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ASTRONOMY

ON THE EXISTENCE OF KUIPER BELT OBJECTS BOUND WITH NEPTUNE BY ORBITAL RESONANCES

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Based on a restricted elliptic three-body problem with due account of secular perturbations caused by Uranus, Saturn, and Jupiter, we have evaluated the orbits and regions of stable existence for Transneptune objects that are in orbital commensurability with Neptune.

INTRODUCTION

The existence of the Transneptune belt was predicted by C. Edgevers (1949) and J. Kuiper (1951). The Kuiper belt lies at a distance of about 40–60 a.u. from the Sun [1, 2]. The total mass of bodies in this belt is comparable with that of the Earth. The first object (1992 QB1) was discovered in 1992 at a distance of 42 a.u. from the Sun. A few more Transneptune objects with major orbital semiaxes $32.3 \le a \le 43.8$ a.u., excentricities $e \le 0.07$, and orbit inclinations $i \le 8$ deg were found in 1993. Their diameters range from 100 to 280 km. In 1997, there were already over 30 known Kuiper belt objects with major orbital semiaxes from 35 to 48 a.u. The orbit excentricities of these bodies turned out to be low, and their diameters were 100–300 km. According to some estimates, the largest Kuiper belt objects may be up to 1000 km in diameter [3, 4].

Under certain assumptions it can be assumed that, owing to gravity influence of the largest bodies in the Transneptune belt and the gravity influence of giant planets, individual Kuiper belt bodies could migrate over the time of the Solar system existence from the central and outer parts of this belt into its inner part. And the majority of bodies with excentricities $e \geq 0.1$ could migrate from the inner part of the Transneptune belt towards Neptune's orbit and further towards the Sun. Specific estimates of the mass of a substance migrating from the Transneptune belt depend on the distribution of bodies in the belt by masses and orbit elements, which is currently unknown [5, 6].

Transneptune objects may exist for a sufficiently long time if their orbit elements correspond to regions of stable motion (orbital stability regions). The existence of such "stability zones" crucially depends on the existence of orbital resonances with giant planets, first of all with Neptune.

Libration stable objects may remain in the Kuiper belt [7, 8]. Their stability is due to the absence of "convergences", i. e., the existence of a nonzero lower bound for the distance between the perturbing body (Neptune) and the "libration object".

The dynamical evolution of Transneptune objects can be described, within the first approximation, in the framework of the external variant of the restricted elliptic three-body problem, with low-order orbital resonances taken into account [9, 10]. For the two-frequency dynamical system considered in this case, a resonance relationship holds between its natural (unperturbed) frequency and the frequency of the external perturbing force. This gives rise to a certain relationship between the integrals of motion, which, in turn, leads to system degeneracy [11, 12].

Even insignificant variations (due to perturbations) in resonance frequencies near the system separatrix, where the libration period tends to infinity, may cause significant changes in the perturbation phase, which causes local instability and high sensitivity of the system to the initial conditions. However, a significant instability may only develop in a dynamical system with a small parameter μ, when influenced significantly by non-gravity interaction effects (for the Sun–Neptune–Transneptune object system, we have $\mu_{\Psi} = 5.17 \times 10^{-5}$, the Neptune mass in terms of the Sun mass), on times much longer than $1/\mu$ ($t_{cr} \leq 1/\mu^2$). For $\mu = \mu_{\Psi}$, the value of t_{cr} is about 10 billion years. Consequently, interpretation of the dynamical evolution of Transneptune bodies is correct in the framework of the "partial determinism" concept on the basis of the gravitational resonance three-body problem, where the use of rigorously justified asymptotic methods allows one to construct an analytical solution that interprets the orbital evolution of specified gravitating celestial-mechanics bodies [13, 14].

