ISSN 0027-1349, Moscow University Physics Bulletin, 2007, Vol. 62, No. 4, pp. 248–254. © Allerton Press, Inc., 2007.
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Deviation of the Earth's Crust and Upper Mantle from Isostatic Equilibrium

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Received May 31, 2006

Abstract—The isostatically unbalanced crust and upper mantle structures conforming to the external gravitational field and minimizing deviation of the internal gravitational field from the isostatic equilibrium field, are determined. A dipole-type distribution of these structures associated with the motion of anomalous masses toward the potential energy minimum is revealed. On the basis of seismic data, it is inferred that the compensation of isostatically unbalanced masses is possible at the core–mantle boundary.

DOI: 10.3103/S002713490704011X

INTRODUCTION

The global tectonics of the Earth, including both horizontal and vertical movements of the crust, is known to arise from an unbalanced state of the interior layers of the Earth. In the external gravitational field, a hydrostatic unbalance manifests itself as the deviation of the measured field from the field of a hydrostatically balanced rotating spheroid. Isostatic unbalance is defined as the deviation from the field of the isostatically balanced Earth.

Our studies of the isostatic equilibrium conditions for the crust and mantle [1, 2] made it possible to determine the contributions from the isostatically balanced masses of the relief, from the density jump at the Mohorovicic discontinuity (M), and from anomalous (inconsistent with the isostatic equilibrium condition) masses of the crust and upper mantle (averaged over five-degree surface elements) into the external and internal gravitational fields of the Earth [3]. These investigations have shown that significant anomalies appearing in the internal gravitational field of the isostatically balanced crust and mantle may lead to considerable gravitational and dynamic instabilities and, consequently, disturb isostatic equilibrium. Moreover, the external gravitational field was found to differ from the isostatically balanced field of the Earth, thus testifying to the disturbance of isostatic equilibrium.

The aim of this work is to find such isostatically unbalanced structures in the crust and mantle that comply with the external gravitational field and minimize the difference between the internal gravitational field and the isostatically balanced field of the Earth (in accordance with the tendency of any closed system (the Earth in our case) toward equilibrium).

ISOSTATICALLY UNBALANCED STRUCTURE THEORY AND COMPUTATION TECHNIQUE

The difference between the external gravitational field and the isostatically balanced field of the Earth can be represented as a series expansion of the isostatic anomalies of the force of gravity in terms of spherical functions:

$$\Delta g(r, \varphi, \lambda) = \frac{fM_0}{r^2}\sum_{n=1}^{N}(n+1)\left(\frac{a}{r}\right)^n \times \sum_{m=0}^{n}(\Delta C_{nm}\cos m\lambda + \Delta D_{nm}\sin m\lambda)\bar{P}_{nm}(\sin\varphi), \quad (1)$$

where M_0 and a are the Earth's mass and average equatorial radius, respectively; $\bar{P}_{nm}$ are the associated Legendre polynomials normalized according to the Kaula rule; and $\begin{Bmatrix}\Delta C_{nm}\\ \Delta D_{mn}\end{Bmatrix} = \begin{Bmatrix}C_{nm} - C_{nm}^{(0)} - C_{nm}^{(s)}\\ D_{nm} - D_{nm}^{(s)}\end{Bmatrix}$, C_{nm} and D_{nm} are the Stokes coefficients for the real Earth, $C_{nm}^{(0)}$ are the Stokes coefficients for the hydrostatically balanced Earth, and $C_{nm}^{(s)}$ and $D_{nm}^{(s)}$ are the contributions of the isostatically balanced anomalous masses of the crust and upper mantle into the Stokes coefficients (s = 1–4, where unity corresponds to the masses of the relief, two refers to the deviations of M from the equilibrium spheroid surface, and three and four correspond to anomalous masses distributed in the crust and upper mantle).

In view of the incorrectness of the problem, the distribution of sources of isostatic anomalies has been ambiguously determined. We have chosen a criterion for minimal deviation of the internal gravitational field in the Earth's crust and mantle from the hydrostatically equilibrium field (in line with the tendency of masses toward the equilibrium state). The sought for distribution of the isostatic anomalies was represented as a sum of L simple layers with continuous density spread on ellipsoids with radii R_l similar to the Earth's reference ellipsoid with an average radius of $R_0 = 6371$ km.

