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ESTIMATING STRING TENSION FROM FINITE-ENERGY SUM RULES

D. V. Meshcheryakov and V. B. Tverskoi

In the framework of the finite-energy sum rules, with the use of the modified expression for the QCD running coupling constant, the string tension parameter is estimated.

A considerable number of papers on the subject of nonperturbative region of QCD have been published [1–7], and yet there still remains much to be clarified. In particular, there cannot be considered determined the values of the most important nonperturbative parameters such as vacuum condensates of various dimensions. The condensate values reported in the literature can roughly be divided into two domains with the corresponding average values differing by a factor of 5 to 10. The domain with lower average values was for the first time mentioned in classical papers [1]. Higher average values were obtained and used in [2, 3, 6]. The status of the present situation can be formulated as follows: the use of the Vainstein–Zakharov–Shifman global-sum rules leads to lower values of condensates [1], while the use of finite-energy sum rules leads to their higher values [6], and this difference cannot be explained only by inaccuracy of the sum rules methods used. Therefore, it seems evident that the global and lower-energy sum rules should be modified so that the two approaches give the results consistent within their accuracy limits. Employing the Schwinger–Dyson equations along with the low-energy sum rule can serve as one of such modifications. Rather than directly considering the values of QCD vacuum condensates of various dimensions, we address in the present paper at the value of hadronic string tension parameter k, or the slope of the Regge trajectory $\alpha' = 1/(2\pi k)$, assuming that a linear confinement takes place, i. e., the confining potential has the form $V(r) \to kr$ as $r \to \infty$.

In [5], the expression for the QCD running coupling constant was modified basing upon the Schwinger–Dyson equations with account of the requirement that nonperturbative contributions are minimal in the ultraviolet region. In this way, the requirements of asymptotic freedom, analiticity and confinement for the running coupling constant were consistently met. In the present paper, we use the modified expression for the running coupling constant to estimate the string tension parameter. We demonstrate that applying the low-energy sum rules gives higher string tension parameter values than when the Schwinger–Dyson equations are used only [5], i. e., the low-energy sum rules with consideration for the Schwinger–Dyson equations lead to higher values of the low-energy parameters.

In [3], in the framework of the finite-energy sum rule method with the help of a simple representation for the pion electromagnetic form factor with four poles, the following equations were obtained:

$$\left(\frac{8\pi^2}{s_0}\right)\Phi = 1 + \frac{\alpha_s}{\pi} - \frac{1}{s_0}\frac{3(\bar{m}_u^2 + \bar{m}_d^2)}{[(1/2)\ln(s_0/\Lambda^2)]^{4/(-b_1)}}, \tag{1}$$

$$\left(\frac{16\pi^2}{s_0^2}\right) s_1\Phi = 1 + \frac{\alpha_s}{\pi} - \frac{2}{s_0}\frac{\pi^2}{3}\left\langle\frac{\alpha_s}{\pi}GG\right\rangle, \tag{2}$$

$$\left(\frac{24\pi^2}{s_0^3}\right) s_1^2\Phi = 1 + \frac{\alpha_s}{\pi} - \frac{3}{s_0^3}\frac{896}{81}\pi^3\alpha_s\langle\bar{q}q\rangle^2, \tag{3}$$

where $\alpha_s/\pi = 2/(-b_1 \ln(s_0/\Lambda^2))$ is the QCD running coupling constant, $\bar{m}_u$ and $\bar{m}_d$ are the u- and d-quark masses, $b_1 = -11/2 + N_f/3$, N_f is the number of quark flavors, s_0 is the continuum threshold, $s_1 = 4m_\pi^2(2u_1^2+1)^2 = 0.63\ \mathrm{GeV}^2$ is the ρ-meson effective mass squared, m_π is the pion mass, $\alpha_s\langle\bar{q}q\rangle^2$ is the four-quark condensate, $\langle(\alpha_s/\pi)GG\rangle$ is the gluon condensate. Here $\Phi = \Phi(u_1, v_1, u_2, v_2)$, where u_i, v_i, $i = 1, 2$, are the poles of the pion electromagnetic form factor defined in the four-sheeted Riemann surface. Consider the estimate obtained in [7]:

$$\bar{m}_u^2 + \bar{m}_d^2 = \frac{2\pi}{3}\frac{64 f_\pi^2 m_\pi^4}{9\langle\alpha_s GG\rangle}. \tag{4}$$

