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About “Black Holes” and Dark Matter

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Abstract—It is shown that the complete system of classical gravitational equations for an isolated centrally symmetric body yields that: (1) in terms of Galilean coordinates all metric coefficients of the Riemannian space induced by the body cannot be equal to zero or infinity anywhere; (2) they, together with the first-order derivatives, should be continuous everywhere. The equations do not contain solutions corresponding to “black holes,” but admit solutions corresponding to objects for which the surface radius (in terms of standard coordinates) is equal to the double mass of matter under this surface. These objects can make the main contribution to the dark matter of the Universe and explain observed effects, such as gravitational microlensing and other effects. Under certain conditions they can become powerful X-ray sources.

Key words: physics of black holes, relativity, and gravitation.

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INTRODUCTION

The problems of “black holes” and dark matter have been attracting the attention of physicists (and not only physicists) for many years. However, a final conclusion on existence of black holes and the nature of dark matter has not been made yet. Cautious and responsible researchers, interpreting those discovered phenomena that are hastily explained by some physicists as a result of the manifestation of black holes, prefer to speak of “candidates” for black holes (using the term black hole in the sense candidate for a black hole) and do not exclude other possible explanations [1]. The discovery of the effect of gravitational microlensing [2–4] extended the capabilities of observers [5], but did not result in the final conclusion that the undoubted cause of this effect is black holes, since it can result from other (yet unknown) supercompact objects which are not observed by conventional tools.

Below, both problems are analyzed in detail, and certain conclusions following from the conventional gravitational equations (and their solutions) are made.

1. BASIC EQUATIONS

In the case of a static spherically symmetric problem the squared interval of the corresponding Riemannian space (the system $c = \hbar = G = 1$ is chosen) is written in the form

$$ds^2 = Bdt^2 - A^{-1}dr^2 - Z^2(d\theta^2 + \sin^2\theta d\varphi^2), \qquad (1)$$

where

$$r^2 = (x^1)^2 + (x^2)^2 + (x^3)^2,$$

x^ε are the Galilean coordinates. The coefficients $B(r)$, $A(r)$, and $Z(r)$ are sought using the complete system of equations

$$R^{\varepsilon\lambda} - \frac{1}{2}g^{\varepsilon\lambda}R = \frac{8\pi}{\sqrt{-g}}T^{\varepsilon\lambda}, \qquad (2)$$

$$\partial_\varepsilon \tilde{g}^{\varepsilon\lambda} = 0, \qquad (3)$$

for the boundary conditions

$$B|_{r\to\infty} \longrightarrow 1, \quad A|_{r\to\infty} \longrightarrow 1,$$
$$\left.\frac{Z}{r}\right|_{r\to\infty} \longrightarrow 1, \quad \left.\frac{rZ'}{Z}\right|_{r\to 0} \longrightarrow 1, \quad Z' \equiv dZ/dr. \qquad (4)$$

Here, $R^{\varepsilon\lambda}$ is the Ricci tensor, $R \equiv R^{\varepsilon\lambda}g_{\varepsilon\lambda}$, $g_{\varepsilon\lambda}$ are the metric coefficients, $g \equiv \det g_{\varepsilon\lambda} = \det\tilde{g}^{\varepsilon\lambda}$, $\tilde{g}^{\varepsilon\lambda} \equiv \sqrt{-g}\, g^{\varepsilon\lambda}$, and the density of the energy-momentum tensor of the matter forming the body (see Section 3) has the form

$$T^{\varepsilon\lambda} = \sqrt{-g}\;[(\rho + p)u^\varepsilon u^\lambda - pg^{\varepsilon\lambda}], \qquad (5)$$

where $p(r)$ and $\rho(r)$ are the matter pressure and scalar density, and $u^\varepsilon \equiv dx^\varepsilon/ds$ are the four-velocities of its elements (in our case, $u^\kappa = 0$, $u^0u^0 = g^{00}$). In the equilibrium state the pressure satisfies the equation

$$\frac{dp}{dr} = -\frac{1}{2}(\rho + p)\frac{d}{dr}\ln B, \quad p|_{r\to\infty} \longrightarrow 0. \qquad (6)$$

