Moscow University Physics Bulletin
Vol. 55, No. 2, pp. 19–22, 2000

Vestnik Moskovskogo Universiteta. Fizika
UDC 530.145

NEUTRAL PARTICLE RADIATION IN ELECTROMAGNETIC FIELD

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The total radiation power of a classical neutral particle having an anomalous magnetic and an electric moment in its motion in an arbitrary electromagnetic field has been found. The result has been obtained on the assumption that the particle spin evolution is determined by the generalized Bargmann–Michel–Telegdi equation.

It is common knowledge that external electromagnetic fields exert considerable effect on the course of processes with participation of neutral particles. This effect may result not only from vacuum polarization but also from the internal electromagnetic structure of heavy particles such as the neutron. This fact makes the study of the external field effect on the processes with participation of neutrons one of the most important problems of high-energy physics.

The internal electromagnetic structure of the neutron manifests itself mainly in its anomalous magnetic moment. Studies show that the consideration of this factor alone makes the results nontrivial. It was thus established that the presence of an anomalous magnetic moment in the neutron results in considerable changes in the beta-decay probability in strong fields [1], which is important for pulsar physics [2]. Another interesting effect is the neutron radiation.

This phenomenon was studied both on the basis of quantum theory and within the framework of classical electrodynamics. Quantum electrodynamics methods were used in studying the neutron in electromagnetic fields of a special kind: homogeneous [3, 4], plane-wave [5–8], and some other [9]. In some works [10, 11], a classical description of neutron radiation was carried out.

In quantum attack of the problem, field models are chosen so that Dirac's equations have the exact solutions in these fields. We should note that the availability of exact solutions is obligatory for the calculation of polarized and nonpolarized particle radiation power. For polarized particles, the classical situation is the same: the availability of exact solutions of the equation of motion (Lorentz equation) and the equation of spin evolution (Bargmann–Michel–Telegdi equation) allows us to calculate the radiation power both for a neutral and a charged particle in its motion in the electromagnetic field.

The problem is considerably simplified when radiation of a neutral nonpolarized particle is being studied. Actually, the classical approach is mainly distinguished from the quantum approach by that radiative recoil is not considered in classical description. Therefore, in quantum electrodynamics, a radiation event brings about a change in the pulse and polarization of the particle, and is thus characterized by the initial and final values of the pulse and polarization, while in classical treatment, the radiation event does not change these quantities, because the recoil is absent. This roughens the picture but, nonetheless, as the comparison with quantum consideration shows, the results agree well as $\hbar \to 0$.

For a neutral particle the solution of the Lorentz equation for the 4-velocity is u^μ = const, and, therefore, the Lorentz transformation that executes the transition to the particle rest system will in this case be determined by the constant operator. In the rest system the only characteristic vector determining the particle is its polarization vector. If we perform averaging over spin directions (which in quantum theory will correspond to a description of a nonpolarized particle radiation), then only the strengths of the electric and magnetic fields in the particle rest system remain the vectors that may enter into the expressions for the radiation energy. This fact suggests the existence of a universal formula for the radiation energy of the neutral nonpolarized particle in arbitrary heterogeneous electromagnetic fields. The calculations given below support the aforesaid.

Let us consider the radiation of a neutral nonpolarized particle caused by its anomalous magnetic moment, which we denote by μ^ν. Classical electrodynamics gives the following formula [12] for the total radiation energy:

$$\mathcal{E} = \int I\,dt, \tag{1}$$

in the covariant form the radiation power can be presented as

$$I = -\frac{2}{3}\langle \ddot{\mu}^\nu \ddot{\mu}_\nu \rangle \tag{2}$$

(the dot denotes differentiation with respect to proper time τ, French quotes denote averaging over spin states, $\hbar = c = 1$).

