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OPTICS AND SPECTROSCOPY

ACOUSTOOPTIC LIGHT MODULATION IN AN ISOTROPIC MEDIUM IN THE CASE OF STRONG ACOUSTOOPTIC INTERACTION

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The results are presented of theoretical studies into the diffraction of light by an amplitude-modulated acoustic wave in conditions of strong acoustooptic interaction. The diffraction efficiency is calculated for a Bragg-type modulator. It is demonstrated that where the modulation frequency is high enough, the symmetry of the acoustooptic coupling is disturbed so that the diffraction efficiency may be as high as 94% even for a focused light beam.

INTRODUCTION

The operation of acoustooptic modulators is based on the fact that acoustic waves can help to control various light wave parameters, such as amplitude, phase, frequency, and polarization [1]. According to which light wave parameter is to be controlled, various types of acoustooptic modulators are possible: amplitude, phase, etc. It is but some types out of the great variety of such modulators that have been studied to date and are finding application. The most important ones include a wide-band amplitude modulator that uses a traveling ultrasonic wave. Such a modulator depends for its operation on the relationship between the intensity of the diffracted light and the amplitude of the acoustic wave.

The rigorous calculation of the modulator reduces actually to the solution of the problem of diffraction of a light beam by an amplitude-modulated acoustic wave. This problem is in itself not a new one: it was considered by many authors [2–7], but all calculations were previously performed in the weak acoustooptic interaction approximation, where the diffraction efficiency was no more than 10%. In this work, we consider light modulation under strong acoustooptic interaction conditions. We have studied the relationship between the diffraction efficiency and the modulator cell parameters for a Bragg-type modulator and analyzed nonlinear distortions developing in the course of modulation.

1. BASIC RELATIONSHIPS

When the diffraction efficiency is low, the acoustooptic interaction is linear. Therefore, in solving the problem of acoustooptic modulation, use can be made of the syperposition principle. The light and acoustic beams are in that case expanded into a spectrum in plane monochromatic waves, the interactions of all spectral components are taken into consideration, and all partial diffracted waves at the exit from the acoustooptic cell are summed up [1, 6, 8]. The strong acoustooptic interaction regime is more interesting from the applied standpoint, but its analysis is very difficult to perform because in this case the diffraction problem becomes nonlinear in sound. The reason is that light beam photons in a strong acoustic field may experience repeated scattering by phonons before they leave the acoustooptic cell. As a result, the diffraction spectrum becomes substantially enriched. Even in the case of Bragg's diffraction, the zeroth and

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first orders may contain many diffraction maxima, whose number increases as the acoustic power increases. All the maxima draw power from one and the same incident light beam, and therefore, there takes place the competition between the diffracted modes, as a result of which the intensity of light in each particular maximum depends on that in the other maxima.

Bearing in mind that the diffraction problem is always linear in light, we will first consider the diffraction of a plane light wave. Assume that in an isotropic medium bounded by the planes $x = 0$ and $x = l$ the amplitude-modulated acoustic wave propagates along the z-axis

$$a(z,t) = a_0[1 + m\cos(\Omega_m t - K_m z)]\exp[j(\Omega_0 t - K_0 z)], \tag{1}$$

where a_0 is the amplitude, m is the depth of modulation, $\Omega_m = 2\pi f_m$ is the modulation frequency, $\Omega_0 = 2\pi f_0$ is the carrier frequency, $K_m = \Omega_m/v$ and $K_0 = \Omega_0/v$ are the wave numbers, and v is the sound velocity. The acoustic field spectrum contains three components: the central component with the amplitude a_0 and frequency Ω_0 and two side components with the amplitudes $a_0 m/2$ and frequencies $\Omega_0 \pm \Omega_m$. While passing through the acoustic field, light is diffracted by all the three components. In this case, at the first stage of scattering, there originate first-order diffracted waves with the frequencies $\omega + \Omega_0$ and $\omega + \Omega_0 \pm \Omega_m$, where ω is the frequency of incident light. At the second Bragg's scattering stage, there develop zero-order waves with frequencies $\omega \pm 2\Omega_m$, $\omega \pm \Omega_m$, and ω. The third scattering stage will add the frequencies $\omega + \Omega_0 \pm 3\Omega_m$ to the first-order spectrum and so on. It is easy to see that with the character of scattering being what it is, an optical spectrum consisting of the equidistant frequencies $\omega \pm i\Omega_m$ in the zeroth order and $\omega + \Omega_0 \pm i\Omega_m$ in first order, where i are integers, will form at the exit from the acoustooptic cell. It can be stated that any diffraction level will only be coupled to three other levels without regard to a particular scattering stage at which it is formed. For example, light will be diffracted to the first-order level $(1, i)$ from the zero-order levels $(0, i)$, $(0, i + 1)$, and $(0, i - 1)$ only, and to the zero-order level $(0, i)$, from the levels $(1, i)$, $(1, i + 1)$, and $(1, i - 1)$. Consequently, the system of equations describing the Bragg diffraction of light by amplitude-modulated sound (1) will have the form