The change to B, A, and Z in (2), (3) results in the system

$$\frac{d}{dr}(ZAZ'^2) = (1 - 8\pi\rho Z^2)Z', \qquad (7)$$

$$\frac{d}{dr}\ln B = \left[\frac{1+8\pi pZ^2}{AZ'^2} - 1\right]\frac{Z'}{Z}, \quad (8)$$

$$\frac{d}{dr}(BAZ^4) = 4BZ^2 r. \quad (9)$$

Equation (7) with account of (4) yields

$$AZ'^2 = 1 - 2y, \quad (10)$$

where

$$y \equiv \frac{M}{Z}, \quad M = 4\pi\int_0^r \rho Z^2 Z' dr. \quad (11)$$

Substituting (10) into (8) and (9), we transform these equations to the form

$$\frac{d}{dr}\ln B = 2\frac{y + 4\pi pZ^2}{1-2y}\frac{Z'}{Z}, \quad (12)$$

$$\frac{(1-2y)}{Z'^2}ZZ'' = 2 - 2y - 2\frac{rZ'}{Z} + 4\pi(p - \rho)Z^2. \quad (13)$$

The equation for A, according to (9), (12) or (10), (13) is written in the form

$$ZZ'A' + 3(1-2y) - 4\frac{rZ'}{Z} + 1 + 8\pi pZ^2 = 0, \quad (14)$$

where asterisks denote derivatives with respect to r. Equations (12), (14), (6), and (13) or (9) determined on the whole interval $0 \le r \le \infty$ can be considered independent.

2. ANALYSIS OF EQUATIONS AND THEIR SOLUTIONS

In nth order differential equations, functions and their derivatives to $(n - 1)$th order inclusive always satisfy the requirement of continuity in the domain of definition. This implies that the functions $p(r)$, $B(r)$, $A(r)$, $Z(r)$, and $Z'(r)$ satisfying corresponding equations, should be continuous everywhere. This results in the fact that the first derivatives B' and A', due to the structure of Eqs. (12), (14), turn out to be continuous (p', B'', A'', and Z'') have a discontinuity at the boundary). Therefore, according to the Weierstrass theorem [6], the functions B and A are bounded on any finite interval. Since for $r \longrightarrow \infty$ they, according to conditions (4), are also bounded, the condition of their boundedness is satisfied everywhere for $0 \le r \le \infty$.

Let us show that the functions B and A do not vanish anywhere and are strictly positive. For this purpose, we consider Eq. (9) and transform it to the integral form,

$$B^2A^2Z^8 = 8\int_0^r B^2AZ^6 r dr. \quad (15)$$

This yields that upon moving away from the point $r = 0$ the function $A(r)$ cannot go into the negative domain, since in this case the right-hand side of (15) would become negative, and the left-hand side should be positive, i.e., Eq. (15) would be violated. Therefore, in some domain $r > 0$ the function $A(r)$ satisfying (15), and therefore, (9), is positive. Can $A(r)$ vanish at some point $r = r_0 > 0$? The answer is obvious: no, it cannot, since in this case, the left-hand side of (15) would vanish at $r = r_0$, and the right-hand side, as the integral of the positive function in the domain $0 \le r \le r_0$, should be positive. Thus, the function A satisfying (9) is strictly positive in the whole domain $0 < r \le \infty$.

Similarly we can prove that there does not exist the point $r > 0$, such that at this point the functions $B(r)$ or $Z(r)$ vanish; due to this and boundary conditions (4) the functions $B(r)$ and $Z(r)$ should be positive for all $0 < r \le \infty$.

Thus, the solutions to Eqs. (6), (9), (12), (14) for the coefficients B and A should satisfy the following strict inequalities in the domain $0 < r \le \infty$:

$$0 < B < \infty, \quad 0 < A < \infty. \quad (16)$$

In this case, the functions B, A, p, Z, B', A', Z' should be continuous everywhere.

Corollary (16) results in the important physical results; let us consider these results.

First, due to inequalities (16), it follows from (10) and (11) that the following inequality is satisfied everywhere:

