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THEORETICAL AND MATHEMATICAL PHYSICS

THE ORIGIN OF MAGNETIC FIELDS OF ASTROPHYSICAL OBJECTS

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In the framework of multidimensional models of the Kaluza–Klein type with consideration for the relation theory of space-time and interactions (binary geometrophysics), explanation of the origin of magnetic fields of the Earth, the Sun and other astrophysical objects is proposed.

1. The proposed explanations for the origin of the magnetic fields of the Earth, the Sun and other astrophysical objects (see, e. g., [1]) suffer from a number of drawbacks. It has been assumed that these magnetic fields are induced by rotation of astrophysical objects. As is known, even P.N. Lebedev [2] carried out experiments to detect a magnetic field of rotating objects. The most appropriate hypothesis is based upon the dynamo theory (see, e. g., [3]).

2. The relation theory of space-time and physical interactions developed by the author (binary geometrophysics) offers an alternative explanation of the origin of the magnetic field of the Earth and of other astrophysical objects [4, p. 238]. The explanation of the magnetic field origin presented in what follows might have been arrived at even in the 1920–1930s in the framework of the multidimensional geometric models of physical interactions that are usually called the Kaluza–Klein theories.

As is known (see, e. g., [4, 5]), physical fields in multidimensional geometric models, e. g. the electromagnetic field, are described by the mixed components $G_{\mu a}$ of the metric tensor. Here the subscript μ takes four values, $0, 1, 2, 3$, and the subscript a may be equal to $4, 5, \ldots$. The charges corresponding to the fields, e. g., the electric charge, are described by the cyclic dependence of the charged particle wave functions on the fifth or another additional coordinate, i. e., this charge is the fifth or other additional component of the momentum.

3. Analysis shows that the models unified as the 5-dimensional Kaluza–Klein theory are in fact two different theories dealing with different additional dimensions and describing different aspects of physical reality.

Thus, in the *5-dimensional Kaluza theory* that unifies gravitational and electromagnetic interactions, the particle wave function Ψ is postulated to depend on the fifth coordinate in the following way:

$$\Psi = \psi(x^\mu)\exp(i\alpha\varepsilon_5 x^5) \equiv \psi(x^\mu)\exp\left(\frac{iec\varepsilon_5}{2\sqrt{k_g}\hbar}x^5\right), \tag{1}$$

where $\psi(x^\mu)$ is the ordinary wave function of a particle depending on four classical coordinates; α determines the period of compactification in x^5; e is the electron charge; ε_5 is the particle charge in units of e; k_g is the Newton gravitational constant; and $\hbar$ is the Planck constant.

In all multidimensional theories, a reduction of the initial geometry to the 4-dimensional space-time cross section is carried out. In 5-dimensional theories, this is made by means of the $1+4$ splitting monad method [3, 4], where the 5-dimensional metric tensor is represented as $G_{AB} = g_{AB} - \lambda_A\lambda_B$. Here g_{AB} is the 4-dimensional metric tensor, λ_A is the 5-dimensional vector (monad) orthogonal to the 4-dimensional cross section.

4. In describing interactions, a key role is played by the monad 4-dimensional differentiation operator (in the gauge analogous to the chronometric one in the general relativity theory), which is invariant under arbitrary transformations of an additional coordinate and covariant under 4-dimensional transformations. It has the form

$$\partial_\mu^\dagger \equiv \frac{\partial}{\partial x^\mu} + \lambda_\mu \lambda^5 \frac{\partial}{\partial x^5} \equiv \frac{\partial}{\partial x^\mu} - \frac{G_{5\mu}}{G_{55}} \frac{\partial}{\partial x^5}. \tag{2}$$

The electromagnetic vector potential A_μ in the Kaluza theory is described by mixed components of the metric according to the formula

$$A_\mu = \frac{c^2}{2\sqrt{k_g}} \lambda^5 \lambda_\mu \equiv -\frac{c^2}{2\sqrt{k_g}} \frac{G_{5\mu}}{G_{55}} \to G_{\mu 5} = \frac{2\sqrt{k_g}}{c^2} A_\mu, \tag{3}$$

where we assumed that $G_{55} = -1$. Acting by operator (2) on the 5-dimensional wave function (1), with regard for (3), one arrives at a known long derivative of electrodynamics, $\partial_\mu^\dagger \Psi = (\partial/\partial x^\mu + (ie/c\hbar) A_\mu)\,\Psi$.

5. In the *5-dimensional theory* considered by O. Klein [6], V. A. Fock [7], and Yu. B. Rumer [8], a different dependence on an additional coordinate was postulated,

$$\Psi = \psi(x^\mu)\exp(i\beta x^4) \equiv \psi(x^\mu)\exp\left(\frac{imc}{\hbar} x^4\right), \tag{4}$$

where m is the particle mass, β determines the compactification period in an additional coordinate denoted as x^4. The coordinate x^4 has, to a constant, the meaning of the particle action.