We restrict the discussion to the assumption that the magnetic moment is proportional to the spin vector

$$\mu^\nu = \mu_0 S^\nu.$$

Then the evolution of the spin vector is described by the BMT equation [13],

$$\dot{S}^\nu = 2\mu_0 \left[F^{\nu\alpha} S_\alpha - u^\nu (u_\alpha F^{\alpha\beta} S_\beta) \right],$$

whose solution satisfies the following additional conditions $(SS) = -1$ and $(Su) = 0$.

For the sake of convenience of comparison with quantum-mechanical description, we use the spin-tensor of rank two $\underset{\sim}{S} = S^0\sigma_0 + \mathbf{S}\boldsymbol{\sigma}$, where σ_ν are the Pauli matrices, instead of the polarization vector S^ν (the notation corresponds to that used in [14]). Then the spin evolution is determined from the formula

$$\underset{\sim}{S}(\tau) = \underset{\sim}{L}\underset{\sim}{R}\underset{\sim}{L}^{-1}\underset{\sim}{S}_0(\underset{\sim}{L}^{-1})^+\underset{\sim}{R}^+\underset{\sim}{L}^+. \tag{3}$$

Here $S_0 \equiv S(\tau_0)$ is the polarization at the initial time τ_0; $\underset{\sim}{L}^{-1}$ is the operator of transition into the particle rest system,

$$\underset{\sim}{L}^{-1} = \frac{1 + \tilde{u}}{\sqrt{2(1+u^0)}}, \quad \underset{\sim}{L} = \frac{1 + \underset{\sim}{u}}{\sqrt{2(1+u^0)}};$$

and $\underset{\sim}{R}$ is the rotation operator that satisfies the equation

$$\dot{\underset{\sim}{R}} = i\mu_0 \underset{\sim}{H}_0 \underset{\sim}{R}, \tag{4}$$

where

$$\mathbf{H}_0(x) \equiv \mathbf{H}_0 = u^0\mathbf{H} - \mathbf{u} \times \mathbf{E} - \frac{\mathbf{u}(\mathbf{uH})}{1 + u^0}$$

is the magnetic field in the particle rest system at the point of its location $x^\nu = u^\nu \tau$.

Using the introduced spin-tensor, formula (2) for radiation power can be rewritten as:

$$I = -\frac{\mu_0^2}{3}\,\mathrm{Sp}\langle \ddot{\underset{\sim}{S}}\ddot{\underset{\sim}{S}} \rangle.$$

Hence, on some simple transformations, the radiation power of a nonpolarized particle can be obtained with regard to (3),

$$I = \frac{\mu_0^2}{3}\,\mathrm{Sp}\sum_i (\underset{\sim}{R}\sigma_i\underset{\sim}{R}^+)^{\cdot\cdot}(\underset{\sim}{R}\sigma_i\underset{\sim}{R}^+)^{\cdot\cdot}. \tag{5}$$

Summing over i in (5) corresponds to an averaging procedure over the final spin states of the particle in quantum theory.

The final result follows from formulas (4) and (5),

$$I = \frac{16}{3}\mu_0^2\{4(\mu_0 \mathbf{H}_0)^4 + (\mu_0 \dot{\mathbf{H}}_0)^2\}. \tag{6}$$

Thus, as pointed out above, the radiation power of nonpolarized neutral particle in an arbitrary electromagnetic field is only expressed in terms of strength of these fields in the particle rest system and their proper-time derivatives.

Some remarks as to formula (1) need to be made. As is customary in quantum dynamics, if in (1) the integral converges on the interval $(-\infty, +\infty)$, then this formula describes the total radiation energy when the particle flies through the region occupied by the field. If there exists the finite limit

$$\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} I\, dt,$$

then this expression defines the radiation energy per unit time in periodic and nonperiodic fields. If the above limit does not exist, then the given formula has no quantum analog, to be more exact, this divergence suggests that, in solving quantum problems, bound states will be observed (discrete energy spectrum) (see, for instance, [15]).