EVOLUTIONARY ORBITS

Let us consider a model, in which a Transneptune object (P) is a material point, passively gravitating in the Sun gravity field (P_0). A perturbing body (Neptune, P') is assumed to be moving along an elliptic orbit with constant excentricity e', whose value is selected, according to [15, 16], from the range $(0.0055, 0.015)$. It is also assumed that P and P' are related by orbital commensurability, so that at the initial moment t_0 the inequality

$$|(k+l)n - kn'| \leq \sqrt{\mu_{\Psi}}\,\mathbf{O}[1],$$

holds, in which n and n' are the average motions of P and P', $k, l \in N$ are respectively the multiplicity and order of the two-frequency resonance (Fig. 1*a*). Since the amplitude of a resonance effect decreases with increasing resonance order [17, 18], we henceforth limit ourselves to the cases of low-order resonances ($l \leq 3$).

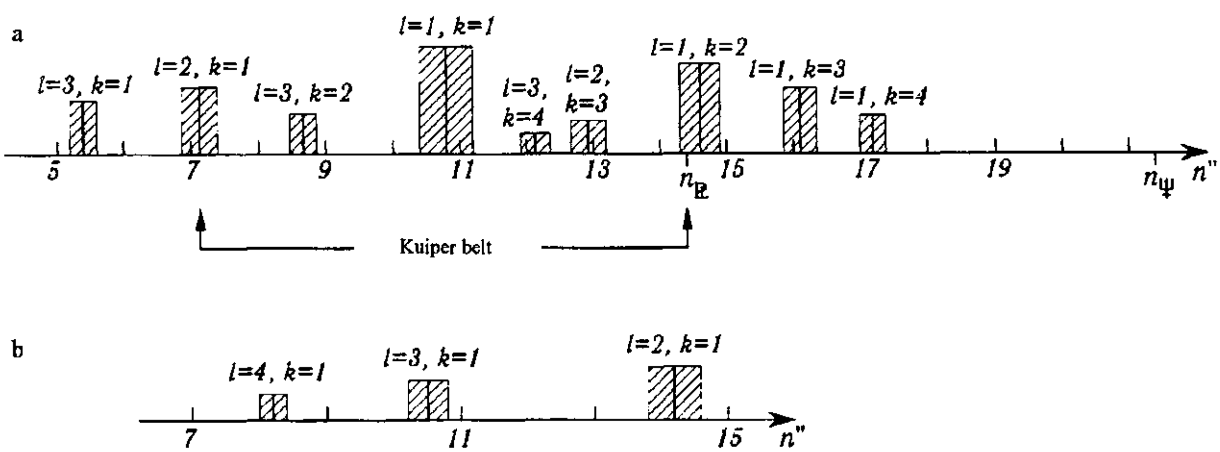

Fig. 1

Arrangement of resonance zones of various order (l) and multiplicity (k), corresponding to "external commensurability" with Neptune (the Transneptune belt) (a) and Uranus (b). Effective resonance zones are hatched.

It shuold be noted that the total span of the resonance effect (Δ_{res}) for $l = \overline{1,3}$, $k = \overline{1,4}$ covers actually the entire Kuiper belt (Δ_K): $\Delta_{res}/\Delta_K = \mathbf{O}$ [1] (see Fig. 1). Therefore the isolation of only resonance Transneptune objects is sufficiently justified.

We consider the motions that occur in one plane, i. e., we neglect the inclination of the instantaneous plane of the orbit of P to the orbital plane of P', and take into account the secular (nonresonance) perturbations caused by the gravitational influence of Uranus, Saturn, and Jupiter, assuming their orbits to be circular.

Let us take the Gauss constant and the major semiaxis (a') of Neptune's orbit to be unity. (We use common notation for the orbital elements of P, priming those that refer to P'.)

An analytical solution that describes the evolution of all orbital elements of a passively gravitating body P (a Transneptune object) in the framework of the restricted elliptic variant of the three-body problem with regard to external perturbations from "N-bodies" was obtained in [13, 19] for the case of Lindblad's resonances. It was found that in terms of the variables x, y related to the orbital elements of P by the expressions

$$x = \sqrt{2\xi}\cos\eta, \quad y = \sqrt{2\xi}\sin\eta, \quad \xi = \frac{1}{2}\frac{(p-p_0)^2+q^2}{E^2},$$
$$\eta = (k+1)(M+\omega) - kM' - \arctan\frac{q}{p-p_0}, \tag{1}$$