As a result of numerical computations, the optimum solution was found to be as follows. The number of layers is $L = 5$: $l = 1$ corresponds to the upper crust layers adjacent to the ground surface ($\bar{R}_1 = R_0 - 5$ km), and $l = 2$ corresponds to the middle crust layers; the third layer lies in the layers of the lower crust and abuts M ($\bar{R}_3 = \bar{R}_M + 5$ km, $\bar{R}_M = R_0 - 22$ km); the fourth layer belongs to the uppermost mantle layer adjacent to M ($\bar{R}_4 = \bar{R}_M - 5$ km); and the fifth layer is situated in the upper mantle beneath the average isostatic compensation depth ($\bar{R}_5 = R_0 - \bar{D}_{\text{comp}} - 10$ km, $\bar{D}_{\text{comp}} = 60$ km). The density distribution within each layer was determined as an expansion in terms of spherical functions [4]:

$$\delta m_l(\varphi, \lambda) = \sum_{n=1}^{N}\sum_{m=0}^{n}(a_{nm}^{(l)}\cos m\lambda + b_{nm}^{(l)}\sin m\lambda)\bar{P}_{mn}(\sin\varphi), \tag{2}$$

where $N = 36$, $\begin{Bmatrix} a_{nm}^{(l)} \\ b_{nm}^{l} \end{Bmatrix} = \frac{2n+1}{3}\left(\frac{R_0}{\bar{R}_l}\right)^2\left(\frac{a}{\bar{R}_l}\right)^n \bar{\sigma} R_0 \begin{Bmatrix} \Delta C_{nm}^{(l)} \\ \Delta D_{nm}^{(l)} \end{Bmatrix}$, $\bar{\sigma}$ is the average density of the Earth, and $l = 1$–5.

For each layer, coefficients $\Delta C_{nm}^{(l)}$ and $\Delta D_{nm}^{(l)}$ were chosen from set $\{\Delta C_{nm}^{(l)}, \Delta D_{nm}^{(l)}\}$ so as to satisfy the criterion for the minimal anomalies in the internal gravitational field of the crust and upper mantle and to ensure proportionality to the corresponding internal gravitational field:

$\begin{Bmatrix} \Delta C_{nm}^{(1,4)} \\ \Delta D_{nm}^{(1,4)} \end{Bmatrix}\begin{Bmatrix} \Delta G_{nm}^{(1,4)} \\ \Delta F_{nm}^{(1,4)} \end{Bmatrix} > 0$ for the upper crust and the uppermost mantle,

$\begin{Bmatrix} \Delta C_{nm}^{(3,5)} \\ \Delta D_{nm}^{(3,5)} \end{Bmatrix}\begin{Bmatrix} \Delta G_{nm}^{(3,4)} \\ \Delta F_{nm}^{(3,4)} \end{Bmatrix} < 0$ for the lower crust and the upper mantle beneath the average isostatic compensation depth, and

$$\begin{Bmatrix} \Delta C_{nm}^{(2)} \\ \Delta D_{nm}^{(2)} \end{Bmatrix}\begin{Bmatrix} \Delta G_{nm}^{(1)} \\ \Delta F_{nm}^{(1)} \end{Bmatrix} < 0 \text{ and } \begin{Bmatrix} \Delta C_{nm}^{(2)} \\ \Delta D_{nm}^{(2)} \end{Bmatrix}\begin{Bmatrix} \Delta G_{nm}^{(3)} \\ \Delta F_{nm}^{(3)} \end{Bmatrix} > 0$$

for the middle crust, where $\{\Delta D_{nm}, \Delta F_{nm}\}$ are the coefficients in the expansion of the anomalies in the internal gravitational field of the upper and lower crust ($l = 1$ and 3) and of the upper mantle ($l = 4$) [3].

The total contribution of the obtained distributions to the internal gravitational field of the central crust ($\delta g^{(2)}$) and the upper mantle ($\delta g^{(4)}$) was computed in a linear approximation [4],

$$\delta g^{(2)} = 4\pi f\sum_{n=1}^{N}\sum_{m=0}^{n}(g_{nm}^{(2)}\cos m\lambda + f_{mn}^{(2)}\sin m\lambda)\bar{P}_{nm}(\sin\varphi), \tag{3}$$

where $\begin{Bmatrix} g_{nm}^{(2)} \\ f_{mn}^{(2)} \end{Bmatrix} = \frac{n+1}{2n+1}\left(\frac{R_3}{r}\right)^{n+2}\begin{Bmatrix} a_{nm}^{(3)} \\ b_{nm}^{(3)} \end{Bmatrix} - \frac{n}{2n+1}\left(\frac{r}{R_1}\right)^{n-1}\begin{Bmatrix} a_{nm}^{(1)} \\ b_{nm}^{(1)} \end{Bmatrix}$.