As Yu. B. Rumer emphasized, geometrization of the electromagnetic field is not the main goal of this theory. This 5-dimensional theory (5-optics) is aimed at the solution of another problem. However, there was a great temptation to describe electromagnetism in terms of the same additional coordinate. The electric charge had to be introduced into the mixed components of the metric that were also related to the electromagnetic field, though, in contrast to (3), by the formula $\tilde{G}_{\mu 4} = (q/mc^2) A_\mu$. As a result, a theory with the configuration space was obtained, i. e., with the metric depending on the charge and mass of the particle in question. This is entirely different from the general relativity theory and from the 5-dimensional Kaluza theory, where universal spaces with the metric independent of an individual particle are used. Yu. B. Rumer (see [8]) directed much effort toward solving problems related to the origination of configuration spaces.

6. Synthesis of the Kaluza theory and the Klein–Fock–Rumer theory that is free from the above difficulties can be performed in the framework of the 6-dimensional geometric model with two additional coordinates x^4 and x^5. The particles wave functions depend on two coordinates and have the form $\Psi = \psi(x^\mu)\exp(i\beta x^4 + i\alpha x^5)$, where the constants α and β correspond to expressions (1) and (4). In this model, the reduction to the 4-dimensional space-time cross section is carried out by means of the dyad $(1+1+4$ splitting) method in the twice chronometric gauge introduced earlier in the general relativity theory. In the 6-dimensional theory, the metric tensor is represented as $G_{MN} = g_{MN} - \xi_M \xi_N - \lambda_M \lambda_N$, where g_{MN} is the 4-dimensional metric tensor, ξ_M and λ_N are the two 6-dimensional dyad vectors orthogonal to the 4-dimensional space-time cross section. They are expressed in terms of the components of the 6-dimensional metric [4, 5].

7. The particle interaction is described by means of a dyad operator of 4-dimensional differentiation that is invariant under the transformation of two additional coordinates and covariant under the 4-dimensional transformations. The operator has the form

$$\partial_\mu^{\dagger\dagger} \equiv \frac{\partial}{\partial x^\mu} + (\xi^4 \xi_\mu + \lambda^4 \lambda_\mu)\frac{\partial}{\partial x^4} + \lambda^5 \lambda_\mu \frac{\partial}{\partial x^5}. \tag{5}$$

Like in [4], the combination $\lambda^5 \lambda_\mu$ is identified with the vector potential A_μ of the electromagnetic field. However, in this theory, there is another combination $\xi^4 \xi_\mu + \lambda^4 \lambda_\mu$ that depends on $G_{4\mu}$ and can hardly be interpreted in the framework of multidimensional geometric models. This can be done in the binary geometrophysics where the appearance of additional dimensions x^4, x^5, etc. [4] is substantiated.

8. The other combination should also be identified with the electromagnetic vector potential according to a formula of the type (3). Then the second term on the right-hand side of the dyad operator (5) will lead to minor corrections to electromagnetic interaction since the value of the charge (mass) appearing upon differentiation with respect to x^4 is many orders of magnitude lower than the electromagnetic charge. In the framework of multidimensional geometric models, one can find from formulas (1) and (4) that the mass m induces an additional ("mass") electric charge $\tilde{q}$ of the form

$$\tilde{q} = 2\sqrt{k_g}m. \tag{6}$$

Taking into account that the electron mass $m_e \cong 9.1 \times 10^{-28}$ g, we get the ratio of the two charges $\tilde{q}/q \equiv \tilde{e}/e \sim 10^{-21}$. It is evident that this correction to electromagnetic interaction is far beyond the limits of experimental accuracy. However, for large masses, when electric charges of oppositely charged particles are compensated, the "mass contribution" to the electromagnetic interaction due to the coordinate x^4 might be substantial.

9. The question arises of the sign of the "mass" electric charge of a massive object. It is evident that the additional electric charge of nucleons is much larger than that of electrons. Assume that protons and neutrons have like mass electric charges, then, regardless of the sign of the electron mass charge, the additional electric charge of massive objects can be estimated by (6), where m is the object mass.

10. We consider massive astrophysical objects like the Earth, planets, and stars. These objects are assumed to be electrically neutral on the average, however, according to the above argument, they are charged. This means that the space-time around "stationary", spherically symmetric objects should ideally be described not by the Schwarzshild metric but by the Reissner–Nordstrom-type metric where the additional constant, the electric charge, is determined by the masses of these objects.