where

$$p = e\cos\omega, \quad q = e\sin\omega, \quad E = \left[\frac{k}{k+1}\sqrt{1+\mu_\Psi}\right]^{1/6}, \quad p_0 = e'\frac{\Phi_{1k}}{\Phi_{2k}},$$
$$\Phi_{1k} = \frac{1}{2}(\alpha D + 2(k+1))L_{1/2}^{(k+1)}(\alpha), \quad \Phi_{2k} = \frac{1}{2}(\alpha D + 2k+1)L_{1/2}^{(k)}(\alpha) - \frac{1}{2}\alpha^{-2}\delta_{k,1},$$

the solution of the original problem is reduced to the integration of an autonomous canonical system of equations with one degree of freedom

$$\frac{dx}{d\tau} = F'_y, \quad \frac{dy}{d\tau} = -F'_x, \tag{2}$$

whose Hamiltonian is

$$F = \left(x^2+y^2\right)^2 + A(x^2+y^2) + Bx. \tag{3}$$

Here $\tau = \tau_0 t$, $\tau_0 = (3/8)[(k+1)/k]^2E^8$, $B = \mu_\Psi(E\alpha/\tau_0)\Phi_{2k}$, $\alpha = E^4$, $D = d/d\alpha$ is the differential operator, $L_{1/2}^{(k)}$ is the Laplace coefficient, $\delta_{k,1}$ is the Kronecker delta, $A = [4/(k+1)]\{\sqrt{\gamma} - E^{-2} + C_0\}$ is the integral of the form

$$\sqrt{\gamma} + C_0 = \sqrt{a} - \frac{k+1}{2}\left[\left(u - \frac{p_0}{E}\right)^2 + v^2\right], \tag{4}$$

$$C_0 = \frac{1}{8\tau_0}\sum_{i=1}^{3}\frac{m_i}{a_i}\beta_i[L_{1/2}^{(0)}(\beta_i) + \beta_i DL_{1/2}^{(0)}(\beta_i)], \quad \beta_i = \frac{a_i}{\gamma},$$

where $u = \sqrt{r}\cos\omega$, $v = -\sqrt{r}\sin\omega$, $r = 2\sqrt{a}(1-\sqrt{1-e^2})$, m_i, and a_i $(i = \overline{1,3})$ are respectively the masses and orbit major semiaxes of Uranus, Saturn, and Jupiter (in the selected system of units)

$$m_1 = 4.36\times10^{-5}, \; m_2 = 2.86\times10^{-4}, \; m_3 = 9.55\times10^{-4},$$

$$a_1 = 0.639, \quad a_2 = 0.316, \quad a_3 = 0.172.$$

Integration of (2) leads to

$$x = \frac{1}{2b_1}[\wp(\tau+w) + \wp(\tau-w) - b_2],$$
$$y = -\frac{i}{2b_1}[\wp(\tau+w) - \wp(\tau-w)],$$

where $i^2 = -1$, $b_1 = 2B$, $b_2 = (2/3)(4C - A^2)$, $C + F = 0$, and $\wp$ is the meromorphic Weierstrass function with real invariants

$$g_2 = 3b_2^2 - 4b_1b_3, \quad g_3 = 2b_1b_2b_3 - b_1^2b_4 - b_2^3,$$

and $b_3 = -AB$, $b_4 = -B^2$, and $w = iw^*$ is a complex constant defined by

$$\wp(2w^*; g_2, -g_3) = -\frac{b_2}{2}, \quad \wp'(2w^*; g_2, -g_3) = -2B^2.$$

The orbital elements a, e, w of the Transneptune object P, according to [14], are expressed in terms of variables (1) as follows:

$$a=\left\{\sqrt{\gamma}+C_0+\frac{k+1}{2E^2}\sum_{j=1}^{2}Z_j^2\right\}^2,\quad e=\left[\sum_{j=1}^{2}(Z_j+\Pi_j)^2\right]^{1/2},\quad \omega=\omega_0+\Theta-\arctan\left[\frac{Z_2+\Pi_2}{Z_1+\Pi_1}\right],$$

$$\Theta=4\pi n+v't+4\arctan\left[\tan\frac{\pi w^*}{2\overline{\omega}}\cot\frac{\pi\tau_0 t}{2\overline{\omega}}\right]+8\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\frac{\overline{q}^{2lm}}{m}\sinh\frac{\pi m w^*}{\overline{\omega}}\sin\frac{\pi m\tau_0 t}{\overline{\omega}}.$$

