Moscow University
Physics Bulletin
Vol. 54, No. 1, pp. 85–88, 1999

Vestnik Moskovskogo
Universiteta. Fizika
UDC 621.315.592

INTERACTION BETWEEN MICROWAVES AND SEMICONDUCTOR WAFER-MIRROR STRUCTURE

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The interference of microwaves in a structure composed of a weakly absorbing wafer, an air gap, and a mirror has been analyzed in a contactless fashion with a view to determining conductivity inhomogeneities in the wafer thickness. It is demonstrated that measuring the reflection coefficient of such a structure at a certain wavelength and an air gap values makes it possible to find the approximate values of the Fourier expansion coefficients of the wafer.

The interaction of microwaves in the one-dimensional approximation with wafer–air gap–mirror structures was investigated for various purposes. For example, the authors of [1, 2] analyzed the reflection from such a structure in connection with the development of photocontrollable microwave modulators and transparencies. The measurement of the time dependence of microwave reflection on a wafer after switching off the illumination is widely used to determine the lifetime τ_r of nonequilibrium charge carriers. It was shown in [3] that the use of a mirror behind the wafer allowed the sensitivity of such measurements to be materially improved. Studied in detail in [4] was the effect of the illumination conditions, the wafer parameters, and the air gap size L_b on the results of measurement τ_r.

The authors of [5] analyzed the possibility of determining the conductivity inhomogeneities σ of a weakly absorbing wafer by measuring the reflectivity R and the phase shift of the reflected wave as a function of L_b at a fixed wavelength λ. It was shown by way of calculations in a one-dimensional approximation that for the structure under consideration such measurements allow the conductivity σ to be determined as a function of the coordinate x in the wave propagation direction, provided that there exists an *a priori* information on the character of the wave.

The goal of this work is to demonstrate that measurements R of the structure of interest at various wavelengths and with the mirror placed in various positions make it possible to determine the Fourier expansion coefficients of the function $\sigma(x)$ for a weakly absorbing wafer, whose optical thickness is a multiple of the whole number of half-waves.

Consider the interaction between the electromagnetic wave and a weakly absorbing wafer of thickness L, for which the conditions $\chi(x)/N \ll 1$, $N \cong \text{const}$ hold true, where $\chi(x)$ and N are the imaginary and the real part of the refractive index of the wafer. In that case

$$R = 1 - \gamma \int\limits_{-L/2}^{L/2} \alpha(x)\varepsilon^2(x)\,dx, \tag{1}$$

where $x = 0$ corresponds to the center of the wafer, α [cm^{-1}] is the linear absorption coefficient, $\varepsilon(x) = E(x)/E_0$, $E(x)$ is the electric field amplitude of the waves interfering inside the wafer, E_0 is the amplitude of the wave incident upon the wafer, $\gamma = N/N_b$, and N_b is the refractive index of the medium adjacent to the wafer. It can be demonstrated that if $\alpha L \ll 1$ and the wavelength values satisfy the condition $\lambda_m = 2NL/m$ $(m = 1, \ldots, M)$, then

$$|\varepsilon_m(x,\varphi)|^2 = B_+(\varphi) + (-1)^m \left[B_-(\varphi)\cos(2k_m x) + 2\gamma^{-1}\sin(2\varphi)\sin(2k_m x)\right], \tag{2}$$

where $B_\pm(\varphi) = 2[\sin^2\varphi \pm \gamma^{-2}\cos^2\varphi]$, $k_m = \pi m/L$, and φ is the phase shift occurring while the wave passes from the wafer to the mirror. Here and in what follows the subscript m means that the corresponding quantity relates to the case $\lambda = \lambda_m$. The dependences ε_m^2 as functions of x calculated by formula (2) at $L = 0.3$ mm, $N = 3, 4$, $m = 1$, and $\varphi = 0, \pi/4, \pi/2$, and $3\pi/4$ are shown in Fig. 1 by dotted curves. Shown by dashed lines in the same figure are the functions $\varepsilon_m^2(x, \pi/2)$ obtained by numerical calculations with recurrent formulas for multilayered structures [6] at the same values of L, N, m, and φ. The wafer was therewith treated as a structure consisting of 160 homogeneous layers. As is seen, the calculation results agree well, the ε_1 values in the interference extrema and the positions of these extrema depend substantially on φ.