For rotating (ideal) objects, the space-time should be described not by the Kerr metric but by the Kerr–Newman-type metric (see, e. g., [9]) with an extra constant (the electric charge) being also expressed in terms of the mass. Additives to the general relativistic effects are also very small in this case, however, both electric and magnetic fields might arise around these objects. As is known, the dipole magnetic moment M_{KN} of the Kerr–Newman source is determined by the formula $M_{KN} = qa$, where q is the source electric charge, a is the source momentum. Assuming the source to be a sphere and substituting the value $\tilde{q}$ from (6), we find the expression for the primary dipole magnetic moment of this (ideal) object:

$$M_{KN} \equiv M_1 = \frac{4\sqrt{k_g}}{5c} mR^2\omega, \tag{7}$$

where m is the mass, R is the radius, and ω is the angular velocity of the source.

11. It is evident that an additional electric charge of real astrophysical objects is compensated for by absorbed oppositely charged particles, so the resultant electric field is absent. However, this does not imply that, in the general case, the magnetic field is also compensated. It should be expected that a certain effective magnetic field arises around these objects, and it is the sum of two components, namely, the primary magnetic field of an additional electric charge stipulated by the mass* and the secondary magnetic field induced by the absorbed charges. These two magnetic fields compensate each other, though there are reasons to assume that they are not completely compensated. The resultant magnetic field depends on the absorbed charge distribution.

12. According to (7), the Earth's primary dipole magnetic moment $M_1 \cong 1 \times 10^{27}$ G cm^3 is close to the known Earth's effective magnetic moment $M_1 - M_2 = M_{\text{exp}} \cong 8 \times 10^{25}$ G cm^3. The latter is 8% of the primary magnetic moment.

13. Now we tabulate the parameters and the values of primary dipole magnetic moments of the Sun and some planets of the Solar system, calculated by (7) and obtained from observations.

Table 1 implies that the primary dipole magnetic moment of all the objects, except for Mercury, exceeds the observed value, and this is in agreement with the hypothesis under discussion. The observed value of the primary magnetic moment is approximately 0.1 and 0.3% for the Sun and Jupiter, respectively.

* Efforts to explain the origin of magnetic fields based solely on (7) are in contradiction with the observational data [10].

Table 1

Object	m (g)	R (cm)	ω (rad s^{-1})	M_1 (G cm^3)	M_{exp} (G cm^3)
Sun	2×10^{33}	7×10^{10}	2.5×10^{-6}	1.7×10^{35}	1.7×10^{32}
Mercury	3.24×10^{26}	2.4×10^{7}	8×10^{-7}	1.0×10^{21}	5×10^{27}
Earth	6×10^{27}	6×10^{8}	7.3×10^{-5}	1.1×10^{27}	8×10^{25}
Mars	6.4×10^{26}	3.4×10^{8}	7×10^{-5}	3.6×10^{25}	2×10^{22}
Jupiter	1.9×10^{30}	7×10^{9}	1.8×10^{-4}	1.2×10^{32}	4×10^{30}
Magnetic moments of Venus and the Moon are negligible.					

14. Using (7), we can estimate the magnetic field of pulsars commonly assumed as neutron stars. They have different parameters. We choose their values within the estimates given in the literature. Taking the pulsar radius $R \cong 10^6$ cm, the pulsar mass of the order of the Sun's mass, $m \cong 10^{33}$ g, and $\omega \cong 10$ rad s^{-1}, we obtain for the primary dipole magnetic moment $M_1 \cong 10^{32}$ G cm^3. This corresponds to the value of the magnetic field strength on the pulsar surface, $B = M_1/R^3 \cong 10^{14}$ G. The pulsar magnetic field is generally estimated as $B_{exp} \cong 10^{12}$–10^{13} G. Hence, our hypothesis of the origin of the magnetic field can be extended to pulsars as well.

15. The proposed method provides a new basis for analyzing the known phenomena such as variation of the dipole moment polarity of the Sun and the Earth, drift of the Earth's magnetic pole, displacement of the magnetic pole from the geographic one and some others. Since the primary electric charge and dipole magnetic moment remain practically unchanged, the above effects may be related to the processes of redistribution of absorbed electric charges.

16. The above proposed explanation of the origin of the magnetic field of the Earth, Sun, and other astrophysical objects does not reject the earlier conjectures, it rather justifies and unifies them to a certain extent. For instance, ideas of P. N. Lebedev, P. M. Blackett and other scientists that rotation of objects is responsible for a magnetic field appearance are developed here. The above arguments are correlated with conjectures of T. Shlomka and V. Swan on changing the law of electromagnetic interaction of two unlike charges. The hypothesis that a magnetic field is stipulated by currents inside objects is not rejected either.

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