Here Z_j and Π_j ($j = 1, 2$) are the components of vectors $\mathbf{Z}$ and $\mathbf{\Pi}$

$$\{\mathbf{Z}\}=(xE,yE),\quad \{\mathbf{\Pi}\}=(p_0\cos\Theta,\,p_0\sin\Theta);\qquad \omega_0=\omega(t=0),$$

$$v'=-2A+4\zeta(w^*;\,g_2,\,-g_3)-\frac{[\wp(w)-\wp(2w)]^2}{B^2}+4\frac{w^*}{\overline{\omega}}\zeta(\overline{\omega})$$

$$+2C_0\left\{4\tau_0(2+C_0)+\sum_{i=1}^{3}\frac{m_i}{a_i}\beta_i^2[2DL_{1/2}^{(0)}(\beta_i)+\beta_iD^2L_{1/2}^{(0)}(\beta_i)]\right\},$$

ζ is the Weierstrass zeta function, $\overline{\omega}$ is the real period of the $\wp$-function, $\overline{q}=q_1=\exp[-\pi\tilde{\omega}/\overline{\omega}]$, when the discriminant Γ of the characteristic equation $4\wp^3-g_2\wp-g_3=0$ is greater than or equal to zero, and $\overline{q}=i\sqrt{q_1}$ ($i^2=-1$), when $\Gamma<0$. The main periods of the $\wp$-function are: $2\omega_1=2\overline{\omega}$, $2\omega_3=i\,2\tilde{\omega}$ at $\Gamma\geq 0$, and $2\omega_1=\overline{\omega}-i\tilde{\omega}$, $2\omega_3=\overline{\omega}+i\tilde{\omega}$ at $\Gamma<0$ (i.e. $\overline{\omega}=\omega_1+\omega_3$ at $\Gamma<0$).

For the variables Z_j ($j=1,2$), as follows from (2) and (3), for $\Re=8A^3+27B^2<0$, i.e., $\gamma<\gamma_1$, where

$$\gamma_1=\left\{E^{-2}-C_0-\frac{3}{8}(k+1)B^{2/3}\right\}^2,$$

there are three stationary solutions: $Z_1^{(1)}>Z_1^{(2)}>0$, $Z_1^{(3)}<0$ ($Z_2^{(i)}\equiv 0$, $i=\overline{1,3}$). The stationary points $(Z_1^{(2)},0)$ and $(Z_1^{(3)},0)$ are Lyapunov stable ("stable centers"), while $(Z_1^{(1)},0)$ is unstable (a "saddle"). For $\Re=0$ ($\gamma=\gamma_1$) $Z_1^{(1,2)}=-Z_1^{(3)}/2=(1/2)EB^{2/3}$, and the stationary point $(Z_1^{(3)},0)$ is stable (of the "center" type), while the points $(Z_1^{(1,2)},0)$ are unstable. In the case $\Re<0$ ($\gamma>\gamma_1$) the stationary points $(Z_1^{(1,2)},0)$ will be complex conjugate roots, and the stationary point $(Z_1^{(3)},0)$ is a "stable center".

The stationary solutions $(Z_1^{(i)},0)$, $i=\overline{1,3}$, in terms of the variables p, q correspond to the solution families determined by the equations

$$(p-p_0)^2+q^2=\left[Z_1^{(i)}\right]^2,\quad i=\overline{1,3}.$$

For $\Re\geq 0$, the stationary solutions to canonic system (2) on the plane ($p=e\cos\omega$, $q=e\sin w$) are the concentric circles centered at p_0 (for $e'=0.015$) with radii $|Z_1^{(i)}|$, $i=\overline{1,3}$ (Fig. 2). Circle *1* describes the domain (the set of points) of the unstable stationary solution, while circles *2* and *3* are the domains of stable solutions (for $\Re=0$, circles *1* and *2* correspond to unstable stationary solutions; for $\gamma>\gamma_1$ the circle of radius $|Z_1^{(3)}|$ describes stable stationary solutions).