$$2M \le Z. \quad (17)$$

This inequality means that the original system of gravitational equations and the boundary conditions admit the existence of such bodies for which the surface radius $Z_0 \equiv Z(r_0)$ cannot be smaller than the double mass of matter $2M_0 \equiv 2M(Z_0)$ within this sphere (hereinafter, all quantities with the index "0" are related to the values of quantities on the body surface). In other words, the original equations do not contain solutions corresponding to black holes (i.e., solutions with the domain in which the inequality $2M > Z$ is satisfied). Upon formal "compression" of the body into an infinitely small domain its mass should also become infinitely small (this possibility was first mentioned in [7]). This implies that the matter collapsing under the action of the self-gravitational field, which possessed the total energy E before the beginning of the collapse, due to the energy conservation law, cannot be concentrated in an infinitely small domain. The value of Z_0 on the surface of the body formed as a result of the collapse, according to (17), is not smaller than $2M_0$; the value of $2M_0$ is determined by the value of E, due to the energy conservation law (with subtracted radiation and other possible losses). Therefore, the collapse into the state of the black hole turns out to be forbidden. This completely agrees with the statement made by Einstein that the main result of the performed study was the clear understanding that Schwarzschild singularities are absent in the real world; that the Schwarzschild singularity does not exist, since matter cannot be concentrated in an arbitrary way; in the opposite case, particles forming clusters would reach the velocity of light [8]; and the statement made by Weinberg that the seeming Schwarzschild singularity can only be the property of the coordinate system [9] (introduced with the violation of the requirements of Jacobians, we should add).

At the same time, the equations and boundary conditions admit solutions corresponding to bodies for which the following relation is satisfied on the surface: $Z_0 = 2M_0$. According to (10), (11), the following equalities are satisfied on such surfaces:

$$\left.\frac{dy}{dZ}\right|_{Z_0-0} = 0, \quad Z'_0 \equiv Z'|_{Z_0} = 0, \quad 8\pi\rho_0^{in} Z_0^2 = 1, \quad (18)$$

where ρ_0^{in} is the value of the internal density ρ on the body boundary[1].

Second, if solutions with $Z_0 = 2M_0$ exist on the body surface, the pressure p on such surface cannot vanish; this will be demonstrated below.

Let us introduce the functions Q and F instead of p and Z as

$$\frac{rZ'}{Z} \equiv Q\sqrt{1-2y},$$
$$y + 4\pi p Z^2 \equiv (F\sqrt{1-2y} - (1-2y)). \quad (19)$$

Then Q and F (13) and (6) yield the following equations:

$$rQ' + 2Q^3 - Q^2F - Q = 0, \quad (20)$$

$$rE' + [F^2 + 1 - 4F\sqrt{1-2y} + 2(1-2y)]Q = 0. \quad (21)$$

It can be seen from (19) that the functions Q and F are continuous everywhere; as a consequence, the derivatives Q' and F' determined in (20), (21) are also continuous. Therefore, according to the Weierstrass theorem, the functions Q and F should be bounded on any finite interval. Therefore, if it is assumed in (19) that $2y_0 = 1$, the following relations are obtained:

$$Z'_0 = 0, \quad 8\pi p_0 Z_0^2 = -1, \quad (22)$$

i.e., $p_0 = -\rho_0^{in}$. The first equality (22) verifies result (18), the second equality seems strange at first sight, but just at first sight. In reality the negative pressure value on the body surface and above it should correspond to physical reality, since inside the body and beyond it there exists a physical gravitational field whose origin is matter (hereinafter, for convenience, matter is understood as all forms of matter except for the related gravitational field). The matter formed by the gravitational field possesses all characteristics inherent to any other matter: a density ρ_f, a pressure p_f, the four-velocities u^ε of its elements, internal interactions, and interactions with the body, and so on (according to Fock [10, p. 446], "the gravitational field itself possesses energy"; Einstein stated earlier [11] that the tensor of gravitational field $\vartheta_{\mu\nu}$ is the field source similar to the tensor of material systems $\theta_{\mu\nu}$; and that the exclusive position of the energy of a gravitational field, as compared to all other types of energy, would result in inadmissible consequences. The gravitational field is manifested in the force action on bodies in this field. Since gravitational forces are attractive forces, the following condition should be satisfied: $\rho_f < 0$, and therefore, $p_f < 0$. On the body's surface the pressure is created by the matter above this surface, i.e., the matter formed by the related gravitational field (if there is no atmosphere), and therefore, it should be negative. In [9] the pressure created by the gravitational field on the body surface was ignored (it was assumed that $p_0 = 0$). Therefore, we disagree with the proof of the constraint $y < 4/9$ given there[2].

If (19) is taken into account in (10) and (12), we obtain the following equalities:

$$AQ^2 = r^2/Z^2,$$

$$B = \exp\left[-2\int_r^\infty (F - \sqrt{1-2y})Q\frac{dr}{r}\right].$$

Since the functions Q and F are bounded, and taking into account conditions (4), the above equalities and (9) imply inequalities (16) which result in (17).