Formulas for the upper mantle are obtained from the above by replacing superscripts 2, 3, and 1 by 4, 5, and 4, respectively. Formulas for the upper and lower crust differ from (3) in the contribution from the anomalous layer of the middle crust:

$$\begin{Bmatrix} g_{nm}^{(1)} \\ f_{mn}^{(1)} \end{Bmatrix} = \begin{Bmatrix} g_{nm}^{(2)} \\ f_{mn}^{(2)} \end{Bmatrix} + \frac{n+1}{2n+1}\left(\frac{R_2}{r}\right)^{n+2}\begin{Bmatrix} a_{nm}^{(2)} \\ b_{mn}^{(2)} \end{Bmatrix},$$

$$\begin{Bmatrix} g_{nm}^{(3)} \\ f_{mn}^{(3)} \end{Bmatrix} = \begin{Bmatrix} g_{nm}^{(2)} \\ f_{mn}^{(2)} \end{Bmatrix} - \frac{n+1}{2n+1}\left(\frac{r}{R_2}\right)^{n-1}\begin{Bmatrix} a_{nm}^{(2)} \\ b_{mn}^{(2)} \end{Bmatrix}.$$

Within a five-degree surface element, the condition of minimal internal field anomalies $\Delta g^{(l)}$ is reduced to the requirement $(\delta g^{(l)}/\Delta g^{(l)})_i < 0$, where $l = 1$–4, $i = 1$–2592. Since the anomalies in the external and internal gravitational fields have different distributions of harmonic amplitudes in n and m, a minor additional correction to the density distributions was required for those five-degree elements where the condition of minimal field anomalies failed. Moreover, correction was performed for several surface elements where the ratio $|\delta g/\Delta g|$ exceeded $|\delta g/\max|\Delta g|| \approx 0.1$.

RESULTS

Figures 1–5 show the obtained lateral distributions of the isostatically unbalanced masses of the crust and upper mantle, which are the source of isostatic anomalies (1) in the external field.

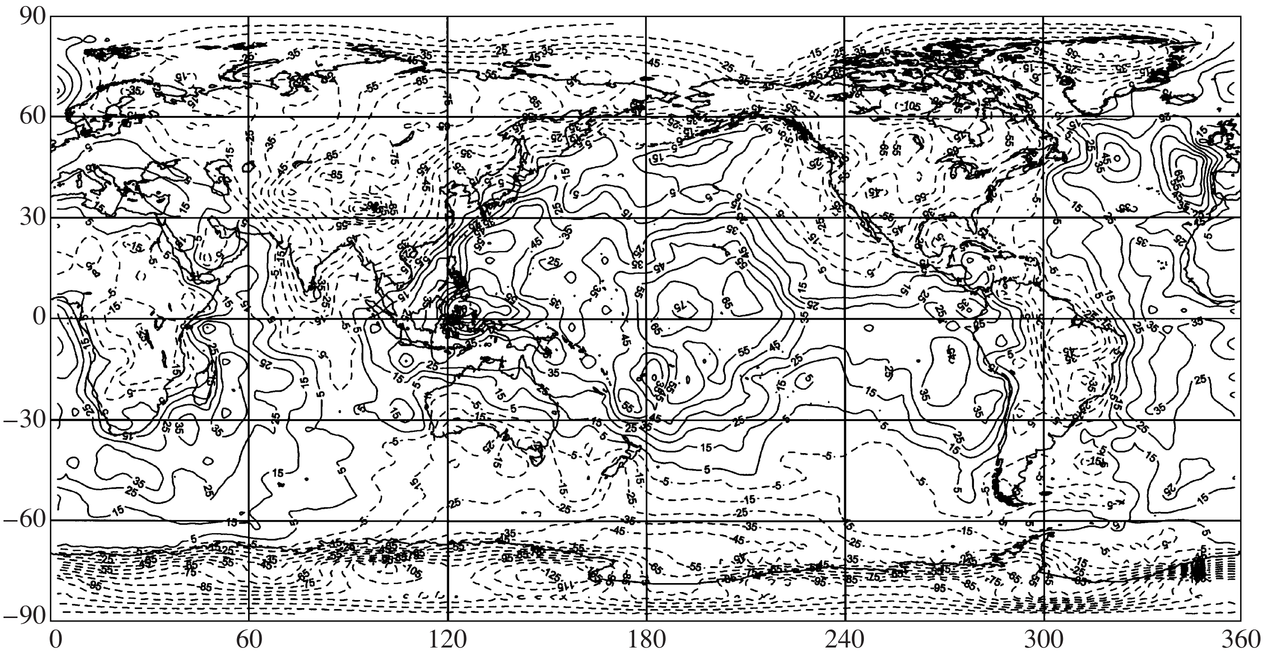

Fig. 1. Isostatically unbalanced mass profiles in the upper crust, 10^4 kg/m^2, mapped in cross section 10^5 kg/m^2. The variation range is $(-138$ to $109) \times 10^4$ kg/m^2.