From integral (4), taking (1) into account, we have for the stationary values of the major semiaxis and excentricity of the orbit of a Transneptune object P

$$a_{i\,\text{stat}}=\left\{\sqrt{\gamma}+C_0+\frac{k+1}{2}\left(\frac{Z_1^{(i)}}{E}\right)^2\right\}^2,\quad e_{i\,\text{stat}}=\left\{\left[Z_1^{(1)}\right]^2+2|Z_1^{(1)}|p_0\cos(S)+p_0^2\right\}^{1/2},\quad i=\overline{1,3},$$

where $\cos(S)=\dfrac{p-p_0}{\sqrt{q^2+(p-p_0)^2}}$.

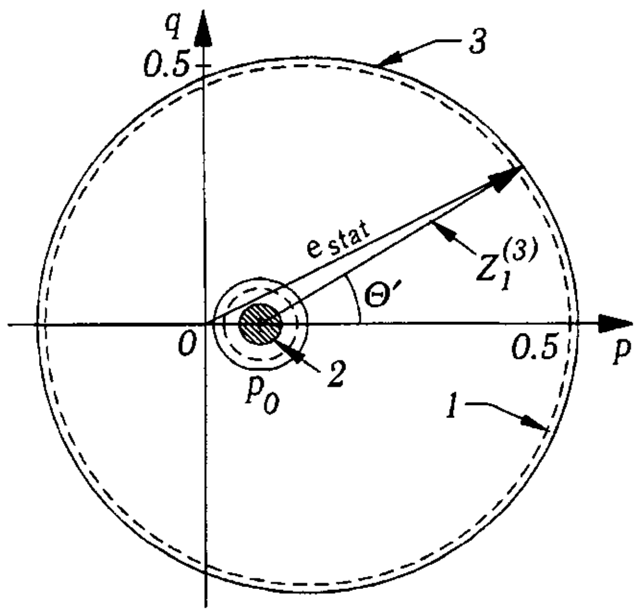

Fig. 2

Stationary solutions on the plane $p = e\cos\omega$, $q = e\sin\omega$ for $\Re > 0$. Circle numbers correspond to superscripts of stationary solutions $(Z_1^{(i)}, 0)$, $i = \overline{1,3}$. For $i = 1, 2$ $\Theta = \Theta'$, for $i = 3$ $\Theta = \pi + \Theta'$, $\Theta' = \arctan[q/(p - p_0)]$. The domain of solutions $Z_1^{(2)}$ is hatched, the domains of $Z_1^{(1,3)}$ are enclosed between the circles with identical line styles.

Consequently, $39.95 \le a_{2\text{stat}} \le 47.20$ a.u. The stationary values of excentricities are 2π-periodic with respect to the variable S, which is a linear function of time. The pericenter argument for the stationary orbit of P, as follows from Fig. 2, may (depending on the initial conditions) execute both a libration and a circulation motions. In the circulation case (solutions $Z_1^{(1,3)}$) the excentricity variation range is $\Delta e_{\text{stat}} = 2p_0$, therefore, according to the table given below, the maximum range of variation Δe_{stat} is realized for the resonance multiplicity $k = 1$.

k	1	2	3	4	5
$p_0 \cdot 10^2$	4.679	1.308	1.362	1.392	1.410

Using (1), it is easy to derive the following expression for the pericenter argument variation rate ($\dot{\omega}$) for the stationary solution $(Z_1^{(i)}, 0)$, $i = \overline{1,3}$

$$\left(\frac{\dot{\omega}}{\omega}\right)_{\text{stat}} = \left(\frac{Z_1^{(i)}}{e_{i\,\text{stat}}}\right)^2 \left(1 + \frac{p_0}{Z_1^{(i)}}\cos(S)\right),$$

which shows that with increasing Neptune orbit excentricity e' the value of $\dot{\omega}_{\max}$ grows, and that, other conditions being equal, makes the "convergences" more probable. On the other hand, the maximum value of $\dot{\omega}_{\text{stat}}$ is reached when the excentricity e is minimal, and this fact decreases the probability of "convergences". When the orbit of P approaches exact commensurability, the pericenter argument is "stabilized" ($\dot{\omega}_{\text{stat}} \to 0$).