$$\begin{cases} 2\dfrac{dC_{0i}}{dx} = q\Big[C_{1i}\exp(j\eta_{ii}x) + \dfrac{m}{2}C_{1,i+1}\exp(j\eta_{i,i+1}x) + \dfrac{m}{2}C_{1,i-1}\exp(j\eta_{i,i-1}x)\Big], & (2) \\ 2\dfrac{dC_{1i}}{dx} = -q\Big[C_{0i}\exp(-j\eta_{ii}x) + \dfrac{m}{2}C_{0,i-1}\exp(-j\eta_{i-1,i}x) + \dfrac{m}{2}C_{0,i+1}\exp(-j\eta_{i+1,i}x)\Big], & (3) \end{cases}$$

where C_{0i} and C_{1i} are the relative amplitudes of the zero- and first-order spectral components, and q is the coupling coefficient proportional to the amplitude a_0 of the acoustic wave [1]. The parameters η_{ik} define the detuning of the ith zero-order level and kth first-order level. In the case of isotropic diffraction, one can obtain the following expression for the detuning parameters

$$\eta_{ik} = \frac{2\pi f_0}{v}[1 + (k - i)F]\left\{\theta + \frac{\lambda f_0}{2nv}[1 + (k + i)F]\right\},$$

where θ is the angle of incidence of the light wave on the acoustooptic cell, λ is the wavelength of light, n is the refractive index, and $F = f_m/f_0$ is the normalized modulation frequency. Equations (2) and (3) are solved subject to the natural boundary conditions $C_{0i}(x = 0) = \delta_{0i}$, $C_{1i}(x = 0) = 0$. The number of equations that must be taken into consideration in calculations depends on the values of the Raman–Nath parameter $V = ql$ and the wave parameter $Q = \lambda l f_0^2/nv^2$ [1].

The quick-acting response of the acoustooptic modulator is governed by the time it takes for the ultrasonic wave to cross the light beam [1]. Consequently, to obtain a fast-acting modulator, it is necessary to reduce as much as possible the size of the acoustooptic interaction region, i. e., to use a highly focused light beam. And to provide for the maximum modulation band, the beam waist must lie at the center of the interaction region in the plane $x = l/2$.

For definiteness, we shall assume that a Gaussian light beam is incident on the acoustooptic cell at an angle θ_0. In that case, the light spectrum at the entrance to the cell will have the form

$$U(\theta) = \frac{\sqrt{\pi}}{2}u_0 d\exp\left[-\frac{\pi^2 d^2 n^2}{4\lambda^2}(\theta - \theta_0)^2\right]\exp\left[-j\frac{\pi n l}{2\lambda}\theta^2\right], \tag{4}$$

where u_0 is the amplitude and d is the beam waist diameter. Where subject to diffraction is a restricted light beam, the diffraction levels broaden. If the beam width d is smaller than the spatial modulation period $\Lambda_m = 2\pi/K_m$, then the diffraction maxima in every order overlap. As a result, there develops a beating at the difference frequencies $i\Omega_m$, and it is exactly this beating that leads to the intensity modulation of the diffracted radiation.

Let us define the integral diffraction efficiency ξ as the ratio between the power of the first-order diffracted radiation and that of the incident light. Then we write

$$\xi(t) = \frac{\int\limits_{-\infty}^{\infty} \left|\sum\limits_i U(\theta^{(i)})C_{1i}(\theta^{(i)})\exp[j\Omega_0 t(1+iF)]\right|^2 d\theta_d}{\int\limits_{-\infty}^{\infty} |U(\theta)|^2 d\theta}, \tag{5}$$

where θ_d are the angles at the exit from the acoustooptic cell. The complex amplitudes $C_{1i}(\theta)$ are found by solving system of equations (1)–(3) at $x = l$. In expression (5), account should additionally be taken of the shift of the spatial light spectrum in the course of diffraction

