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RADIOPHYSICS

SUPPRESSION OF OPPOSITE-WAVE GENERATION IN CYCLOTRON-RESONANCE MASERS WITH LONGITUDINAL PROFILING

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Opposite-wave generation in cyclotron-resonance masers with linear longitudinal variation of the waveguide radius and the leading magnetic field is considered. Conditions for the generation suppression are obtained.

Cyclotron-resonance masers (CRM) are promising as high-intensity generators and amplifiers of microwave radiation in the millimeter and submillimeter bands [1–5]. In comparison with their most extensively studied options, the gyrotrons, at a fixed magnetic field, they can use shorter waves owing to the Doppler frequency shift. Moreover, the CRMs require lower electron energies for their operation than ubitrons because the cyclotron oscillation pitch can be made smaller than the undulator period.

The first experimental investigations of the CRM generators [2] and amplifiers [3] have been mostly carried out in the so-called grazing synchronism mode where the longitudinal velocity of electrons coincides with the group velocity of the electromagnetic wave which is the lowest mode of a round waveguide H_{11}. This ensures the single-mode interaction, but necessitates the use of waveguides with low cross sections, with diameters on the order of the radiation wavelength.

It is clear that, at short wavelengths, this mode imposes hard-to-satisfy requirements both on the shaping and transportation of a helical relativistic electron beam and on the heat removal from the walls at high levels of intensity. Therefore, a more natural approach is to increase the waveguide diameter and, accordingly, to pass to higher modes. However, in this case, in addition to the beam's synchronism with the forward wave, also occurs a synchronism with lower-frequency opposite waves, whose excitation is undesirable. Therefore, in order to implement the efficient forward-wave CRMs, techniques should be developed for suppressing spurious generation at opposite waves. Among these are the use of waveguides with absorbing walls or waveguides consisting of short sections separated by absorbers [1, 4]. The section length must be shorter than the starting length for spurious mode generation.

The method proposed below is based on longitudinal variation of the waveguide radius [2] and the leading magnetic field, which allows one to vary the phase relations between the beam and the opposite wave and, in some modes, to suppress its generation. It is possible to select the CRM parameters so that longitudinal nonuniformity has no effect on forward wave amplification.

We describe the interaction of the helical relativistic electron beam with the opposite electromagnetic wave in the maser in the same way as in [5], neglecting the drift of the electron beam's leading center and small transverse components of a weakly nonuniform external magnetic field. Moreover, we assume that the wave frequency differs significantly from the mode's critical frequency, so that the excitation equation can be used in a single-wave approximation. Then the self-consistent system of equations for the motion of electrons and wave excitation has the form

$$\frac{du}{dz'} = \frac{1}{1+b(1-|u|^2)}\left\{F(u^*)^{s-1} - \frac{i}{s}u\left[\Delta - \mu(1-|u|^2)\right]\right\}, \tag{1}$$

$$\frac{dF}{dz'} = I_0 \frac{1}{2\pi} \int_0^{2\pi} \frac{u^s}{1+b(1-|u|^2)} d\vartheta_0, \tag{2}$$

where

$$u = \left[1 - \frac{2(1+h\beta_{z0})}{\beta_{\perp 0}^2} \frac{\gamma_0 - \gamma}{\gamma_0}\right]^{1/2} e^{i\vartheta/s}$$

is a complex variable describing the variations of energy (γ is the relativistic factor) and slow electron phase $\vartheta = s\theta - (\omega t + k_z z)$, θ is the phase of cyclotronic rotation, s is the harmonic number, ω, k_z are the frequency and longitudinal wave number of the wave; $\beta_{\perp 0}, \beta_{z0}$ are the initial transverse and longitudinal electron velocities normalized by the velocity of light, γ_0 is the initial relativistic factor; $z' = \omega z/(c\beta_0)$ is the longitudinal coordinate; $h = k_z c/\omega$ is a quantity inversely proportional to the wave's phase velocity; F is its normalized complex amplitude; the asterisk denotes complex conjugation.

The system of equations (1), (2) contains four parameters: b, μ, Δ, and I_0. The normalized interaction length is determined after solving the boundary value problem in the case of absolute instability under study.

The recoil parameter

$$b = \frac{h\beta_{\perp 0}^2}{2\beta_{z0}(1+h\beta_{z0})} \tag{3}$$

determines the variation of longitudinal velocity with the electron energy in accordance with the "autoresonance" integral of motion: $\gamma(1+\beta_z/h) = \text{const}$. The inertial grouping parameter

$$\mu = \frac{\beta_{\perp 0}^2(1-h^2)}{2(1+h\beta_{z0})} \tag{4}$$

characterizes the electrons' nonisochronism. The cyclotron resonance detuning has the form

$$\Delta = 1 + h\beta_{z0} - s\Omega_0/\omega, \tag{5}$$

where $\Omega_0 = eB_0/(m\gamma_0)$ is the cyclotron frequency of electrons.

