Moscow University
Physics Bulletin
Vol. 55, No. 2, pp. 92–94, 2000

Vestnik Moskovskogo
Universiteta. Fizika
UDC 517.9

STUDYING PHYSICAL PROCESSES IN NONPERIODIC HIGHLY HETEROGENEOUS MEDIA

G. N. Medvedev and B. I. Morgunov

Mathematical models of electrodynamical and thermomechanical processes in highly heterogeneous nonperiodic media have been considered. An iterative procedure for determining the asymptotic expansion coefficients of solutions of electrodynamics and thermoelasticity equations has been described.

In order to determine the coefficients of asymptotic transformations of equations of heat conduction, elasticity and viscoelasticity by the asymptotic method suggested in [1], a procedure based on the periodicity cell numerical integration has been used.

The methods for determining the coefficients of asymptotic transformations of electrodynamics and thermoelasticity equations in highly heterogeneous periodic media based on the iterative procedure were discussed in [2, 3].

In the present work the iterative procedure for determining the coefficients of asymptotic expansions is also being realized for the case of heterogeneous nonperiodic media with the structure analogous to that considered in [2, 3].

We determine the coefficients of asymptotic transformations of electrodynamics and thermoelasticity equations for the case of a heterogeneous medium, whose properties (electrical, magnetic, mechanical and thermodynamic) vary rapidly in two directions, $\xi_\alpha = x_\alpha/\varepsilon$, $\alpha = 1, 2$. All physical characteristics of the medium have a structure similar to that considered in [2, 3], for instance, $\tau(\xi) = \tau_0 + \tau_1(\xi)$ for magnetic permeability.

The equations for the coefficients of asymptotic expansions of solutions of mathematical physics fundamental equations are obtained by the procedure described in [4] and have the form discussed in [2, 3].

For instance, in order to determine the magnetic field components, we need to find the solutions of equations in partial derivatives of the form

$$\frac{\partial}{\partial \xi_\alpha}\left[\tau(\xi)\left(\frac{\partial \Phi_\beta}{\partial \xi_\alpha} + \delta_{\alpha\beta}\right)\right] = 0, \quad \beta = 1, 2. \tag{1}$$

To solve equations (1), the iterative procedure similar to that used in [3] should be applied

$$\tau_0 \frac{\partial^2 \Phi_\beta^{n+1}}{\partial \xi_\alpha \partial \xi_\alpha} = -\frac{\partial}{\partial \xi_\alpha}\left[\tau_1(\xi)\left(\frac{\partial \Phi_\beta^n}{\partial \xi_\alpha} + \delta_{\alpha\beta}\right)\right] \equiv -F_\beta^n(\xi),$$
$$\Phi_\beta^0 \equiv 0, \quad \beta = 1, 2, \quad n = 0, 1, 2 \dots . \tag{2}$$

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Representing the functions involved in [4] as the Fourier integrals

$$\Phi^n_\beta(\xi) = \frac{1}{2\pi}\int \varphi^n_\beta(\eta)\, e^{i\xi\eta}\, d\eta,$$
$$\tau_1(\xi) = \frac{1}{2\pi}\int T(\eta)\, e^{i\xi\eta}\, d\eta, \tag{3}$$
$$F^n_\beta(\xi) = \frac{1}{2\pi}\int f^n_\beta(\eta)\, e^{i\xi\eta}\, d\eta, \quad \beta = 1,2,$$

(here and in what follows $\int = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}$), we find that the Fourier images $\varphi^n_\beta(\eta)$ are defined from the equations

$$\tau_0\eta_\alpha\eta_\alpha\varphi^{n+1}_\beta(\eta) = f^n_\beta(\eta)$$

or, in more detail,

$$\varphi^{n+1}_\beta(\eta) = \frac{1}{\tau\eta_\alpha\eta_\alpha}\left(i\eta_\alpha T(\eta)\delta_{\alpha\beta} - \frac{\eta_\alpha}{2\pi}\int T(\eta-\zeta)\zeta_\alpha\varphi^n_\beta(\zeta)\, d\zeta\right).$$

Then from the first formula of (3) we find $\Phi^n_\beta(\xi)$.

There are similarly solved the equations of type (1) with physical characteristics other than $\tau(\xi)$ that have the same structure for determining the electric field components and also the temperature and transverse displacements in thermomechanical problems.