We introduce the following notation:

$$y_1(x) = \frac{1}{4}[\gamma^2\varepsilon_m^2(x,0) + \varepsilon_m^2(x,\pi/2)], \tag{3}$$

$$y_2(x) = \frac{(-1)^m}{4}[\varepsilon_m^2(x,\pi/2) - \gamma^2\varepsilon_m^2(x,0)], \tag{4}$$

$$y_3(x) = \frac{(-1)^m}{4}\gamma[\varepsilon_m^2(x,\pi/4) - \varepsilon_m^2(x,3\pi/4)]. \tag{5}$$

Then $y_1(x)=1$, $y_2(x)=\cos(2k_m x)$, and $y_3(x)=\sin(2k_m x)$. Consequently, the integrals $\frac{2}{L}\int_{-L/2}^{L/2} \alpha_m(x) y_i(x)\,dx$ at $i = 1, 2, 3$ are the Fourier expansion coefficients of $\alpha_m(x)$ (a_{0m}, a_m, and b_m, respectively). If absorption is due to free charge carriers and the condition $\omega_m\tau \ll 1$ is satisfied, where ω_m is the angular frequency and τ is the average relaxation time of the current carrier pulse, then $\alpha_m(x) = Z\sigma(x)/N$, where $Z = 377\ \Omega$ is the

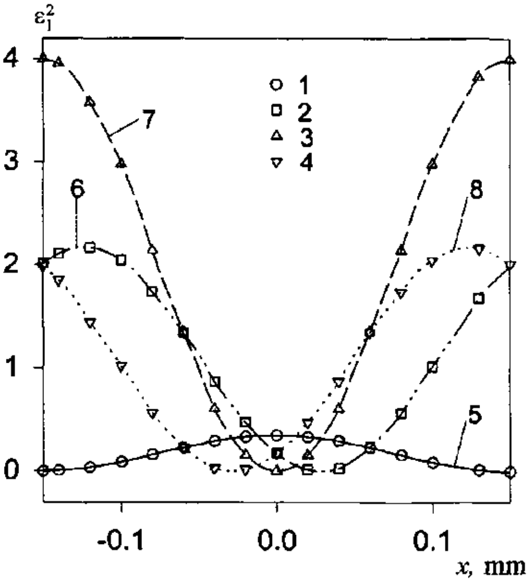

Fig. 1

Functions $\varepsilon_1^2(x)$ for a standing wave in a nonabsorbing silicon wafer at various values of the phase shift φ obtained by formula (2) (dotted curves *1* through *4*) and by numerical calculations from the recurrent formulas for multilayered structures [6] (curves *5* through *8*): $\lambda = 2NL$, $N = 3, 4$, $L = 0.3$ mm, $\varphi = 0$ (curves 1 and *5*), $\pi/4$ (*2* and *6*), $\pi/2$ (*3*, *7*), and $3\pi/4$ (*4* and *8*).

wave impedance of a vacuum. In this case, the quantities a_{0m}, a_m, and b_m are equal, within the constant factor Z/N, to the Fourier expansion coefficients of $\sigma(x)$. So, measuring the R values under the conditions considered above enables one to find a sufficiently smooth function $\sigma(x)$ with an accuracy determined by the value of M.