It is easy to verify that when substituting into (6) the fields, which were earlier used for calculations both by quantum and classical methods, a complete agreement is observed with the obtained results if we omit the terms that characterize the recoil, i. e., the terms which have the next order of smallness in Planck's constant.

Let us discuss the possibility of using the classical approximation in more detail. As noted above, we can neglect quantum corrections if the energy of radiated photons is small as compared to the energy of radiating particle. This requirement is responsible for two types of restrictions. First, the bond energy in the rest system must be considerably less than the particle rest energy, $\mu_0 H_0/m \ll 1$. Second, the field must slightly vary over the distance of the order of the particle Compton wavelength, $\dot{H}_0/mH_0 \ll 1$. There is good reason to rewrite these conditions in Gaussian units. Then, considering that $\mu_0 \sim e\hbar/mc$ and introducing the so-called critical magnetic field $H_{cr} = m^2c^3/e\hbar$, we get the above-mentioned conditions as $H_0/H_{cr} \ll 1$ and $(H_0/H_{cr})(\omega/\omega_c) \ll 1$. Here ω is the characteristic frequency of external field variation, and $\omega_c = eH/mc$ is the cyclotron frequency. Hence it follows that for the neutron these conditions are *a fortiori* satisfied at reasonable strengths of the fields and their gradients. An important point is that the reported estimates are not weaker than those that allow us to disregard the effective mass variation in the external field and the Stern–Gerlach effect when deriving the BMT equation [16, 17]. The abovesaid confirms the fact that our approach can be used when solving the problem on the radiation of a neutral nonpolarized particle in an external field.

It is of interest to compare, in the order of magnitude, the radiation power I of a neutral particle (the neutron) defined by formula (6) with the classical radiation power of a charged particle I_0 (see, for instance [12]) which has close values of mass and energy (the proton). It is easy to find that

$$\frac{I}{I_0} \sim \max\left\{\left(\frac{H_0}{H_{cr}}\right)^2, \left(\frac{u^0\hbar\omega}{mc^2}\right)^2\right\}.$$

Therefore, the powers of charge radiation and magnetic moment can be compared either in superstrong fields or in very high-frequency fields, which can exist in the vicinity of astrophysical objects of pulsar type.

In conclusion, it should be pointed that obtained formula (6) can be extended to the case of the existence, along with the anomalous magnetic moment, of an electric moment (naturally, the static limit of electric moment ϵ_0 only exists in theories with T-invariance violation). In this case, the evolution of a spin vector is described by the generalized BMT equation [13, 18]

$$\dot{S}^\nu = 2\mu_0\{F^{\nu\alpha}S_\alpha - u^\nu(u_\alpha F^{\alpha\beta}S_\beta)\} + 2\epsilon_0\{H^{\nu\alpha}S_\alpha - u^\nu(u_\alpha H^{\alpha\beta}S_\beta)\},$$

where $H^{\nu\alpha} = -\frac{1}{2}e^{\nu\alpha\beta\gamma}F_{\beta\gamma}$ is the dual tensor.

The generalized formula (6) has the form

$$I = \frac{16}{3}(\mu_0^2 + \epsilon_0^2)\{4(\epsilon_0\mathbf{E}_0 + \mu_0\mathbf{H}_0)^4 + (\epsilon_0\dot{\mathbf{E}}_0 + \mu_0\dot{\mathbf{H}}_0)^2\}, \tag{7}$$

where

$$\mathbf{E}_0(x) \equiv \mathbf{E}_0 = u^0\mathbf{E} + \mathbf{u}\times\mathbf{H} - \frac{\mathbf{u}(\mathbf{uE})}{1+u^0}$$

is the electric field in the rest system of a particle at the point of its location. The results obtained with formula (7) agree with those obtained earlier in [19].

The authors are deeply indebted to V.A. Bordovitsyn, A. V. Borisov, and V. Ch. Zhukovskii for their discussion of the work.

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