[1] We point out that dr/dZ is the Jacobian of transition from the variable r to the variable Z, if this transition was considered. However, due to (18) and the requirements on Jacobians (on open coordinate sets Jacobians should not be equal to zero or infinity), this transition turns out to be forbidden if the states of the bodies with $Z = 2M$ are realized; it is possible only if $Z > 2M$ everywhere. Therefore, the conclusions obtained by the transition from r (without considering the Jacobian behavior) to concomitant coordinates in the cases where the state $Z = 2M$ is admissible cannot be considered to be physically consistent.

Some authors state that in (2) the coordinates x^ε can be arbitrary. This is a deep delusion. The admissible class of coordinates should be determined by the requirements for Jacobians connecting two sets of coordinates, including Galilean coordinates (in which the physical meaning of the determined quantities is clear). Ignoring these requirements results in the appearance of different ambiguities, singularities, and nonphysical results (which excite people's minds and assist in obtaining ample financing from credulous governments and companies; the conclusions obtained in this paper may turn out to be a more ponderable argument in favor of receiving funding, since they are mathematically grounded and more interesting from the point of view of physics

[2] If it is assumed, regardless of the above said, that beyond the body the values of ρ and p are equal to zero, the following strict inequality follows from (19): $2y < 1$; this inequality agrees with [9]; in this case, the derivative Z' does not vanish anywhere. However, the pressure p_0 on the body surface can be assumed to be equal to zero only if there is no matter beyond the body. If the material character of the field and the universal character of gravitational interactions are accepted to be a physical reality, it is impossible to assume p_0 to be equal to zero, since beyond the body matter exists formed by the external gravitational field. In this paper, the approach based on this idea is considered (see Section 3).

3. THE ROLE OF THE GRAVITATIONAL FIELD

Thus, the scalar ρ included in $T^{\varepsilon\lambda}$ is formed, both due to the self matter, and the gravitational field induced by it, i.e., it can be represented (written) in the form

$$\rho \equiv \rho|_{G=0} + (\rho - \rho|_{G=0}) \equiv \rho_s + \rho_f, \qquad (23)$$

where ρ_s is the part formed by the "bare" (without the gravitational "coat") matter, and ρ_f is the part due to the gravitational field alone. The problem consists in the search for the scalar ρ_f. Here, Eqs. (2) are useful; using identical transformations (with account for (3)) these equations can be reduced to a form that should be called the field form for reasons following from further consideration,

$$g^{\alpha\beta}\partial_\alpha\partial_\beta\tilde{g}^{\varepsilon\lambda} = 16\pi(T^{\varepsilon\lambda} + \tau^{\varepsilon\lambda}). \qquad (24)$$

Here, the structure $\tau^{\varepsilon\lambda}$, which has the meaning of the density of the energy-momentum tensor of the gravitational field, is determined by the expression

$$16\pi\sqrt{-g}\tau^{\varepsilon\lambda} \equiv \frac{1}{2}\left(\tilde{g}^{\varepsilon\alpha}\tilde{g}^{\lambda\beta} - \frac{1}{2}\tilde{g}^{\varepsilon\lambda}\tilde{g}^{\alpha\beta}\right)$$

$$\times\left(\tilde{g}_{\nu\sigma}\tilde{g}_{\tau\mu} - \frac{1}{2}\tilde{g}_{\tau\sigma}\tilde{g}_{\nu\mu}\right)\partial_\alpha\tilde{g}^{\tau\sigma}\partial_\beta\tilde{g}^{\nu\mu}$$

$$+\tilde{g}^{\alpha\beta}\tilde{g}_{\tau\sigma}\partial_\alpha\tilde{g}^{\varepsilon\tau}\partial_\beta\tilde{g}^{\lambda\sigma} - \tilde{g}^{\varepsilon\beta}\tilde{g}_{\tau\sigma}\partial_\alpha\tilde{g}^{\lambda\sigma}\partial_\beta\tilde{g}^{\alpha\tau} \qquad (25)$$

$$-\tilde{g}^{\lambda\alpha}\tilde{g}_{\tau\sigma}\partial_\alpha\tilde{g}^{\beta\sigma}\partial_\beta\tilde{g}^{\varepsilon\tau} + \frac{1}{2}\tilde{g}^{\varepsilon\lambda}\tilde{g}_{\tau\sigma}\partial_\alpha\tilde{g}^{\sigma\beta}\partial_\beta\tilde{g}^{\alpha\tau}$$

$$+\partial_\alpha\tilde{g}^{\varepsilon\beta}\partial_\beta\tilde{g}^{\lambda\alpha},$$

where $\tilde{g}_{\tau\sigma} \equiv g_{\tau\sigma}/\sqrt{-g}$.

