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WHY DOES THE MEISSNER EFFECT ARISE?

N. B. Brandt, G. A. Mironova, and V. V. Rzhevskii

A possible experiment that would allow one to determine the necessary and sufficient conditions for the Meissner effect to arise in classical superconductors is described.

The Meissner effect discovered in 1933 [1] is known to refer to massive simply connected superconductors. When such a superconductor is cooled in a static magnetic field whose strength is lower than the critical field strength (H_c and H_{c1} for superconductors of the first and second kinds, respectively), the transition to the superconducting state results in that the field is forced out of the bulk of the sample and a superconduction current begins to flow in the surface layer of thickness λ (λ is the depth of magnetic field penetration into the superconductor). This current balances the applied field in such a way that the magnetic flux density $\mathbf{B}$ within the superconductor is always zero. Note that the Meissner effect cannot be explained in terms of classical Maxwell electrodynamics, according to which conduction currents are created by applied forces (by chemical current sources, induction electromotive forces, etc.). No applied force is involved in the Meissner effect. Strictly, the current that arises should not be identified with conduction current; to stress this point, it should be called the Meissner current. Formally, the Meissner current appearance can be treated as a particular case of the self-organization phenomenon, although this can hardly make things clearer.

Why does the Meissner effect arise? More exactly, what properties should an ensemble of particles possess for the Meissner effect to arise, and what is the nature of the state in which induction within a system of particles turns zero? These questions have no generally accepted answers. There are three different approaches to the explanation of the Meissner effect.

1. P. de Gennes [2] suggested that a free energy of a superconductor be written in the form

$$F = \int\limits_V F_s dV + E_J + E_M, \tag{1}$$

where V is the sample volume; F_s is the condensation energy density of superconducting electrons; $E_J = \frac{1}{2}\int\limits_V n_s \cdot mv_s^2\, dV$ is the kinetic energy of superconduction currents $\mathbf{j}_s(\mathbf{r}) = n_s e\mathbf{v}_s$; n_s is the concentration of superconducting electrons; e is their charge; $v_s(\mathbf{r})$ is the drift velocity of superconducting electrons; and $E_M = \int\limits_V \frac{h^2}{8\pi} dV$ is the magnetic energy related to the $\mathbf{h}(\mathbf{r})$ macroscopic magnetic field within the sample created by the $\mathbf{j}_s(\mathbf{r})$ currents. The minimization of (1) with respect to the $\mathbf{h}(\mathbf{r})$ field distribution leads to the London equation

$$\mathbf{h} + \lambda^2{}_L \,\mathrm{curl}\,\mathrm{curl}\,\mathbf{h} = 0, \quad \lambda_L = \left(\frac{mc^2}{4\pi n_s e^2}\right)^{1/2}, \tag{2}$$

which formally describes the Meissner effect. Using the London calibration for the vector potential $\mathbf{A}(\mathbf{r})$, ($\operatorname{curl}\mathbf{A} = \mathbf{h}$): $\operatorname{div}\mathbf{A} = 0$, $\mathbf{A}_n = 0$ on the sample surface, where $\mathbf{A}_n$ is the component of $\mathbf{A}$ normal to the sample surface, we obtain

$$\mathbf{j} = -\frac{n_s e^2}{mc}\mathbf{A}. \tag{3}$$

The conclusion is drawn that the state of a superconductor with zero induction in the bulk and with screening Meissner current (3) corresponds to its free energy minimum.

One interesting circumstance should be mentioned. In the suggested derivation of (2), it is nowhere assumed that the system of superconducting electrons is coherent. The expression for the kinetic energy of current in (1) has exactly the same form for a system of charged Fermi particles, which, for some reason, create a nondissipative current; that is, have a constant $\mathbf{v}_s$ velocity component.