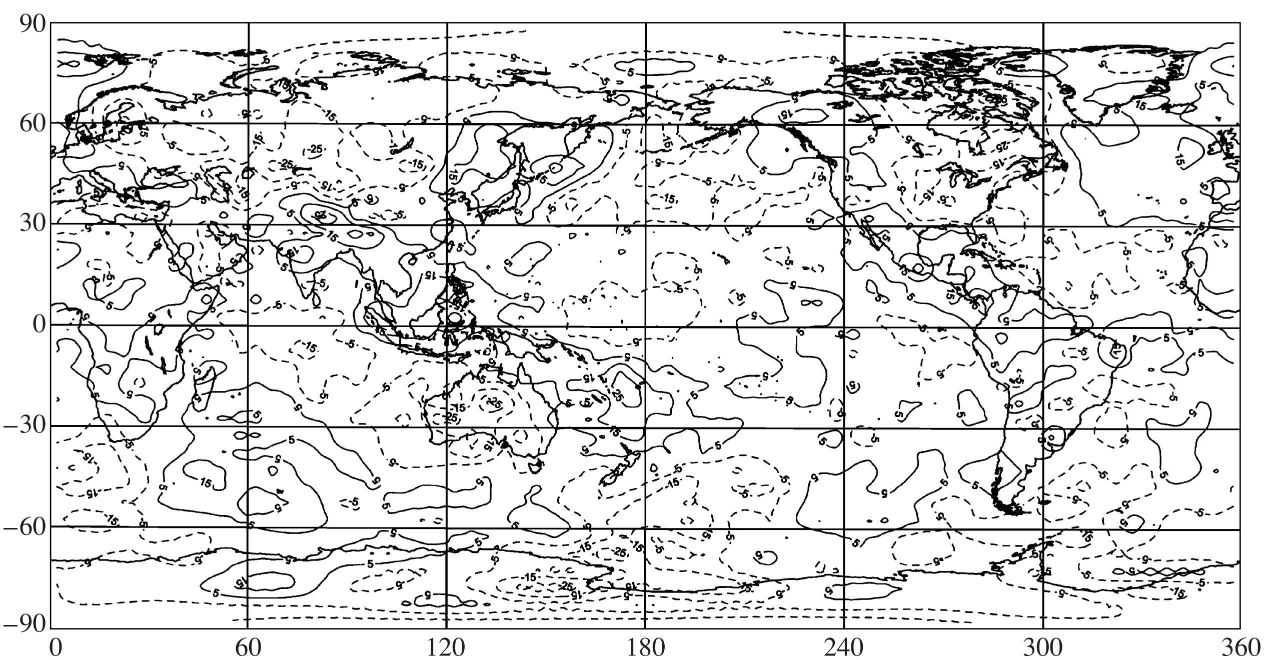

Fig. 2. Isostatically unbalanced mass profiles in the middle crust, 10^4 kg/m^2, mapped in cross section 10^5 kg/m^2. The variation range is $(-37$ to $38) \times 10^4$ kg/m^2.

Some average characteristics of the anomalous structures in the crust and mantle are presented in the table. The data were obtained from various regions of the Earth with similar properties of the isostatically balanced crust and upper mantle [2].

Here, h is the average altitude of the relief relative to the geoid; d is the depth of the M; D is the compensation depth; $\Delta m^{(k)}$ and $\Delta m^{(m)}$ are the isostatically balanced anomalous masses of the crust and mantle; $\Delta p^{(k)}$ and $\Delta p^{(m)} = -\Delta p^{(k)}$ are the isostatically balanced verti-