$$\theta^{(i)} = \theta_d - \frac{\lambda}{\Lambda_0 n}(1+iF), \tag{6}$$

where $\Lambda_0 = v/f_0$ is the length of the ultrasonic wave. Substituting (4) and (6) into (5), we get

$$\begin{aligned}\xi(t) = \frac{D}{2}\sqrt{\frac{\pi}{2}}\int\limits_{-\infty}^{\infty}\Bigg|\sum_i \exp\left[-\frac{\pi^2 D^2}{16}(\Theta_d - \Theta_0 - 2 - 2iF)^2\right] \\ \times \exp\left[-j\frac{\pi Q}{8}(\Theta_d - 2 - 2iF)^2\right] C_{1i}(\Theta^{(i)})\exp[j\Omega_0 t(1+iF)]\Bigg|^2 d\Theta_d.\end{aligned} \tag{7}$$

Here the following dimensionless parameters are introduced to facilitate numerical computations

$$D = \frac{d}{\Lambda_0}, \quad \Theta_0 = \frac{\theta_0}{\left|\theta_B^{(\mathrm{is})}\right|}, \quad \Theta_d = \frac{\theta_d}{\left|\theta_B^{(\mathrm{is})}\right|},$$

where $\theta_B^{(\mathrm{is})} = -\lambda/2n\Lambda_0$ is the Bragg angle in the case of isotropic diffraction. Expression (7) enables one to analyze in detail the operation of the acoustooptic modulator.

2. COMPUTATION RESULTS

The Bragg diffraction regime is usually defined by the condition $Q > 2$, for if this condition is satisfied, more than 95% of the incident light can be scattered to the first order levels [1]. The results presented below were obtained at $Q = 8$, and so the neglect in calculations of higher orders (2nd, 3rd, etc.) is quite justifiable. The most interesting case has been considered, where the angle of incidence θ_0 is equal to the Bragg angle at the frequency F_0: $\theta_0 = -\lambda f_0/2nv$, $\Theta_0 = -1$. In calculating the diffraction spectrum, use was made of up to 17 equations of system (2)–(3), which provided for a high calculation accuracy (error is no more than 1%).

Figure 1 presents the integral diffraction efficiency ξ as a function of the Raman–Nath parameter V in the absence of modulation ($m = 0$). As distinct from the Bragg scattering of a plane light wave [1], the diffraction efficiency here is no longer equal to unity, and the higher the focusing of the light beam, the lower is the maximum value of ξ_m. The reason is the violation of the phase-match condition in acoustooptic interaction. In a divergent light beam, there exist plane-wave components that differ in the direction of the wave normal within the limits of the divergence angle φ_L, and the phase-match condition can only be fulfilled for one of them. For the rest of the components, there will exist a divergence η, which reduces the

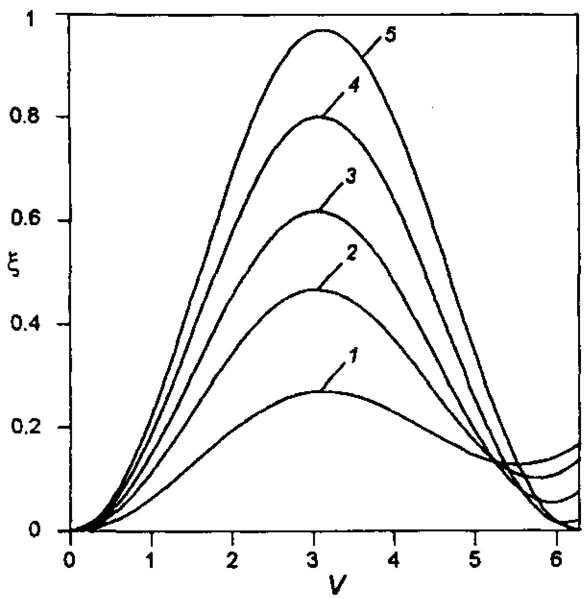

Fig. 1

Integral diffraction efficiency as a function of the Raman–Nath parameter in the absence of modulation: *1* — $D = 2$; *2* — $D = 4$; *3* — $D = 6$; *4* — $D = 10$; *5* — $D = 30$.

scattering efficiency. The greater the angle φ_L, the stronger this effect manifests itself. Important in this case is the ratio between the divergence angles of the light beam (φ_L) and the acoustic (φ_S) beam, i. e., the Gordon parameter $G = \varphi_L/\varphi_S$ rather than the absolute values of the angle φ_L [1]. Here $G = 4Q/\pi D$. The plane-wave approximation occurs, when $G \to 0$. And at $G > 1$ the diffraction efficiency drops as $\xi \sim G^{-1}$.