A classification of the phase trajectories of the system in question is given in [14].

REGIONS OF TRANSNEPTUNE LIBRATION OBJECTS EXISTENCE

Using (2), (3) and acting in the same way as in [20], it is possible to determine the stability/instability domains for the orbital motions of Transneptune objects in the e–a diagram. The corresponding instability domains for first-order commensurability and different resonance multiplicities, $k = \overline{1,4}$, are shown in Fig. 3.

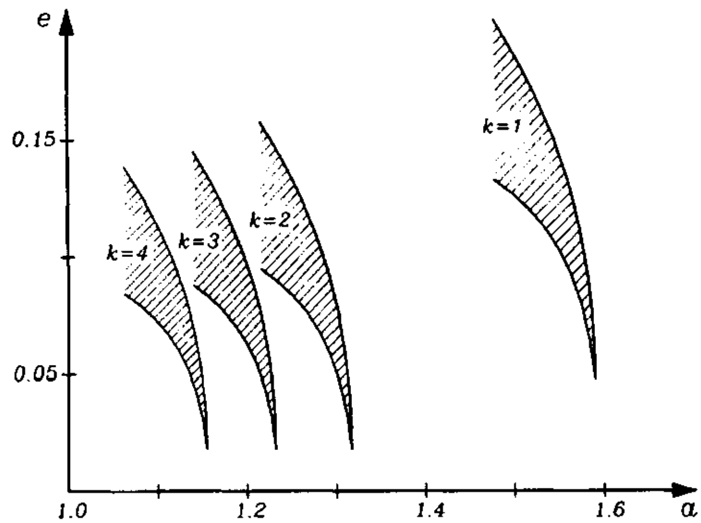

Fig. 3

Instability regions in the e–a diagram for first-order resonances of multiplicities $k = \overline{1,4}$ with $\mu_\Psi = 5.17 \times 10^{-5}$.

It is obvious that Transneptune objects can exist for sufficiently long times if their orbital elements e and a are located outside the hatched regions in Fig. 3.

Since the "instability zones" (in Fig. 3) are asymmetric with respect to the points of exact commensurability $n_0(k) = kn'/(k+1)$ $(k = \overline{1,4})$, it can easily be concluded that in the resonance zones (other conditions being equal) the existence of libration Transneptune objects (and with high excentricities) is more likely for $n < n_0(k)$ than for $n > n_0(k)$.

Thus, in the Kuiper belt, Transneptune objects in the vicinities (the resonance effect's span is about $2\sqrt{\mu_\Psi}/(k+l)$) of average motions $n = 10, 77''$ $(l = k = 1)$, $n = 14, 35''$ $(l = 1,\ k = 2)$ may be located outside the regions shown hatched in Fig. 3 (the "avoidance zones") clustering near the boundaries of these zones.

At the same time, as shown in [21], for first-order resonances the shortest distance between P and P' corresponds to the following correlation condition:

$$S^* \cong k\omega,$$

where $S^* = (k+1)M - k(M' - \omega)$ is the "Delauney anomaly" (the critical argument). Therefore, if the perihelion longitude (ω) of a Transneptune object executes slow motions (oscillations) in the vicinity of $\omega = \pi$, then for odd k the "convergences" on Neptune are possible only for Transneptune objects with "apocentric libration" $(S^* = \pi)$; for even k and at perihelion $\omega = 0$ they are possible for "pericentrically librating" $(S^* = 0)$ Transneptune objects. Consequently, in the $2:1$ resonance area among the Transneptune objects with high excentricities, one should expect (other conditions being equal) the existence of predominantly "apocentrically librating" objects with $\omega \cong 0$, while in the $3:2$ commensurability zone, one should expect "apocentrically librating" Transneptune objects both with $\omega \cong 0$ and with $\omega \cong \pi$. It must be noted that, according to B. V. Chirikov's stochasticity criterion (the "resonance overlapping criterion"), for $l = 1$ the condition of stochasticity has the form [22]

$$(k+1) \geq 0.4\mu_\Psi^{-2/7}.$$

In the case at hand, $(k+1) \geq 6.711$, therefore for $k < 5$ resonance zones do not overlap, and the "isolated resonance" model, which was used above, is correct.