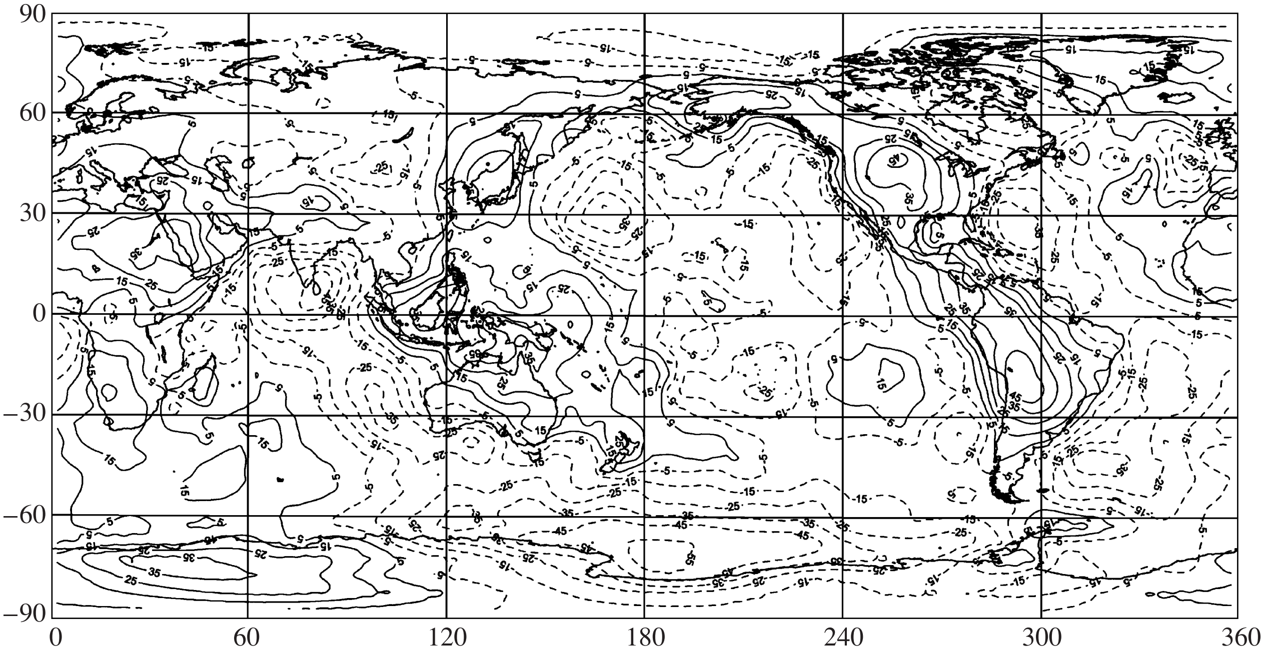

Fig. 3. Isostatically unbalanced masses in the lower crust, 10^4 kg/m^2, mapped in cross section 10^5 kg/m^2. The variation range is $(-62$ to $54) \times 10^4$ kg/m^2.

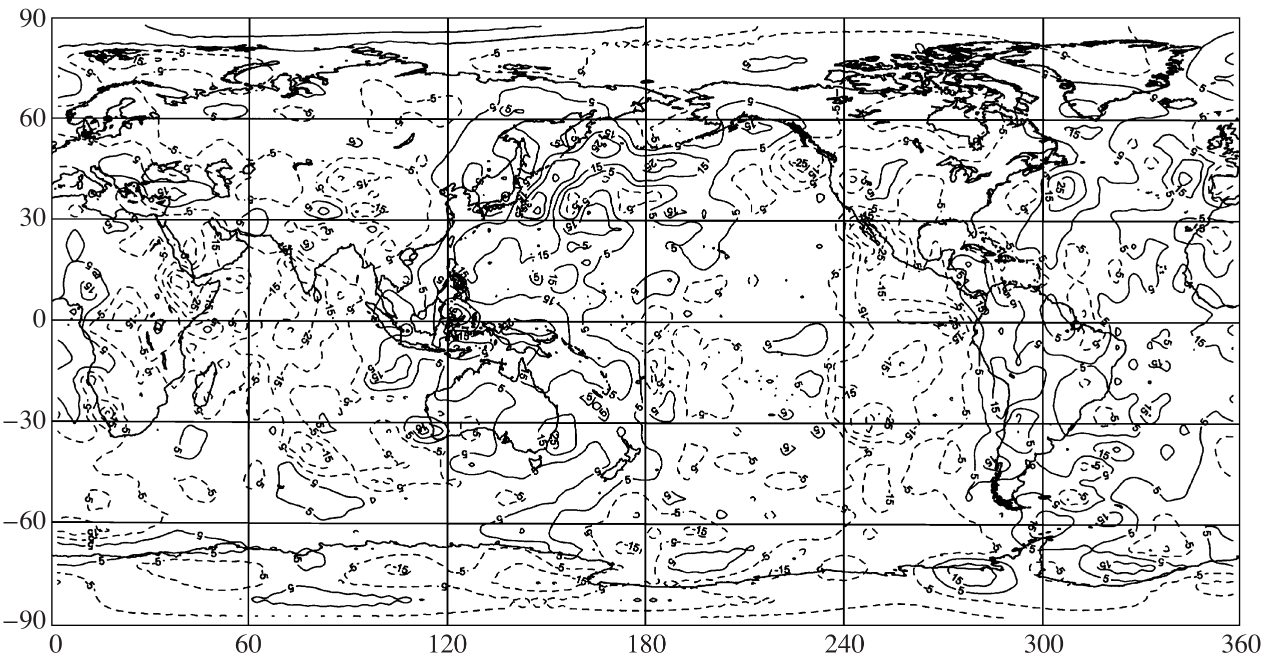

Fig. 4. Isostatically unbalanced masses in the uppermost mantle, 10^4 kg/m^2, mapped in cross section 10^5 kg/m^2. The variation range is $(-49$ to $55) \times 10^4$ kg/m^2.