Of practical importance, however, is the case when the condition $\alpha L \ll 1$ is not strictly satisfied. For example, semiconductor electronics makes wide use of silicon wafers with $\rho = 20\text{–}4\ \Omega \cdot \text{cm}$ ($\rho = 1/\sigma$), which corresponds to $\alpha_1 L = 0.16\text{–}0.83$ ($\chi_1 = 0.09\text{–}0.47$) at $\omega_m \tau \ll 1$ and $L = 0.3$ mm. It can be shown that considering the absorption, the quantity $\varepsilon_m^2(x,\varphi)$ for a homogeneous wafer is given by

$$\varepsilon_m^2(x,\varphi) = \frac{1}{C(m,\varphi)}\Big\{B_+(\varphi)\cosh(\alpha_m x - \beta_m) + (-1)^m\left[B_-(\varphi)\cos(2k_m x) + 2\gamma^{-1}\sin(2\varphi)\sin(2k_m x)\right]\Big\}, \quad (6)$$

where $\beta_m = \alpha_m L/2$, B_+, B_- are the same as before, and

$$C(m,\varphi) = (\cosh\beta_m + \gamma^{-1}\sinh\beta_m)^2\cos^2\varphi + (\cosh\beta_m + \gamma\sinh\beta_m)^2\sin^2\varphi. \quad (7)$$

To illustrate the accuracy of these expressions, the dotted curves of Fig. 2 show the functions $\varepsilon_1^2(x,\pi/2)$ calculated by formula (6) at $\rho = 1000$, 100, 10, and 4 $\Omega \cdot$cm (curves *1* through *4*). The same figure presents the functions $\varepsilon_1^2(x,\pi/2)$ (curves *5* through *8*) obtained numerically in the same way as curves *5* through *8* of Fig. 1 at the same values of L, N, m, and ρ. As is seen at $\rho \geq 10\ \Omega \cdot \text{cm}$ ($\alpha_m L \leq 0.3$), the results of both calculations agree well enough. Considering the absorption,

$$y_1(x) \cong \frac{1}{4}[\gamma^2 C(m,0)\varepsilon_m^2(x,0) + C(m,\pi/2)\varepsilon_m^2(x,\pi/2)], \quad (8)$$

$$y_2(x) \cong \frac{(-1)^m}{4}[C(m,\pi/2)\varepsilon_m^2(x,\pi/2) - \gamma^2 C(m,0)\varepsilon_m^2(x,0)], \quad (9)$$

$$y_3(x) \cong \frac{(-1)^m}{4}\gamma[C(m,\pi/4)\varepsilon_m^2(x,\pi/4) - C(m,-\pi/4)\varepsilon_m^2(x,-\pi/4)]. \quad (10)$$

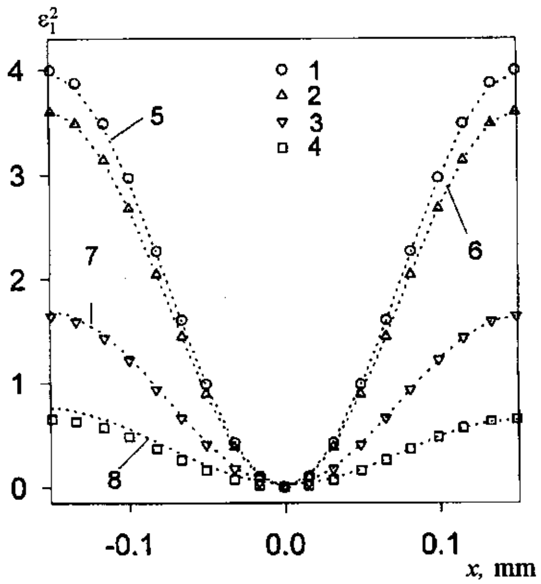

Fig. 2

Functions $\varepsilon_1^2(x)$ for a standing wave in an absorbing silicon wafer, obtained approximately by formula (6) (dotted curves *1* through *4*) and by numerical calculations from the recurrent formulas for multilayered structures [6] (dotted curves *5* through *8*) at $\rho = 1000\ \Omega \cdot \text{cm}$ (curves *1* and *5*), 100 $\Omega \cdot$ cm (*2* and *6*), 10 $\Omega \cdot$ cm (*3* and *7*), and 4 $\Omega \cdot$ cm (*4* and *8*) for $L = 0.3$ mm, $N = 3, 4$, $m = 1$, and $\varphi = \pi/2$.

