eremin, N.V. Orlov, and A.G. Sveshnikov, *Zh. Vychisl. Mat. Mat. Fiz.,* vol. 35, no. 6, p. 918, 1995.
6. L. Felsen and N Markuvits, *Emission and Scattering of Waves* (in Russian), vol. 1, Moscow, 1978.

8 December 1997

Department of Mathematics