cal tensile (positive) and compressive (negative) stresses, and $\Delta g^{(k)}$ and $\Delta g^{(m)}$ are the anomalies of the gravity force in the isostatically balanced crust and upper mantle; δm, δp, and δg are similar characteristics due to the contribution of the isostatically unbalanced sources of the isostatic anomalies; S is the percentage of the region's area; σ is the root-mean-square deviation; and $c(x, y)$ is the correlation $c(x, y) = (\sum_{i=1}^{I}(x_i y_i \cos\varphi_i)/\sum_{i=1}^{I}|x_i y_i \cos\varphi_i|) \times 100\%$, where $I = 2592$).

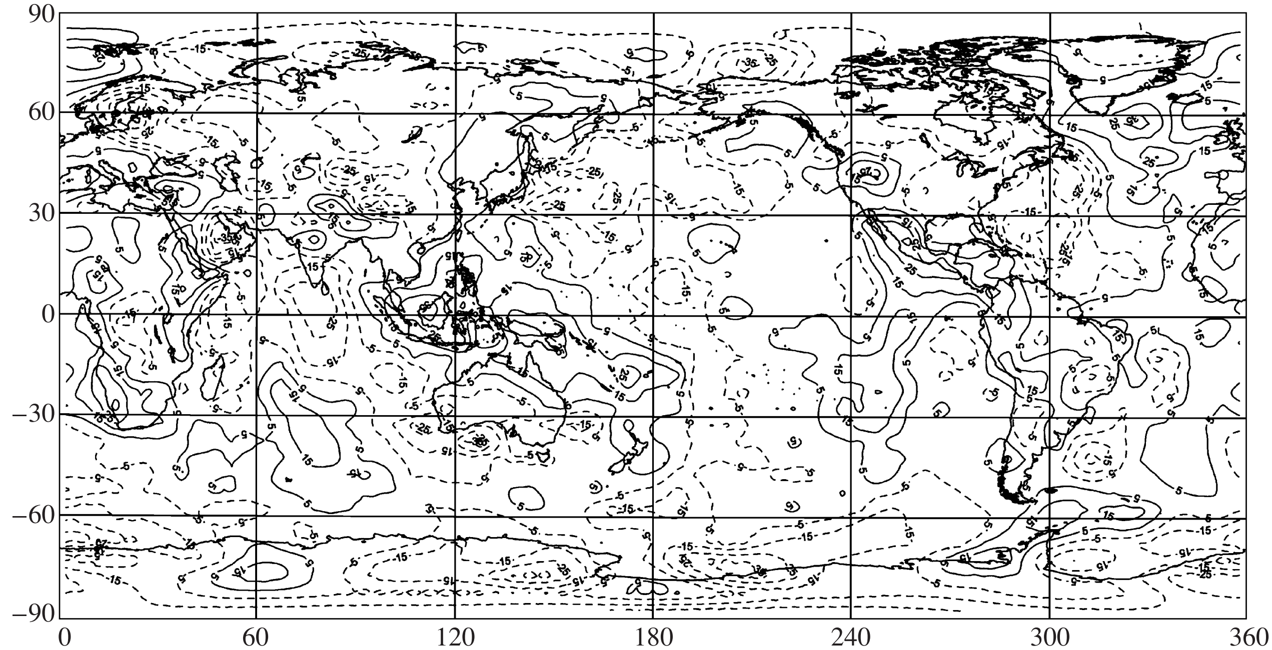

Fig. 5. Isostatically unbalanced masses in the upper mantle below the compensation depth, 10^4 kg/m^2, mapped in cross section 10^5 kg/m^2. The variation range is (–49 to 45) × 10^4 kg/m^2.