For the case of a tubular electron beam (with current I_b and radius R_b) in a circular waveguide (with radius R and mode $H_{m,p}$), the current parameter I_0 is given by

$$I_0 = 2\frac{eI_b}{mc^3} \frac{1+h\beta_{z0}}{\gamma_0} \frac{\varkappa^2}{h} \left[\frac{1}{(s-1)!\,2^s}\left(\varkappa \frac{s\beta_{\perp 0}}{1+h\beta_{z0}}\right)^{s-1}\right]^2 \frac{J_{m\mp s}^2(k_\perp R_b)}{(\nu^2 - m^2)J_m^2(\nu)}, \tag{6}$$

where $k_\perp = \nu/R$ is the transversal wave number, ν is the mode eigenvalue; $\varkappa = k_\perp c/\omega$.

The boundary conditions for the system of equations (1), (2), corresponding to a single-velocity, unmodulated beam at the input and no opposite wave at the output, are given by

$$u(0) = e^{i\vartheta_0/s}, \quad 0 \le \vartheta_0 < 2\pi; \qquad F(L') = 0. \tag{7}$$

Let us employ a normalization that allows us to eliminate the current parameter I_0 from equations (1), (2),

$$\zeta' = \sqrt{I_0}\, z', \quad F' = \frac{F}{\sqrt{I_0}}, \quad \mu' = \frac{\mu}{\sqrt{I_0}}, \quad \Delta' = \frac{\Delta}{\sqrt{I_0}}; \tag{8}$$

the primes are henceforth omitted.

In terms of these variables, the energy conservation law has the form

$$|F|^2 - \frac{1}{2\pi}\int_0^{2\pi} |u|^2 d\vartheta_0 = \text{const}.$$

This allows us to express the equivalent efficiency $\eta_\perp$ in terms of the input wave amplitude $F_0 = F(0)$,

$$\eta_\perp = 1 - \frac{1}{2\pi}\int_0^{2\pi} |u(\zeta)|^2 d\vartheta_0 = |F_0|^2. \tag{9}$$

Then the electron efficiency of the interaction is

$$\eta = \frac{\beta_{\perp 0}^2}{2(1-1/\gamma_0)(1+h\beta_{z0})}\eta_\perp = \frac{\beta_{\perp 0}^2}{2(1-1/\gamma_0)(1+h\beta_{z0})}|F_0|^2. \tag{10}$$

For a linear variation of the magnetic field and the waveguide radius,

$$B = B_0(1+\delta Bz/L), \quad R = R_0 - z\tan\varphi,$$

the cyclotron resonance detuning also becomes a function of coordinate ζ with a certain effective profiling coefficient δ,

$$\Delta = \Delta_0 - \delta\zeta,$$
$$\delta = \frac{\beta_{z0}}{2\pi I_0}\frac{\lambda}{L}\left\{(1+h\beta_{z0})\delta B + \frac{\beta_{z0}\nu^2\lambda L}{2\pi k_{z0}R_0^3}\tan\varphi\right\}. \tag{11}$$

Formula (11) was derived under the assumption that the operating frequency different sufficiently from the critical value, $(k_{z0}L)^2 \gg 2\nu^2(L/R_0)^3\tan\varphi$.

Note that the interaction efficiency may significantly increase for $\delta > 0$ (a growing magnetic field, a narrowing waveguide) [5]. Then we consider the opposite case $\delta < 0$ corresponding to a decreasing magnetic field and an expanding waveguide.

We limit ourselves to the fundamental cyclotron resonance, $s = 1$. Isolating the dependence on the wave number h in (6),

$$I_0 = \frac{(1+h\beta_{z0})\varkappa^2}{h}I,$$
$$I = \frac{eI_b}{2mc^3\gamma_0}G, \quad G = \frac{J_{m\mp 1}^2(k_\perp R_b)}{(\nu^2-m^2)J_m^2(\nu)},$$

where I is the new beam current parameter, we represent the normalized inertial grouping parameter μ of (4), (8) as

$$\mu = \frac{\beta_{\perp 0}^2}{2\sqrt{I}}\frac{\sqrt{(1-h^2)h}}{(1+h\beta_{z0})^{3/2}}. \tag{12}$$

This demonstrates that $\mu = 0$ at $h = 0$, $h = 1$, and the function $\mu(h)$ has a single maximum at $h = \left(\sqrt{\beta_{z0}^2+3} - \beta_{z0}\right)/3$, which lies in the range from $h = 1/\sqrt{3}$ (at $\beta_{z0} = 0$) to $h = 1/3$ (at $\beta_{z0} = 1$).