2. Feynman [3] explained the Meissner effect with the use of the general quantum-mechanical equation for current density

$$\mathbf{j} = \frac{1}{2}\left\{\left[\frac{\hat{p} - q\mathbf{A}/c}{\mu}\psi\right]^* \psi + \left[\frac{\hat{p} - q\mathbf{A}/c}{\mu}\psi\right]\psi^*\right\}, \tag{4}$$

where $\hat{p} = \frac{\hbar}{i}\nabla$ is the particle momentum operator, μ and q are the mass and the charge, and $\psi(\mathbf{r}, t)$ is the wave function of the particle. The square of the wave function magnitude gives the probability for the particle to occur at point $\mathbf{r}$ at time t. For charged Bose particles, the $\psi\psi^*$ product can be treated as the density $\rho(\mathbf{r})$ of the charge of particles at point $\mathbf{r}$ and time t. Assuming that $\psi(\mathbf{r}) = \sqrt{\rho(\mathbf{r})/2}\exp i\theta(\mathbf{r})$, the expression for stationary current can be rewritten as

$$\mathbf{j} = \frac{\hbar}{2\mu}\left(\nabla\theta - \frac{q}{\hbar c}\mathbf{A}\right)\rho(\mathbf{r}). \tag{5}$$

For a simply connected superconductor, the divergence of current should be zero. Assuming $\operatorname{div}\mathbf{A} = 0$, we find that $\nabla^2\theta = 0$. For a uniform three-dimensional material, $\nabla^2\theta$ is only zero if $\theta = \text{const}$. Substituting (5) into (4) yields the London equation $\mathbf{j} = -\rho(\mathbf{r})\frac{q}{2\mu c}\mathbf{A}$. Taking into account that $q = 2e$, $\mu = 2m$, and, in a uniform superconductor, $\rho(\mathbf{r}) = qn_s$, we find

$$\mathbf{j} = -\frac{n_s e^2}{mc}\mathbf{A} = -\frac{c}{4\pi\lambda_L^2}\mathbf{A}.$$

It follows that, according to Feynman, the Meissner effect is a consequence of coherence of a nondissipative Bose system of charged particles. In this approach, thermodynamics is excluded from consideration.

3. The London equation also follows from the second equation of Ginzburg–Landau theory [4],

$$\mathbf{j}_s(\mathbf{r}) = \frac{c|\phi(\mathbf{r})|^2}{4\pi\lambda_L^2}\left(\frac{\Phi_0}{2\pi}\nabla\theta - \mathbf{A}\right),$$

where $\phi(\mathbf{r}) = |\phi|\exp i\theta(\mathbf{r}) = \psi(\mathbf{r})/\psi_0$ is the dimensionless wave function, ψ_0 is the equilibrium ordering parameter, $|\psi(\mathbf{r})|^2 = n_s/2$, and Φ_0 is the magnetic flux quantum. Renormalizing the $\mathbf{A} = \mathbf{A}' + \Phi_0/2\pi\nabla\theta$ vector potential, we find

$$\mathbf{j}_s(\mathbf{r}) = -\frac{c|\phi(\mathbf{r})|^2}{4\pi\lambda_L^2}\mathbf{A}.$$

From this point of view, the Meissner effect corresponds to a Gibbs energy minimum of a nondissipative coherent system of charged Bose particles.

The question arises: Can we experimentally determine whether free energy minimization accompanying the transition to the Meissner state is a necessary condition for arising of the Meissner effect? So far as we know, this issue has not been discussed in the literature. Below, we describe an experiment that allows one to determine which of the conditions specified above are necessary and sufficient for the Meissner effect to arise.

Consider a long thin cylindrical sample of radius r_0 in a constant magnetic field $\mathbf{H_0}$ parallel to its axis. Let this field be lower than the H_c or H_{c1} critical value. Compare the free energy values for the state when the magnetic field penetrates the sample and the Meissner current is absent and the state when the applied field $\mathbf{H_0}$ is screened by Meissner current.