Note that Eqs. (2), (3), (24) can be represented in the covariant form, changing the partial derivatives d_ε with respect to the Galilean coordinates x^ε to covariant derivatives D_α in the metric $\gamma_{\alpha\beta}(\xi)$ with arbitrary chosen coordinates ξ^α and transforming all quantities included in the equations to the new coordinates; all quantities included in the equations possess the truly tensor properties (including the Christoffel symbols, see [12, 13] for detail). If instead of $\tilde{g}^{\varepsilon\lambda}$, following [14], we introduce $\tilde{\Phi}^{\varepsilon\lambda} \equiv \tilde{g}^{\varepsilon\lambda} - \tilde{\gamma}^{\varepsilon\lambda}$, where $\tilde{\Phi}^{\varepsilon\lambda} \equiv \sqrt{-\gamma}\Phi^{\varepsilon\lambda}$, $\tilde{\gamma}^{\varepsilon\lambda} \equiv \sqrt{-\gamma}\gamma^{\varepsilon\lambda}$, $\gamma \equiv \det\gamma_{\alpha\beta}$, and $\gamma_{\alpha\beta}(x)$ are the metric coefficients of the Minkowski space, the derivatives $\partial_\alpha\tilde{g}^{\varepsilon\lambda}$ can be replaced by the covariant derivatives $D_\alpha\tilde{\Phi}^{\varepsilon\lambda}$ everywhere, since $D_\alpha\tilde{\gamma}^{\varepsilon\lambda} \equiv 0$. Then it becomes clear that $\Phi^{\varepsilon\lambda}$ represents the gravitational potentials of the (rank 2) tensor gravitational field, and $\tau^{\varepsilon\lambda}$, which is, according to (25), the quadratic form of first order derivatives of gravitational potentials, is the density of the energy-momentum tensor of the gravitational field. In this case, Eq. (3) is written in the form[3]

$$D_\varepsilon\tilde{\Phi}^{\varepsilon\lambda} = 0. \qquad (3a)$$

If, using [15, 16], the tensor $\Phi^{\varepsilon\lambda}$ is decomposed in terms of irreducible representations with the spins S = 2, 1, 0, 0', it turns out, as was shown in [14], that (3) or (3a) do not admit nonphysical states of the field with the spins S = 1, 0' and admit real states with S = 2, 0. It can be seen that Eqs. (3) or (3a) are fundamentally important and they cannot be ignored if we want to avoid nonphysical admixtures.

The satisfaction of conditions (3) or (3a), eliminating the nonphysical admixtures with S = 1, 0' from the field states results, as was shown in Section 2, to the positive definiteness of the function $A(r)$ and correspondingly, the satisfaction of inequalities (17). If (3a), and therefore, (9), are ignored, i.e., the contribution of nonphysical admixtures into the field is taken into account, negative values of $A(r)$ in solution (10) to Eq. (7) are not eliminated, i.e., y can be both smaller and larger than 1/2. Therefore, the contribution to the gravitational field of nonphysical admixtures with S = 1, 0' results in negative values of $A(r)$ and solutions with $2M > Z$ corresponding to black holes. This cannot be admitted!

Equations (24) are valid for any gravitational field $\Phi^{\varepsilon\lambda}$, both static, and nonstatic. Since they are second-order equations, it is required that the potentials $\Phi^{\varepsilon\lambda}$ and their first order derivatives $\partial_\alpha\Phi^{\varepsilon\lambda}$ satisfying these equations are continuous on the whole interval $0 \le r \le \infty$; infinitely far from the source, $\Phi^{\varepsilon\lambda}$ should satisfy the following conditions: $\Phi^{\varepsilon\lambda}|_{r\to\infty} \longrightarrow 0$. This yields that, due to the Weierstrass theorem, all components of the potential $\Phi^{\varepsilon\lambda}$, and therefore, $\tilde{g}^{\varepsilon\lambda}$, are always everywhere bounded, and the density $\tau^{\varepsilon\lambda}$, is continuous.

In our case,

$$\tilde{g}^{00} = \frac{Z^2}{r^2\sqrt{BA}},$$

$$\tilde{g}^{kn} = \sqrt{\frac{B}{A}}\left[\gamma^{kn} + \left(1 - \frac{Z^2A}{r^2}\right)\frac{x^kx^n}{r^2}\right].$$

The boundedness of these quantities implies the following inequalities obtained earlier: $0 < A < \infty$, $0 < B < \infty$. Let us consider the search of ρ_f.