At the fixed parameters b, μ, δ and the given input wave amplitude F_0, the system of equations (1), (2) was integrated for different values of the detuning Δ_0, until the output wave amplitude $F(\zeta_0)$ vanished to within a specified accuracy ($F(\zeta_0) < 10^{-2}F_0$). The corresponding values of Δ_0 and ζ_0 play the role of eigenvalues of the boundary value problem and determine the generation frequency and the interaction length. As follows from (9), this solution corresponds to the generation of an opposite wave with the equivalent efficiency $\eta_\perp = |F_0|^2$. The maximum efficiency is determined by the value F_{0m}, beginning with which (for $F_0 > F_{0m}$) no solution exists.

Our modeling showed that with increasing absolute value of the profiling coefficient the maximum efficiency decreases monotonically and vanishes at a certain critical value $\delta_c(\mu, b)$ (Fig. 1). This means suppression of opposite-wave generation for $|\delta| > |\delta_c(\mu, b)|$ ($\delta < 0$). Note that, as the wave number increases from 0 to 1, the recoil parameter b (3) grows monotonically from 0 to the maximum value $b = \beta_{\perp 0}^2/[2\beta_{z0}(1+\beta_{z0})]$, which does not exceed 0.1 in the practically important cases ($\beta_{\perp 0} \le 1/\gamma_0$, $\gamma_0 \ge 2$).

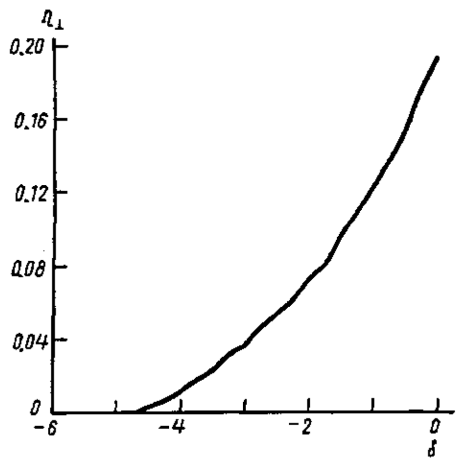

Fig. 1

Typical dependence of maximum equivalent efficiency on the profiling parameter δ; $b = 0$, $\mu = 3$.

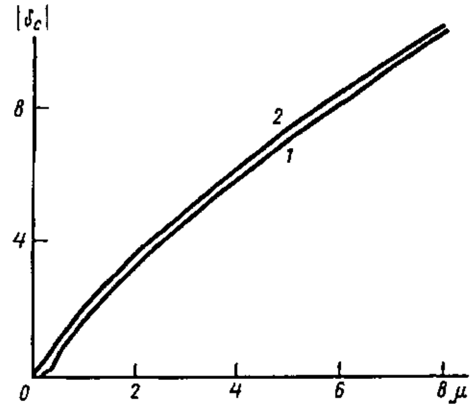

Fig. 2

Absolute value of critical profiling parameter $|\delta_c|$ as a function of inertial grouping parameter μ at $b = 0$ (*1*) and 0.1 (*2*).

Therefore, the critical profiling parameter δ_c was plotted as a function of the inertial grouping parameter μ at $b = 0$ and $b = 0.1$ (Fig. 2). These data allow us to determine the conditions for the waveguide aperture φ and the magnetic field variation δB, which are necessary for suppressing opposite-wave generation.

Let us first consider the case of a uniform magnetic field ($\delta B = 0$). In accordance with (11), (12), the expression for the angle can be represented as ($\tan\varphi < 0$)

$$|\tan\varphi_0| = \frac{|\delta_c(\mu, b)|}{\mu}\nu\sqrt{I}\frac{\beta_{\perp 0}^2}{2\beta_{z0}^2}\sqrt{\frac{h}{1 + h\beta_{z0}}}. \tag{13}$$

The quantity δ_c/μ is shown in Fig. 3 as a function of μ. Noting that $|\delta_c|/\mu \leq 2$, we obtain the following estimate:

$$|\tan\varphi_0| \leq \nu\sqrt{I}\frac{\beta_{\perp 0}^2}{\beta_{z0}^2}\sqrt{\frac{h}{1 + h\beta_{z0}}}. \tag{14}$$