In the former state, the free energy per unit sample length is $F_1 = \frac{H_0^2}{8\pi}\pi r_0{}^2$. In the latter state,

$$F_2 = \int\limits_V \left(\frac{B(\mathbf{r})^2}{8\pi} + \frac{n_s m v_s(\mathbf{r})^2}{2} \right) dV.$$

The energy difference $\Delta F = F_1 - F_2$ for a cylindrical sample has the form

$$\Delta F = \frac{r_0^2}{2}\left(\frac{2\pi\lambda}{c}\right)^2 C_0^2 I_0(r_0/\lambda)\left\{I_0(r_0/\lambda) - \frac{2\lambda}{r_0}I_1(r_0/\lambda)\right\}, \qquad (6)$$

where λ is the magnetic field penetration depth measured experimentally. If $r_0 \gg \lambda$, then $F_2 \ll F_1$, and, according to de Gennes, the Meissner state is energetically favorable. A decrease in r_0 causes ΔF to decrease. The question therefore arises if ΔF can be made negative by decreasing r_0; that is, if the Meissner state can have a higher free energy (at the expense of current energy) and be energetically unfavorable.

An analysis of equation (6) for the energy difference shows that, no matter what the sample radius r_0, the transition of a superconductor to the Meissner state remains energetically favorable.

It appears that the question of whether or not the requirement of energy minimization is the necessary condition of the Meissner effect existence can be answered as follows.

We compared above the current and magnetic field energies under given temperature T, penetration depth λ, and applied field H conditions when the sample radius r_0 was varied. Now consider free energy variations at the given applied field H and the sample radius r_0 and variable $\lambda(T)$. Let the sample have the radius $r_0 \gg \lambda(0)$. According to the dependence of λ on T shown in Fig. 1 (curve *2*), switching on a magnetic field or placing the superconductor into a magnetic field at $T < T_c$ induces screening the superconduction current $\mathbf{j}_s$ flowing in the superconductor in the surface layer of thickness $\lambda(T)$, which corresponds to the given temperature T.

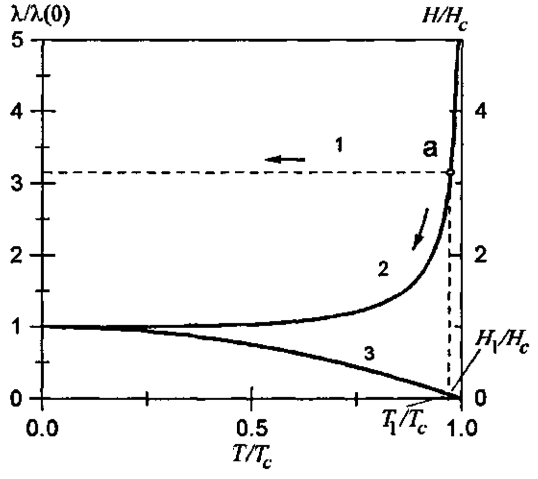

Fig. 1

Characteristic temperature dependences $\lambda(T)$ (curves *1* and *2*) and $H_c(T)$ (curve *3*). Arrows indicate how λ changes during cooling in magnetic field H_1: dashed line *1* (case 1) and solid line *2* (case 2).

Consider the temperature dependence of the depth of magnetic field penetration into the superconductor under the Meissner effect conditions when the superconductor is cooled in a constant magnetic field

$H_1 \ll H_c(0)$. At T_1, magnetic field is forced out of the superconductor bulk. Cooling generates the Meissner current flowing in the surface layer $\lambda(T_1)$ thick. The $\lambda(T_1)$ value ($\lambda(T_1) \gg r_0$) is determined by point "a" of the $\lambda(T)$ curve. The effect of further cooling may be twofold.

1. The depth λ of magnetic field penetration may turn "frozen" during cooling below T_1 given by

$$T_1 = T_c - \frac{H_1}{(dH_c/dT)_{T_c}}, \quad (7)$$

where H_1 is the field in which the sample is cooled. In Figs. 1 and 2, this situation is shown by dashed lines.