[3] The gravitational potentials $\Phi^{\varepsilon\lambda}(x)$ represented in terms of Galilean coordinates x^ε correspond to the realistic physical gravitational field. Upon transition from x^ε to arbitrary coordinates ξ^α, the physical components $\Phi^{\varepsilon\lambda}(x)$ mix into $\Phi^{\alpha\beta}(\xi)$ with noninertial admixtures, which appear due to the transformations

$$\Phi^{\alpha\beta}(\xi) = \Phi^{\varepsilon\lambda}(x)\frac{\partial\xi^\alpha}{\partial x^\varepsilon}\frac{\partial\xi^\beta}{\partial x^\lambda}.$$

Therefore, if we want to consider the gravitational field alone, we should use Galilean coordinates, which is done in this paper.

The unique tensor scalar connected with the gravitational field, and just with the gravitational field, is the everywhere continuous spur

$$\tau \equiv \tau^{\varepsilon\lambda}\tilde{g}_{\varepsilon\lambda} = \rho_f - 3p_f. \tag{26}$$

There are no other scalars determined by the gravitational field. Therefore, it is natural to put forward the hypothesis that the part ρ_f included in (23) is the scalar ρ_f which is determined by expression (26), i.e., assume that

$$\rho \equiv \rho_s + \tau + 3p_f, \tag{27}$$

where

$$16\pi\tau \equiv \frac{1}{2}\left(\tilde{g}_{\varepsilon\sigma}\tilde{g}_{\lambda\tau} + \frac{1}{2}\tilde{g}_{\tau\sigma}\tilde{g}_{\varepsilon\lambda}\right)g^{\alpha\beta}\partial_\alpha\tilde{g}^{\tau\sigma}\partial_\beta\tilde{g}^{\varepsilon\lambda} + \frac{1}{\sqrt{-g}}\tilde{g}_{\varepsilon\lambda}\partial_\alpha\tilde{g}^{\varepsilon\beta}\partial_\beta\tilde{g}^{\lambda\alpha}. \tag{28}$$

Taking into account (10)–(13), we find

$$4\pi\rho_f Z^2 = -\frac{7q^2 - 6qf + 2f^2}{1-2y} + 4\left(1 + q - \frac{1}{2}f\right) - \frac{1}{q^2}(4 - 4q + 3q^2)(1-2y) + 12\pi p_f Z^2, \tag{29}$$

where

$$q \equiv \frac{rZ'}{Z}, \quad f \equiv y + 4\pi p Z^2 + (1-2y).$$

Since according to hypothesis (27), the value of ρ turns out to be nonzero everywhere, the mass $M(Z)$ of the body is not constant at $Z > Z_0$, the mass of the matter formed by the gravitational field makes the contribution to this mass (the total mass $E = M(\infty)$ is always smaller than M_0, and in the case of the strong field, several times smaller, see [13]). Similarly, in all equations related to the external region of the body, terms with ρ and p do not vanish. For example, Eq. (6) in the domain $Z > Z_0$ is the equation of equilibrium of the matter formed by the field.

The contribution of ρ_f into ρ plays the basic role in the gravitational mass defect, it is this contribution that makes it impossible that the double mass $2M_0$ goes beyond the value Z_0, however small this value becomes (see [13] for detail). In the Newtonian limit hypothesis (27) results in correct (and well-known) results, both for the total mass of an isolated static body and for the energy of a system of bodies [13]. The total mass E of the isolated body is determined, according to (11), (27), by the expression

$$E = 4\pi\int_0^\infty(\rho_s + \rho_f)Z^2Z'dr. \tag{30}$$

For a homogeneous (ρ_s = const) sphere with the radius r_0 in the Newtonian limit (29) and (13) yield

$$4\pi\rho_f Z^2 \simeq -3y^2/(1-2y) \simeq -3M_s^2/r^2,$$

$$M_s = 4\pi\int_0^r \rho_s r^2 dr = y_0\frac{r^3}{r_0^2},$$

$$Z_{\text{ext}} \simeq r + M_s^0, \quad Z_{in} \simeq r\left(1 + \frac{3}{2}y_0 - \frac{1}{2}y_0\frac{r^2}{r_0^2}\right),$$

$$y_0 \equiv \frac{4\pi}{3}\rho_s r_0^2.$$

Taking this into account in (30), we obtain, as expected, the value

$$E = M_s^0 - \frac{3}{5}\frac{(M_s^0)^2}{r_0},$$

where M_s^0 is the total mass of the sphere matter (in this case, the mass M_0 turns out to be equal to $M_0 = E + 3(M_s^0)^2/r_0 > E$).