The terrestrial zones considered are classified into the following types [2]: (1) mid-oceanic ridges (MORs); (2) flanks and adjoining trenches ($d < 13$ km and $\Delta = (\Delta m^{(k)} + \Delta m^{(m)})g < -15$ MPa) with vertical (a) compressive and (b) tensile stresses in the upper crust; (3) low-anomalous ocean zones (the mantle and crust stresses are less than 1 and 5 MPa, respectively, $\Delta < 15$ MPa); (4) deep-water trenches ($h < -3.1$ km, $\Delta > 15$ MPa); (5) ocean–continent transition zones ($h \geq -3.1$ km) with (a) vertical tensile stress ($h \leq -0.3$ km) and (b) vertical compressive stress ($h \leq 0.2$ km) in the upper crust; (6) platforms I ($d \geq 32$ km, $h > -0.3$ km, $\Delta p^{(k)} \geq 0$); (7) platforms II ($d \geq 32$ km, $h > 0$ km, $\Delta p^{(k)} < 0$, $\Delta > 0$); (8) high-mountain zones and plateaus ($h > 0.2$ km, $\Delta p^{(k)} < 0$, $\Delta < 0$); and (9) the entire surface of the Earth.

CONCLUSIONS

Analysis of the data presented in the table and comparison of distributions in Figs. 1–5 lead to the following conclusions.

(i) The distribution of isostatically unbalanced masses in the upper and lower crust, the uppermost mantle, and the lower part of the upper mantle has a mainly dipole character, which is especially pronounced for the regions of central and northeastern Pacific Ocean, deep-water trenches, some sectors of MORs, and the mountain and coastal zones of the continental crust. Such a distribution gives rise to additional, isostatically unbalanced stresses, which are compressive in the upper mantle beneath deep-water trenches and in the ocean crust (and increase with distance from MORs) and tensile in the uppermost mantle of MORs and in the continental crust. These additional stresses generally intensify the isostatically balanced stress of the MOR crust and the ocean's uppermost mantle (where correlation $c(\Delta p^{(k)}, \delta p^{(k)})$ is positive) and weaken the stress of the continental crust and ocean trenches (where the correlation is negative).

(ii) The contribution of the dipole distribution of isostatically unbalanced masses to the gravitational field may lead to a discrepancy between the anomalies in the force of gravity derived from analytic extension of a satellite-based model and from ground-based measurements. In the linear approximation, this contribution to the nth power harmonic is proportional to $(\Delta/R_l)(n + 2)\delta m_n^{(l)}(R_l/r)^{n+2}$, where $\Delta = R_l - R_{l+1}$ and l corresponds to the upper layer of the dipole. It follows that, on the Earth's surface, the contribution of the upper layer prevails, growing with the harmonic number (for $r \approx R_l$); the lower harmonics become more significant with altitude since the contribution to higher harmonics tends to zero for $r > R_l$.

(iii) Nonequilibrium structures lead to a certain redistribution of isostatically balanced masses, specifically, the continent crust is mostly decompressed in the upper layers and compacted in the lower layers, while the ocean crust is compacted in the upper layers and decompressed in the lower layers. However, there are some exceptions. For example, the lower crust in the

Average anomalous characteristics of the crust and mantle for various Earth regions