Substituting (4) into (1)–(3) and excluding Φ from these equations yield the following relation:

$$\alpha\langle\bar{q}q\rangle^2\left(\frac{896\pi^3}{27s_0^3} - \frac{R}{s_0\alpha\langle\bar{q}q\rangle^2\langle(\alpha/\pi)GG\rangle}\right) = \frac{\tau^2-3}{\tau(\tau-2)}\langle(\alpha/\pi)GG\rangle\left(\frac{2\pi^2}{3s_0^2} - \frac{R}{s_0\langle(\alpha/\pi)GG\rangle^2}\right), \tag{5}$$

where

$$\tau = \frac{s_0}{s_1}, \quad R = \frac{128 f_\pi^2 m_\pi^2}{9[(1/2)\ln(s_0/\Lambda^2)]^{-4/b_1}}.$$

The modified expression for the running coupling constant obtained in [5] has the form

$$\alpha_s(q^2) = \frac{4\pi}{b_1}\left(\frac{1}{\ln(q^2/\Lambda^2)} - \frac{\Lambda^4}{(q^2)^2} + O((q^2)^{-3})\right), \tag{6}$$

where Λ is the QCD fundamental dimensional parameter related to the scale of strong interactions. This form of the coupling constant leads to the following dependence of Λ on the string tension parameter k [5]:

$$\Lambda^2 = \frac{3b_1}{8\pi}k. \tag{7}$$

In order to estimate k, we use (3) with regard to (4), (5) and the low-energy relation [1]

$$(m_u + m_d)\langle\bar{q}q\rangle = -f_\pi^2 m_\pi^2,$$

and arrive at the equation

$$AK - \frac{R}{K} = \frac{\tau^2-3}{\tau(\tau-2)}K\left(B - \frac{R}{K}\right), \tag{8}$$

where

$$A = \alpha_s(q^2)\frac{7\pi^2 f_\pi^2}{3s_0^2}, \quad B = \frac{2\pi^2}{3}, \quad K = \frac{243}{16\pi^4}k^2\ln\left(\frac{8\pi}{27}\frac{\mu^2}{k^2}\right).$$

In the latter expression, μ is the regularization parameter [5]. Numerical study of (8) with the continuum threshold value $s_0 = 1.3\ \mathrm{GeV}^2$ taken from [3] demonstrates that there exist two solutions for k. One of them is close to zero and must be rejected on physical grounds (it actually means that there is no confinement). The other one increases with growth of the cuttoff parameter μ, and reaches its maximum value

$$k = 0.97\ \mathrm{GeV}^2, \tag{9}$$

which we take as the string tension parameter. Thus, the method based upon the low-energy sum rules with consideration for the consequences of the Schwinger–Dyson equation for the gluon propagator gives the value of the parameter k approximately five times greater than the estimate of [5], where it was obtained basing upon the Schwinger–Dyson equation without using the finite-energy sum rules. Thus, our result (9) is in agreement with our earlier estimates [2, 3] as well as with the conclusions of other authors [6] who used the low-energy sum rules as a basis for their study of QCD nonperturbative region. In the present paper, we have shown that consideration for the Schwinger–Dyson equation along with the use of the low-energy sum rules lead not to the "standard" low numerical estimates, but to their increase. And, as it was mentioned above, this is generally characteristic of the approach based upon the low-energy sum rules [3, 6].

In order to find a numerical value, we use the analytical expression for the running coupling constant found in [8] based upon spectral representation without subtraction, with the finite infrared limit $\alpha_s(0) = 1.40$. As a result, we have $\sigma_0 = 0.47$ GeV^2 which is roughly by a factor of two lower than our estimate and by a factor of three greater than the estimate of [5].

Taking into account the above results, it seems reasonable to find a modification of the QCD global and finite-energy sum rules, which will give the coincidence of the QCD low-energy parameters determined in its framework (within the error limits). This could possibly be achieved by imposing stabilizing conditions as it was done in [9].

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