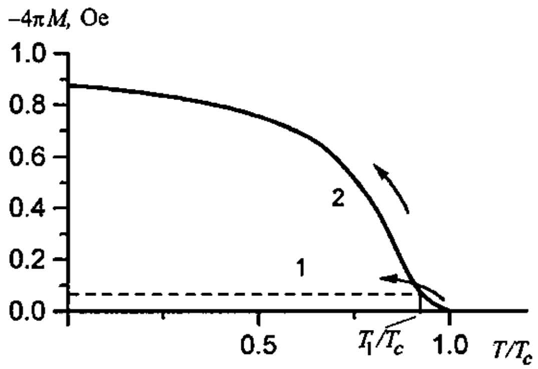

Fig. 2

Temperature dependence of the magnetization of a cylindrical sample during cooling: dashed line *1* (case 1) and solid line *2* (case 2).

2. Irrespective of the applied magnetic field value, the penetration depth λ decreases with cooling following curve *2* in Fig. 1. The magnetic moment of the sample then changes during cooling as is shown by curve *2* in Fig. 2.

In case 1, λ is determined by the superconducting electron concentration $n_s(T_1)$ corresponding to the temperature T_1 at which the $H_1 = \text{const}$ straight line parallel to the abscissa axis intersects the $H_c(T)$ curve of critical fields; this concentration determines the Meissner screening superconduction current value. The radial distribution and the Meissner current value in the sample do not change with cooling, although the $n_s(T)$ equilibrium concentration of superconducting electrons increases.

In case 2, some part of superconducting electrons, $n_s(T) - n_s(T_1)$, become involved in the Meissner current as temperature decreases (although the mechanism of such involvement remains to be discovered). As a result, the density of superconducting electrons creating the screening current increases, and $\lambda(T)$ decreases to λ_0 as $T \to 0$.

Now compare the free energies of the sample at $T < T_c/2$ for cases 1 and 2. In case 1 ($\lambda(T_1) > r_0$ is frozen), magnetic field exists throughout the sample volume; its value is only insignificantly lower than H_1. Simultaneously, "weak" Meissner current flows in the whole sample volume. The free energy per unit length is the sum of the magnetic field and current energies.

In case 2, $\lambda = \lambda_0$ at $T < T_c/2$, and magnetic field H is displaced from the bulk and only penetrates to the depth λ_0 from the sample surface. The free energy is then the sum of the energies of "strong" Meissner current and magnetic field created by this current in the layer λ. Calculations show that, as would be expected, the situation with magnetic field forced out of the bulk is more favorable energetically.

It follows that obtaining experimental evidence in favor of case 2 would mean that the displacement of magnetic field from the bulk of the sample minimizes the free energy, and the transition to the Meissner phase is the transition from a state with a higher free energy to a state with a lower energy. The Meissner current always ensures maximal lowering of magnetic field within a sample (also, when temperature changes the superconducting state being already attained). That is, the requirement of minimum free energy is the necessary condition for the Meissner effect to arise.

The behavior corresponding to case 1 would mean that the superconducting state with a higher free energy value, that is, with a very weak Meissner effect can exist. To put it differently, such a behavior would be evidence for the existence of a superconducting state in which the majority of superconducting electrons does not participate in Meissner current.

To summarize, if the penetration depth gets "frozen" during cooling, then the predominant role is played by coherence of the system of superconducting electrons, because the Meissner effect then arises when the energy of the sample is not minimum.

The suggested experiment can most conveniently be conducted with superconductors of the second kind characterized by large depths λ of magnetic field penetration. Measurements should be performed under strictly static conditions with the use of constant magnetic field $H_0 < H_{c1}$ to exclude generation of screening currents on the surface of the sample. For instance, a sample can be suspended on a thin elastic thread at the middle in a uniform magnetic field $H_0 < H_{c1}$ oriented normally to the thread, and sample rotation during cooling below T_c can be measured. The axis of the sample should lie in the horizontal plane and make a small angle with the direction of field $\mathbf{H_0}$.

In this work, we considered various aspects of the physical nature of the Meissner effect such as its nondissipative character, the condition of a free energy minimum, and the condition of coherence of the system of superconducting electrons. Clearly, the absence of dissipation is the key condition of the existence of a stationary Meissner effect. The question of which of the rest two conditions is necessary, or all three conditions are such remains open. We suggested an experiment that would allow us to determine whether or not the Meissner effect arises because the sample undergoes the transition to a state with a minimum free energy in an applied magnetic field.

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