	1	2a	2b	3	4	5a	5b	6	7	8	9
h, 10^2 m	–22	–23	–23	–28	–34	–14	–8	2	2	12	–17
σ	3	2	3	5	3	9	9	6	7	10	15
d, 10^2 m	104	96	93	133	147	244	262	399	440	391	220
σ	16	10	15	35	33	67	68	46	56	84	124
D, 10^3 m	90	53	82	70	100	39	45	47	54	50	59
σ	16	15	33	28	29	11	22	10	16	14	28
$\Delta m^{(k)}$, 10^4 kg/m^2	–87	–60	–25	13	62	14	–58	57	129	–141	3
σ	62	56	54	89	62	124	110	156	109	106	120
δm_1, 10^4 kg/m^2	8	15	11	18	27	–2	–8	–31	–28	–46	1
σ	21	17	22	25	28	29	30	30	35	40	35
$c(\Delta m^{(k)}, \delta m_1)$, %	–33	–69	–48	11	71	10	25	–27	–74	92	12
δm_2, 10^4 kg/m^2	2	1	0	0	–2	2	2	–2	–8	3	0
σ	7	6	8	8	8	10	9	12	9	10	9
$c(\Delta m^{(k)}, \delta m_2)$, %	–49	–47	–62	–70	–52	–54	–64	–79	–91	–72	–69
δm_3, 10^4 kg/m^2	–8	–4	–3	–8	–10	4	6	9	11	11	–1
σ	16	13	16	17	17	19	19	17	17	17	19
$c(\Delta m^{(k)}, \delta m_3)$, %	70	66	51	2	–70	–16	–31	39	83	–69	2
$\Delta m^{(m)}$, 10^4 kg/m^2	–142	–162	–214	14	184	–7	–29	7	57	–84	–3
σ	123	84	107	66	138	139	148	58	75	139	135
δm_4, 10^4 kg/m^2	–7	–11	–13	1	9	2	1	–1	0	–4	0
σ	2	11	11	9	13	11	12	9	9	12	11
$c(\Delta m^{(m)}, \delta m_4)$, %	91	91	97	87	92	85	87	11	27	88	86
δm_5, 10^4 kg/m^2	7	6	10	0	–9	3	4	–3	–9	2	0
σ	10	9	11	11	13	13	14	12	12	15	13
$c(\Delta m^{(m)}, \delta m_5)$, %	–88	–74	–85	–80	–78	–77	–81	–70	–87	–83	–80
$\Delta p^{(k)}$, 10^5 Pa	–18	–23	14	0	33	23	–18	10	–15	–20	2
σ	28	23	19	1	40	29	24	10	15	19	27
$\delta p^{(k)}$, 10^5 Pa	–3	–3	–2	–5	–6	1	2	6	6	9	–1
σ	6	6	4	7	8	6	7	10	10	11	9
$c(\Delta p^{(k)}, \delta p^{(k)})$, %	94	100	–100	39	–92	20	–94	100	–100	–100	–48
$\Delta p^{(m)}$, 10^5 Pa	18	23	–14	0	–33	–23	18	–10	15	20	–2
σ	28	23	19	1	40	29	24	10	15	19	27
$\delta p^{(m)}$, 10^5 Pa	5	4	7	0	–5	0	1	0	–1	2	0
σ	7	6	7	3	7	5	6	1	2	6	5
$c(\Delta p^{(m)}, \delta p^{(m)})$, %	98	100	–100	–90	94	43	66	–57	–100	98	72
$\Delta g^{(k)}$, mGal	156	170	147	245	357	–64	–152	–441	–471	–562	0
σ	66	44	62	94	69	172	169	100	138	189	323
$\delta g^{(k)}$, mGal	–7	–8	–5	–10	–15	3	6	16	16	23	–1
σ	6	6	6	8	9	10	10	10	13	14	15
$\Delta g^{(m)}$, mGal	–115	–132	–177	10	140	–7	–21	19	69	–45	0
σ	96	70	83	55	107	109	121	52	55	115	108
$\delta g^{(m)}$, mGal	6	6	9	–1	–7	1	1	–1	–4	3	0
σ	6	5	6	3	6	6	7	3	4	7	6
Δg, mGal	1	3	3	5	7	3	2	–13	–15	–16	0
σ	21	15	21	24	26	28	29	27	28	32	27
S, %	6	4	2	33	9	12	11	12	5	5	100

ocean regions adjacent to continents and microcontinents usually has a higher density, similar to that underlying continents (Fig. 3). This may be attributed to an incomplete oceanization of the crust. The lower crust of northern and central Eurasia has a density comparable with that of ocean crust. This may either testify to the presence of oceans there in the past or indicate a forthcoming oceanization. As for the middle crust, separating it into a particular structure is fairly formal for oceans. Generally, it contributes to a decrease in the anomaly (correlation $c(\Delta m^{(k)}, \delta m^{(2)}) < 0$). The unbalanced structures of the upper mantle beneath oceans exhibit a pronounced dipole behavior; they mainly increase the nonuniformity of distributions in the layers adjacent to M (the regions of positive correlation $c(\Delta m^{(k)}, \delta m^{(4)})$) and decrease the nonuniformity near the bottom of the isostatic compensation region (the regions of negative correlation $c(\Delta m^{(k)}, \delta m^{(4)})$).

(iv) The analysis shows that isostatic equilibrium in the crust and upper mantle is disturbed by the redistribution of mass. Computation of the potential energy in neighboring regions of continents and oceans shows that redistribution of mass minimizes the differences between the internal gravitational fields and the field of the hydrostatically balanced Earth and is in accordance with the tendency of the system toward minimum potential energy. Thus, to reach a more favorable energy state, the light components of the ocean's upper crust should move upward and into the upper part of the continental crust; and the heavier components of the ocean lower crust, into the lower parts of the continental crust and into the upper layers of the mantle underlying deep trenches.

In conclusion, note that the revealed isostatically unbalanced structures must be compensated somewhere in the deep layers of the Earth since the pressures at the center of the Earth must be equalized. If the compensation takes place at the core–mantle boundary, the possible distribution of the compensation masses agrees well with seismic data [5]. In the future, we propose analyzing in detail the distribution of compensation masses, which is consistent with the external gravitational field and seismic data.

ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research, project no. 04-02-16681.

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