In order to determine the functions entering the expressions for the displacement vector longitudinal components, we have to solve a system of equations of the form

$$\frac{\partial}{\partial\xi_1}\left(\nu + \nu\frac{\partial U}{\partial\xi_1} + \lambda\frac{\partial V}{\partial\xi_2}\right) + \frac{\partial}{\partial\xi_2}\left(\mu\frac{\partial U}{\partial\xi_2} + \mu\frac{\partial V}{\partial\xi_1}\right) = 0,$$
$$\frac{\partial}{\partial\xi_1}\left(\mu\frac{\partial V}{\partial\xi_1} + \mu\frac{\partial U}{\partial\xi_2}\right) + \frac{\partial}{\partial\xi_2}\left(\lambda + \lambda\frac{\partial U}{\partial\xi_1} + \nu\frac{\partial V}{\partial\xi_2}\right) = 0.$$

The procedure

$$\nu_0\frac{\partial^2 U^{n+1}}{\partial\xi_1^2} + (\nu_0 - \mu_0)\frac{\partial^2 V^{n+1}}{\partial\xi_1\partial\xi_2} + \mu_0\frac{\partial^2 U^{n+1}}{\partial\xi_2^2} = -F^1(U^n, V^n),$$
$$\mu_0\frac{\partial^2 V^{n+1}}{\partial\xi_1^2} + (\nu_0 - \mu_0)\frac{\partial^2 U^{n+1}}{\partial\xi_1\partial\xi_2} + \nu_0\frac{\partial^2 V^{n+1}}{\partial\xi_2^2} = -F^2(U^n, V^n), \quad n = 0, 1, \dots, \tag{4}$$

is the generalization of the iterative procedure (2), where

$$F^1(U^n, V^n) = \frac{\partial}{\partial\xi_1}\left(\nu_1 + \nu_1\frac{\partial U^n}{\partial\xi_1} + \lambda_1\frac{\partial V^n}{\partial\xi_2}\right) + \frac{\partial}{\partial\xi_2}\left(\mu_1\frac{\partial U^n}{\partial\xi_2} + \mu_1\frac{\partial V^n}{\partial\xi_1}\right),$$
$$F^2(U^n, V^n) = \frac{\partial}{\partial\xi_1}\left(\mu_1\frac{\partial V^n}{\partial\xi_1} + \mu_1\frac{\partial U^n}{\partial\xi_2}\right) + \frac{\partial}{\partial\xi_2}\left(\lambda_1 + \lambda_1\frac{\partial U^n}{\partial\xi_1} + \nu_1\frac{\partial V^n}{\partial\xi_2}\right).$$

We represent the functions involved in (4) as the Fourier integrals

$$U^n(\xi) = \frac{1}{2\pi}\int u^n(\eta)\, e^{i\xi\eta}\, d\eta, \quad V^n(\xi) = \frac{1}{2\pi}\int v^n(\eta)\, e^{i\xi\eta}\, d\eta,$$
$$\lambda_1(\xi) = \frac{1}{2\pi}\int \Lambda(\eta)\, e^{i\xi\eta}\, d\eta, \quad \mu_1(\xi) = \frac{1}{2\pi}\int M(\eta)\, e^{i\xi\eta}\, d\eta, \tag{5}$$
$$\nu_1(\xi) = \frac{1}{2\pi}\int N(\eta)\, e^{i\xi\eta}\, d\eta, \quad F^n_\beta(\xi) = \frac{1}{2\pi}\int f^n_\beta(\eta)\, e^{i\xi\eta}\, d\eta,$$

where

$$f_1^n(\eta) = i\eta_1 \left(N + \{N; i\eta_1 u^n\} + \{\Lambda; i\eta_2 v^n\}\right) + i\eta_2 \left(\{M; i\eta_2 u^n\} + \{M; i\eta_1 v^n\}\right),$$

$$f_2^n(\eta) = i\eta_1 \left(\{M; i\eta_1 v^n\} + \{M; i\eta_2 u^n\}\right) + i\eta_2 \left(\Lambda + \{\Lambda; i\eta_1 u^n\} + \{N; i\eta_2 v^n\}\right),$$

$$\{F(\eta); G(\eta)\} = \frac{1}{2\pi} \int F(\zeta) G(\eta - \zeta) d\zeta \quad \text{(convolution)}.$$

Substituting the indicated transformations into iterative equations (4), we find that the Fourier images u^{n+1}, v^{n+1} are found from the system of equations

$$(\nu_0 \eta_1^2 + \mu_0 \eta_2^2) u^{n+1} + (\nu_0 - \mu_0) \eta_1 \eta_2 v^{n+1} = f_1^n(\eta),$$

$$(\nu_0 - \mu_0) \eta_1 \eta_2 u^{n+1} + (\mu_0 \eta_1^2 + \nu_0 \eta_2^2) v^{n+1} = f_2^n(\eta)$$

with the nonzero determinant $\mu_0 \nu_0 (\eta_1^2 + \eta_2^2)$.

Then we find the functions $U^{n+1}(\xi)$, $V^{n+1}(\xi)$ from formulas (5).

The physical processes in three-dimensional finely divided nonperiodic media are studied in a similar way.

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22 October 1999 Department of Mathematics