The curves of Fig. 1 can be treated as the modulation characteristics of the acoustooptic modulator calculated for the given light beam width D. The position of the operating point in the characteristic curve is determined by the parameter V. It is clear from Fig. 1 that the least distortion in analog light modulation should be expected, when $V \cong \pi/2$. This case is illustrated by Fig. 2 that presents time responses of the modulator to a harmonic stimulus at various depths of the acoustic wave modulation amplitude. The computations were made for a highly focused light beam ($D = 2$) and two modulation frequencies — a low frequency ($F = 1/1000$, dashed curves a) and a high frequency ($F = 1/15$, solid curves b). Lines 1 and 2 indicate the diffraction efficiency levels corresponding to $V = \pi$ and $V = \pi/2$.

The case $F = 1/1000$ can be regarded as a quasistatic operating regime of the modulator, for the spatial modulation period Λ_m is in this case 500 times the width of the light beam. The shape of the curves a relating to the given case is quite explicable. At a small depth of modulation the intensity of the diffracted light varies harmonically (curve *3a*). As m is increased, there occur the distortions caused by the nonlinearity of the modulation characteristic (curve *4a*), and at $m = 1$ the shape of light modulation curve approaches a meander (curve *5a*). However, no matter what the values of V and m, the diffraction efficiency does not exceed the maximum value of ξ_m in the modulation characteristic (i. e., the level of line *1*).

A different situation obtains in the case of high modulation frequency. Low modulation depth values have the same effect as in the case of a quasistatic regime (curve *3b*). But at high m values, the function $\xi(t)$ has a substantially different form (curve *5b*). The main specific feature is that the diffraction efficiency in a portion of the modulation period can substantially exceed ξ_m. It especially manifests itself in Fig. 3, which illustrates how $\xi(t)$ varies as the function of the operating point position at $m = 1$. If the operating point is chosen to lie in the maximum of the modulation characteristic, then, as might be expected, the shape of the light modulation curve in the quasistatic case is far from sinusoidal: strong harmonics are present in the output signal spectrum (curve *4a*). In the case of high modulation frequency, distortions are less, but the main peak diffraction efficiency $\xi_{\max}$ reaches 0.6, which is 2.3 times the value of ξ_m (curve *4b*). One may also note that this value is reached before the instant $f_m t = 1$, at which the acoustic beam power in

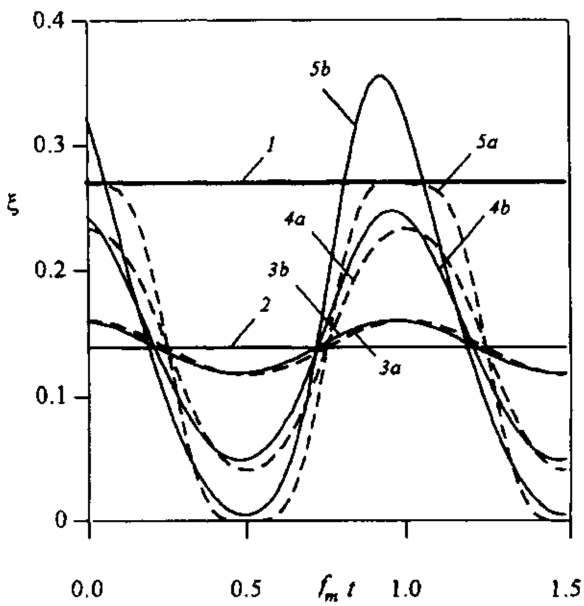

Fig. 2

Integral diffraction efficiency as a function of time for (a) $F = 1/1000$ and (b) $F = 1/15$: *1* — $m = 0$, $V = \pi$; *2* — $m = 0$, $V = \pi/2$; *3* — $m = 0.1$, $V = \pi/2$; *4* — $m = 0.5$, $V = \pi/2$; *5* — $m = 1$, $V = \pi/2$.

the cross section of the light beam reaches its maximum. A similar effect was discovered by the authors of [9] to occur in the course of pulsed light modulation under strong acoustooptic interaction conditions. The reason is the disturbance of the acoustooptic coupling symmetry between the first- and the zero-order wave in the nonstationary acoustic field. If light is scattered into the +1st order, then as the ultrasound power is raised, light is diffracted into a region with a lower acoustic power density. The reverse pumping of light to the zeroth order is impeded here. As a result, the diffracted light intensity rises faster and for some time may materially exceed the stationary value. But if light is scattered into the −1st order, then a similar effect must be observed in that portion of the modulation period, where the ultrasound power decreases. Curve *4c* relating to the −1st order supports this conclusion. It is seen that this curve is the mirror image of curve *4b*.