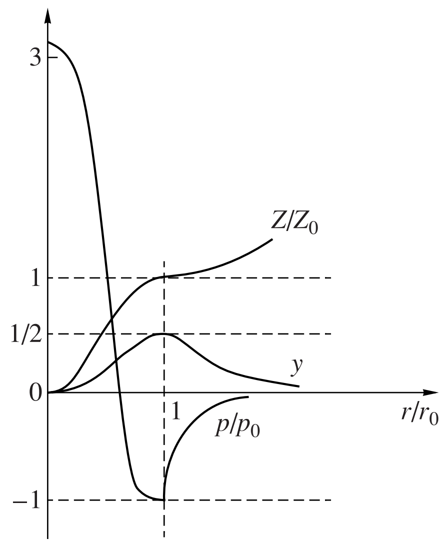
In the case of an arbitrary field, expression (11) represents, due to (27), (29), an integral equation for the mass $M(r)$. It is better to pass over from this equation to the differential equation for $y \equiv M/Z$,

$$y' = \left\{-y + 4\pi Z^2\left[\rho_s\Theta\left(1 - \frac{Z}{Z_0}\right) + \rho_f\right]\right\}\frac{Z'}{Z}, \tag{31}$$

where $\Theta(1-x) = 1$ for $x < 1$ and $\Theta(1-x) = 0$ for $x > 1$. The term with $\rho_f < 0$ in the right-hand side of (31), due to its structure, does not allow the quantity y to exceed the value $y_0 = 1/2$.

4. PHYSICAL CONSEQUENCES

Thus, the system of gravitational equations admits solutions with $y_0 = 1/2$, i.e., admits the possibility of the existence of objects with $\rho_0^{in} = 1/8\pi Z_0^2$ and $p_0 = -\rho_0^{in}$ on their surfaces. In the case of large matter concentration in the body when $8\pi\rho_s Z^2(r)$ turns out to be much higher than unity in the region $r_1 \le r \le r_0$, the value of $y \equiv M(r)/Z(r)$ can become very close to 1/2 already for $r \sim r_1$. The increment of y in the layer $r_1 \le r \le r_0$, due to (17), (29), (31) is insignificant (see Figure), i.e., the approximate equality $y \sim 1/2$ is satisfied in the whole layer. This, according to (11), (19), provides other approximate equalities $8\pi\rho Z^2(r) \sim 1$, $4\pi p Z^2(r) \sim -y \sim 1/2$, i.e., $p \sim -\rho$. On the body boundary the pressure $p = p_s + p_f$ reaches the lowest possible value, which is completely determined by the field pressure $p_f^0 = -\rho_0^{in}$; the pressure of the matter here is equal to zero, $p_s^0 = 0$. In this layer the value of the quantity $Z(r) \equiv r(1 + \tilde{\Phi}(r))$



Plots of $y(r)$, $Z(r)/Z_0$, and $p(r)/p_0$ for a body with $\rho_s/\rho_0^{in} \gg 1$.

connected with the gravitational field remains close to $Z(r_1) \sim Z(r_0) \equiv Z_0$, due to (10) or (19), which transforms the above approximate equalities to the form $p \sim -\rho \sim -\rho_0^{in}$. This implies that the pressure p_s of the matter is close to zero in the whole layer (it can be very wide). This can mean that the object temperature T in the near-surface region practically does not differ from absolute zero ($T \sim 0$). Therefore, no noticeable radiation can arise from the near-surface region. Radiation from the central region, if it is capable of going outside, is strongly energy-suppressed by a large gravitational red shift (due to the smallness of g_{00} in the central region where the temperature can turn out to be high). Therefore, these objects are invisible and can make the main contribution to the dark mass of the Universe. But these objects should be manifested in dynamic effects, those in which, as was assumed earlier, black holes are manifested [17, 18]. As well, they should be manifested in gravitational microlensing effects [19–22], replacing black holes.