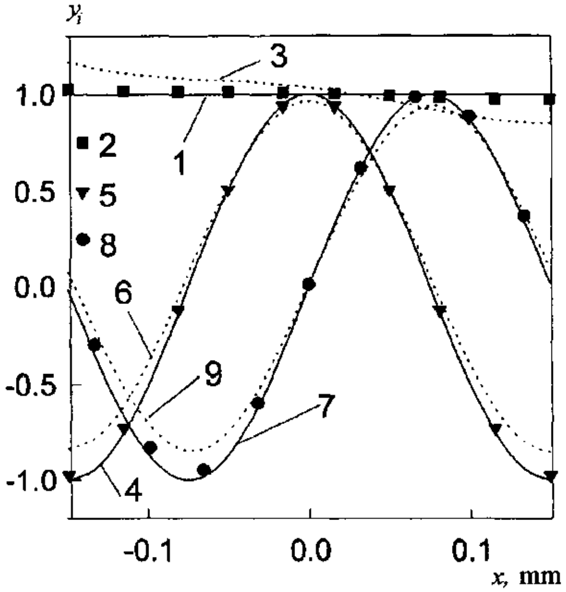

Fig. 3

Functions $y_1(x)$ (curves *1* through *3*), $y_2(x)$ (*4* through *6*), and $y_3(x)$ (*7* through *9*) obtained from formulas (8), (9), and (10) at $\rho = 1000\ \Omega\cdot\text{cm}$ (curves *1*, *4*, and *7*), $10\ \Omega\cdot\text{cm}$ (*2*, *5*, and *8*), and $4\ \Omega\cdot\text{cm}$ (*3*, *6*, and *9*) for $L = 0.3$ mm, $N = 3, 4$, and $m = 1$.

Figure 3 presents the functions $y_1(x)$, $y_2(x)$, and $y_3(x)$ calculated from formulas (8), (9), and (10) at the $\varepsilon_1^2(x,\varphi)$ values corresponding to $\rho = 1000$, 10, and $4\ \Omega\cdot\text{cm}$ and the same L, N, and m values. Actually, at $\rho = 1000\ \Omega\cdot\text{cm}$ $y_1(x) = 1$, $y_2(x) = \cos(2\pi x/L)$, and $y_3(x) = \sin(2\pi x/L)$. It can be seen that at $\rho = 10\ \Omega\cdot\text{cm}$ the functions $y_1(x)$, $y_2(x)$, and $y_3(x)$ are almost the same as at $\rho = 1000\ \Omega\cdot\text{cm}$ (the difference in the maxima is less than 5%, $\alpha_m L \leq 0.2$), though the values of $\varepsilon_1^2(x,\varphi)$ differ more than by a factor of two. Similar functions are also obtained, when $\rho(x) \neq \text{const}$, provided that $\rho(x) > 10\ \Omega\cdot\text{cm}$. The discrepancy becomes important at $\rho \leq 4\ \Omega\cdot\text{cm}$. Consequently, if $\alpha_m L \leq 0.3$,

$$\alpha_{0m} \cong \frac{1}{2L}[\gamma C(m,0)A(m,0) + \gamma^{-1}C(m,\pi/2)A(m,\pi/2)], \tag{11}$$

$$\alpha_m \cong \frac{(-1)^m}{2L}[\gamma^{-1}C(m,\pi/2)A(m,\pi/2) - \gamma C(m,0)A(m,0)], \tag{12}$$

$$b_m \cong \frac{(-1)^m}{2L}[C(m,\pi/4)A(m,\pi/4) - C(m,3\pi/4)A(m,3\pi/4)], \tag{13}$$

where $A(m,\varphi) = 1 - R(m,\varphi)$.

Thus, by measuring the values of R under the conditions considered above we can find the approximate function $\sigma(x)$ in a weakly absorbing wafer. This method makes it possible to construct $\sigma(x)$ profiles by taking a comparatively small number of measurements.

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8 July 1998

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