Curves *1* through *4* in Fig. 4 present the function $\xi_{\max}(F)$ for various values of V and m. At a small modulation depth (curve *1*) the frequency characteristic is uniform throughout the operating range of the modulator up to frequencies, at which the spatial modulation period Λ_m becomes comparable with the light beam width d. The dip of the characteristic curve is due to the averaging of modulation over the beam aperture. At $d \doteq \Lambda_m$, the quantity $\xi_{\max}$ decreases down to the stationary value ξ_m, and the light modulation depth M (curve *5*) drops to zero. Increasing m makes the frequency characteristic nonuniform in the high-frequency region and enhances the high-frequency components of the optical spectrum (curves *2* through *4*).

CONCLUSION

The calculation of light diffraction by an amplitude-modulated acoustic wave has shown that under strong acoustooptic interaction conditions the frequency characteristic of the modulator becomes nonuniform. At high frequencies near the modulation band edge the diffraction efficiency rises and may exceed several times the stationary level. The higher the quick responce of the modulator, the stronger it manifests itself. This feature of frequency characteristic is due to the disturbance of symmetry of the acoustooptic coupling between the zeroth and the first diffraction order in the nonstationary acoustic field.

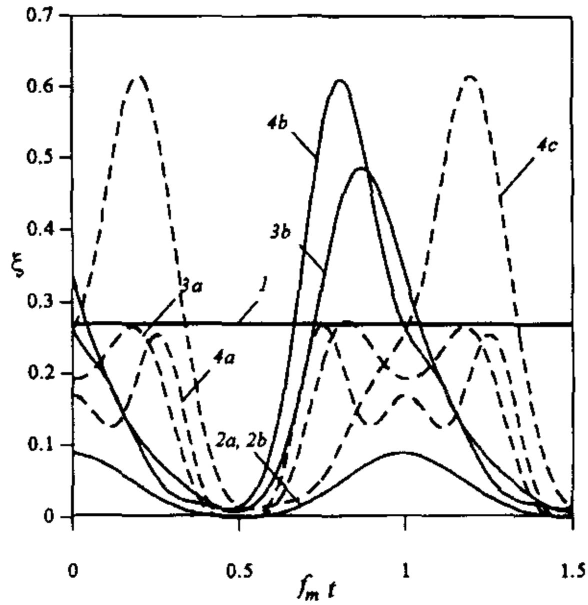

Fig. 3

Time response of the modulator at (*a*) $F = 1/1000$ and (*b*) and (*c*) $F = 1/15$: *1* — $V = \pi$, $m = 0$; *2* — $V = 0.2\pi$, $m = 1$; *3* — $V = 0.7\pi$, $m = 1$; *4* — $V = \pi$, $m = 1$.

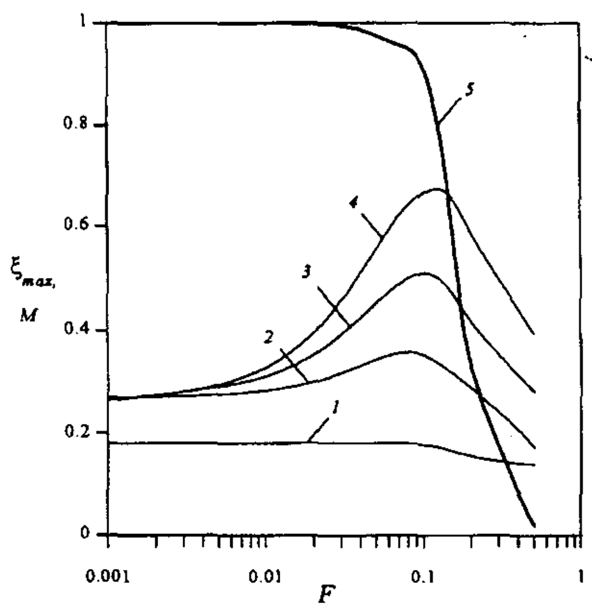

Fig. 4

$\xi_{\max}$ (curves *1* through *4*) and M (curve *5*) as a function of the normalized modulation frequency: *1* —$V = \pi/2$, $m = 0.2$, *2* — $V = \pi/2$, $m = 1$, *3* — $V = 0.7\pi$, $m = 1$, *4* — $V = \pi$, $m = 1$.

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