Let us now evaluate the probability of "capture into resonance" (and of "escaping from resonance") for Transneptune objects. To this end, we determine the probabilities for a trajectory to pass, under the influence of various perturbation factors (characterized by independent parameters $\delta = \{\delta_1, \delta_2, \ldots, \delta_n\}$), from one region of the phase space (phase plane) to another.

As shown in [23], the probability of trajectory transition (under the influence of perturbation factor δ; henceforth, without loss of generality, we set $\delta = \delta_1$) from region i into region j $(i, j = \overline{1,3},\ i \neq j)$ on the phase plane is given by

$$W_{ij} = \frac{(-1)^j \dfrac{2}{4-j}\Delta + (j-3)\dfrac{\pi}{2}}{(-1)^i \dfrac{\pi}{2} - \Delta}, \tag{5}$$

where

$$\Delta = \arcsin\varepsilon_1 + \psi_1\varepsilon_2,$$

$$\varepsilon_1 = \frac{1}{2}\left\{\frac{\psi_2}{1 - \mathrm{sign}(1-\psi_2)\varepsilon_3}\right\}^{1/4}, \quad \varepsilon_2 = \frac{1}{\sqrt{3}}(1 - 4\varepsilon_3^2)^{1/2}, \quad \varepsilon_3 = \cos\left(\frac{\pi + \arccos|1-\psi_2|}{3}\right).$$

In turn, ψ_1 and ψ_2, which are the independent parameters, are related to the coefficients of the Hamiltonian (3) by

$$\psi_1 = -\frac{A}{B}\frac{\partial B/\partial\delta}{\partial A/\partial\delta}, \quad \psi_2 = -\frac{27}{4}\frac{B^2}{A^3}.$$

According to (1)–(4), the coefficient B for a fixed Neptune mass depends only on the resonance multiplicity, and the integral A, a function of the system state, may in the general case depend on various perturbation factors. For instance, with increasing m_i (the giant planet masses) the value of A grows proportionally.

Formula (5) implies, in particular, that when B and A vary similarly (so that $B'_\delta/A'_\delta > 0$), a phase trajectory located in the "resonance zone" (between the two branches of the separatrix) is more likely to pass into the "outer zone" than into the "inner zone" (which immediately surrounds the stable stationary point $(Z_1^{(2)}, 0)$): $W_{31} > 1/2$, $W_{32} < 1/2$ (Fig. 4).

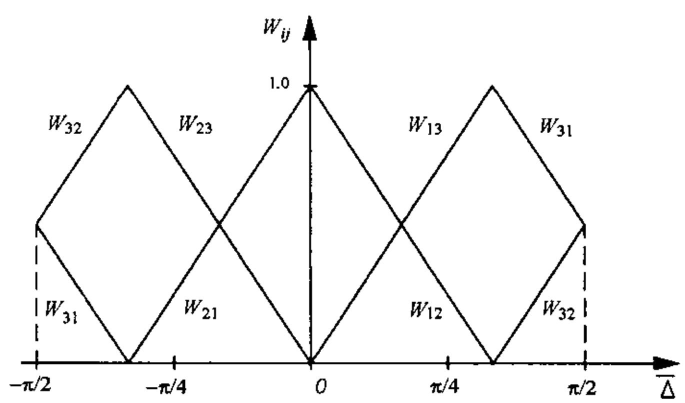

Fig. 4

Probabilities W_{ij} of phase trajectory transitions from zone i to zone j $(i, j = \overline{1,3},\ i \neq j)$ as functions of $\overline{\Delta} = \arctan[\Delta(\psi_1, \psi_2)]$; *1* — outer, *2* — inner, *3* — phase plane resonance zones.