We see that the maximum of the right-hand side is attained at $h = 1$. Thus, in order to suppress generation in the entire variation range of the normalized wave number, from 0 to 1, the waveguide angular aperture must satisfy the condition

$$|\tan\varphi_0| = \nu\sqrt{I}\frac{\beta_{\perp 0}^2}{\beta_{z0}^2\sqrt{1+\beta_{z0}}}. \tag{15}$$

For example, at $\beta_{\perp 0} = 0.5$, $\beta_{z0} = 0.71$, $\nu = 1.841$ (mode H_{11}), we obtain $|\tan\varphi_0| \cong 0.7\sqrt{I}$, which yields acceptable angle values for moderate currents (roughly, $I \leq 10^{-2}$). The results of calculations by formula (13) are compared with estimates (15) in Fig. 4. The noticeable difference for small wave numbers is due to the reduction of the recoil parameter and, in accordance with Fig. 3, the quantity $|\delta_c|/\mu$ in this region.

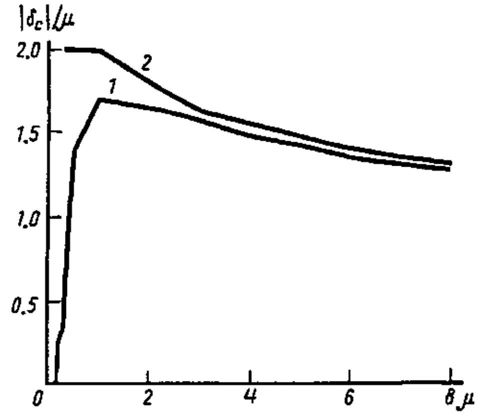

Fig. 3

Dependences of $|\delta_c|/\mu$ on inertial grouping parameter μ at $b = 0$ (*1*) and 0.1 (*2*).

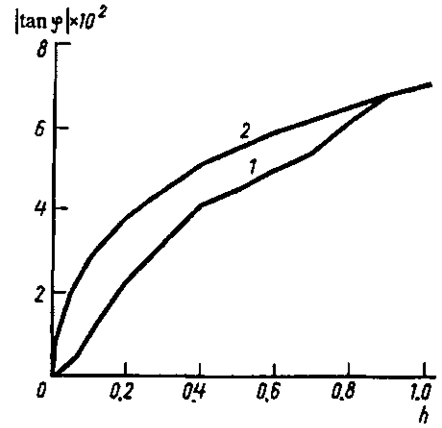

Fig. 4

Dependence of $|\tan\varphi|$ on normalized wave number h at $\beta_{\perp 0} = 0.5$, $\beta_{z0} = 0.71$, $I = 10^{-2}$: curve *1* — calculation by formula (13), curve *2* — estimate by formulas (15).

For the relative longitudinal variation of the waveguide radius, one can obtain the following relation:

$$\frac{\Delta R}{R_0} = \frac{L|\tan\varphi_0|}{R_0} = \frac{|\delta_c|}{\mu}\zeta_0 b.$$

The variation of the waveguide radius affects detuning of both the opposite and the forward waves, which depends on the coefficient δ_f that can be obtained from (11) by replacing the wave number and the wavelength. This variation is undesirable for efficient amplification of the forward wave, and can be compensated by choosing the appropriate magnetic field nonuniformity $\delta B \neq 0$, subject to the conditions

$$\delta = \delta_c = a_0 \delta B + d_0 \tan\varphi, \qquad \delta_f = 0 = a_f \delta B + d_f \tan\varphi, \tag{16}$$

where a_0, d_0 and a_f, d_f are the coefficients in (11) for the opposite and forward waves, respectively. Formulas (16) are a system of two linear equations by which we determine the required values of δB and $\tan\varphi$,

$$|\tan\varphi| = \frac{|\tan\varphi_0|}{(1+Q)}, \quad Q = \frac{h(1-h_f\beta_{z0})}{h_f(1+h\beta_{z0})},$$

$$h_f = \frac{(1+\beta_{z0}^2)h + 2\beta_{z0}}{(1+\beta_{z0}^2) + 2\beta_{z0}h},$$

$$\delta B = \beta_{z0}\frac{\varkappa^2(1-h_f\beta_{z0})}{h_f(1+h\beta_{z0})^2}\,\frac{|\delta_c|\zeta_0 b}{\mu(1+Q)},$$

where h_f is the normalized wave number of the forward wave.

For example, with the values of $\beta_{\perp 0}$, β_{z0}, and I that were used in Fig. 4, the required magnetic field reduction is small, amounting to a few percent.

Thus, the use of an expanding waveguide in the CRM allows one to suppress opposite-wave generation at moderate beam currents. With appropriate choice of the magnetic field variation, waveguide profiling does not affect forward-wave amplification.

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