If the substance falls on objects with a large matter concentration, in these objects (as was considered above) the accretion substance does not undergo noticeable deceleration due to collisions in the layer $r_1 \le r \le r_0$, since in this layer the body mass turns out to be almost completely "eaten up" by the gravitational field. Indeed, the approximate equalities $M(r)/Z(r) \sim 1/2$, $Z(r) \sim Z_0$ satisfied on the interval $r_1 \le r \le r_0$ yield the following approximate equality $M(r) \sim M(r_1) \sim M(r_0) \equiv M_0$, which means that in this layer the increment of M due to the body matter is practically completely compensated by the gravitational mass defect. In the region $r < r_1$ the metric coefficient g_{00} becomes extremely small, which results in a large gravitational radiation frequency shift. All this in aggregate should result in a considerable softening of the radiation spectrum of the accretion substance. With increasing body mass, radiation of the accretion substance should shift more and more toward the soft region (the radiation of the accretion disk alone can remain noticeable). With decreasing body mass, the gravitational mass defect decreases to an insignificant Newtonian correction. In the case of the substance accretion to such bodies the radiation spectrum has a sharp boundary vanishing (or considerably softened) in the case of accretion to bodies with large mass.

Another situation takes place if such an object, for example, with a mass equal to several solar masses, captures plasma at the external near-surface region. Then plasma particles captured by the strong gravitational field can reach the velocities $v^2 \sim 0.1$ in the quasi-equilibrium state. Therefore, such objects are the sources of powerful X-radiation.

The equations admit solutions corresponding to equilibrium giant bodies with a mass equal to several hundred solar mass and higher (a giant with $M \sim 10^4 M_\odot$ has dimensions close to those of the Earth). It is natural to expect that they are concentrated in the central region of the galaxy.

The possibility of the existence of structures with $\rho_0^{in} \simeq 1/8\pi Z_0^2$ in the cores of common stars, in particular, the core of the Sun, is not forbidden.

Some such objects could form as the result of substance collapse to the state $Z_0 = 2M_0$ (let us call such objects "collapsars"). But it is also possible to assume that they were born at the early stage of the Universe's evolution due to fluctuations of its substance at super-high densities (let us call such objects "relics"). If they were formed[4], for example at densities 10^{50} g/cm^{-3} < $\rho < 10^{60}$ g/cm^{-3} (higher densities are also assumed to be admissible), such relics possess, according to (18), the linear dimension 3×10^{-17} cm $< Z_0 < 3 \times 10^{-12}$ cm and the mass 2×10^{11} g $< M < 2 \times 10^{16}$ g. At the distance r cm from the center they cause the acceleration $(10^4/r^2)$ cm s^{-2} < $|\vec{a}| < (10^9/r^2)$ cm s^{-2}. Flowing through a gaseous or liquid medium, relics intensely absorb its closely situated elements, creating a glowing plasma cloud in their neighborhood. They can pierce solid bodies practically without hindrance, tearing out and absorbing the closest atoms and molecules and imparting to the body as a whole some acceleration (moving it).

These are the consequences following from the analysis of the complete system of gravitational equations.

[4] It is not clear, however, what such relics consist of; but if the initial equations, including equilibrium equation (6), do not forbid corresponding solutions (which is indeed so), theoretically, their existence can be considered admissible; whether they exist in nature should be found out by observations.

CONCLUSIONS

The performed analysis of the complete system of classical gravitational equations and their solutions resulted in the following important conclusions. (1) All metric coefficients turn out to be bounded and do not vanish anywhere; they and their first derivatives are continuous everywhere for $0 \leq r \leq \infty$. (2) Black holes turn out to be physically unfeasible, the equations do not contain solutions corresponding to them. (3) Bodies whose surface radius (in terms of standard coordinates) is equal or larger than the double mass of the matter within this sphere turn out to be physically admissible. (4) Bodies with $2M_0 = Z_0$ have negative pressure and a temperature close to zero at the surface. Such bodies can make the main contribution to the dark matter of the Universe and explain the phenomena which were earlier interpreted as a result of manifestation of black holes. (5) Under certain conditions they can create powerful sources of X-rays. (6) Under definite conditions, they can form powerful sources of X-ray radiation. (7) The existence of both microscopic (with respect to dimensions, but not mass) and macroscopic objects (up to giants) with $Z_0 = 2M_0$ on the surface is not forbidden. (8) The gravitational field of the body plays the key role in all the results.

Of course, all this requires further comparison with observations. However, no contradictions with the results presented above were found yet, both from the theoretical point of view and from the point of view of the available experimental data. If new problems arise (which is quite possible), they should be considered based on the material character of the existing world and without going beyond the framework of strict mathematical laws in order to not obtain just another mystic consequence, as was the case with black holes.

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