If $A = \mathrm{const}_\delta$, $B = \mathrm{var}$ (when $\psi_1 = \infty$), no "capture into resonance" (transition from the inner or outer zone into the resonance one) is possible for a Transneptune object. For a fixed resonance multiplicity $\psi_1 = 0$, therefore in this case a Transneptune object is most likely to be captured into the "inner" zone than into the "resonance" one. The changes in the sizes (s_j) of the corresponding phase plane regions ($j = \overline{1,3}$), or the widths of the "stochastic layers", prove to be the order of the perturbation parameter δ. If $\partial A/\partial\delta > 0$ ($B = \mathrm{const}_\delta$), then the "inner" and "resonance" zones (s_2, s_3) grow in size, while the "outer" zone (s_1) diminishes; for $\partial A/\partial\delta < 0$ the values of s_i $(i = 2, 3)$ decrease and s_1 increases.

The probabilities for Transneptune objects to be captured into the "resonance" (W_{13}) and into the "inner zone" (W_{12}) are shown in Fig. 5 for fixed resonance multiplicities ($k = 1, 2$) as functions of the

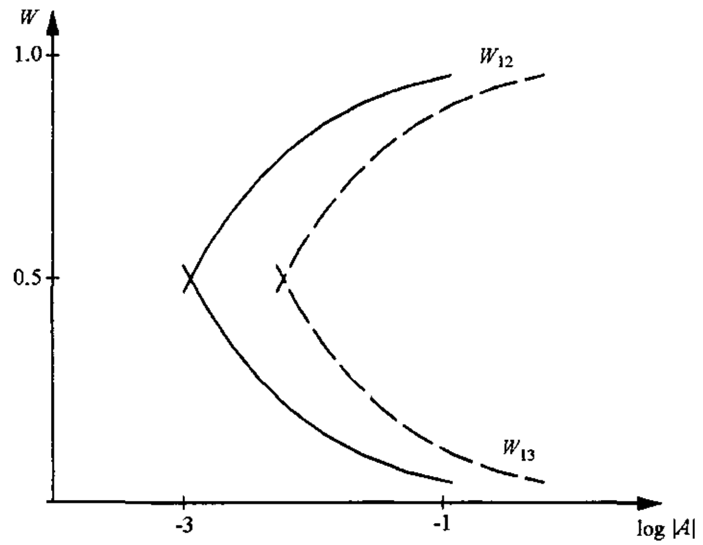

Fig. 5

Probabilities W_{12} and W_{13} as functions of $\log|A|$ for resonance multiplicities $k = 1$ (solid curve) and $k = 2$ (dashed curve).

integral A. These curves imply that a Transneptune object is most likely to be "captured into resonance" at $47.9 \leq a \leq 50.4$ a.u. ($k = 1$) and $39.3 \leq a \leq 40.6$ a.u. ($k = 2$).

Taking the influence of Uranus's resonance zones into account (see Fig. 1*b*), the existence of libration Transneptune objects in the Kuiper belt is only possible in the following ranges of average motions: $6.92''$–$7.44''$, $8.59''$–$8.77''$, $10.74''$–$11.15''$, $12.19''$–$12.41''$, $12.76''$–$13.07''$, $14.31''$–$14.61''$.

CONCLUSION

Orbital resonance effects cause stability of the orbits that have the libration type of motion. In resonance zones, libration orbits turn out to be close to the stable stationary solution, which ensures their "survival". Despite the secular perturbations from the giant planets Uranus, Saturn, and Jupiter (as well as Uranus's "resonance" influence) and gravitational interaction between Transneptune bodies, these bodies can be captured by Neptune into an orbital resonance and possessing orbital stability can exist for a long time.

The analytical solution considered in the present study, which interprets in the framework of "partial determinism" the dynamical evolution of a Transneptune object bound with Neptune through a first-order orbital resonance, can be used as an intermediate orbit. The case of second- and third-order commensurability was studied in detail in [24] on the basis of the restricted circular three-body problem.

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