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Effect of Tensor g-Factor on the Spectrum of Eigen Modes in Magnetoactive Plasma

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Abstract—The influence of the tensorial nature of the gyromagnetic coupling on a spectrum of eigen modes in magnetoactive plasma is studied. A dispersion equation for waves propagating in such a medium is obtained in the approximation of frigid hydrodynamics.

Key words: plasma hydrodynamics, plasma spectrum eigenmodes, gyromagnetic ratio, g-factor.

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To describe the behavior of the classic spin systems it is sometimes enough to introduce an effective anisotropic field as is done, for instance, in magnetostatics. However, there are quite a number of physical phenomena that cannot be described within such an approximation, for example, splitting of EPR spectra [1, 2], dynamics of polymer structures, etc. These phenomena are brought about because, in general, the anisotropy of a system's features implies, not only the emergence of additional effective fields but also a tensorial connection between the mechanical L and magnetic M moments in the systems considered. To describe these, a tensor gyromagnetic ratio is introduced.

In this work the influence of the tensorial nature of the gyromagnetic coupling on a wave propagating in magnetoactive plasma is investigated. The existence of the tensor g-factor (or gyromagnetic ratio) in the equations may lead to an equation of magnetic moment precession (in the classic theory, it is the Landau–Lifshitz–Gilbert equation) that appears to contradict the energy conservation law [2]. Let us demonstrate that imposing constraints on a tensorial connection between magnetic and mechanical moments prevents such a contradiction from arising. Indeed, let us write down the Landau–Lifshitz equation with no account for dissipative forces [3]:

$$\frac{dL}{dt} = \left[M\frac{\delta W}{\delta \mathbf{M}}\right], \tag{1}$$

where W is the total magnetic energy of a system, $\mathbf{M}$ is the magnetization vector, and $\mathbf{L}$ is the total mechanical moment of a system. The last two are connected by the relation (2):

$$M_i = \gamma_{ij}L_j, \tag{2}$$

where γ_{ij} is the tensor gyromagnetic ratio. Let the magnetic energy of a system depend only on its magnetization and not explicitly depend on time: $W = W(\mathbf{M})$. Then for the energy conservation law to hold it is required that:

$$\begin{aligned} \frac{dW}{dt} &= \frac{\partial W}{\partial t} + \sum_j \frac{\partial W}{\partial M_j}\frac{dM_j}{dt} \\ &= \sum_{j,\,i} \frac{\partial W}{\partial M_j}\gamma_{ji}\left[\mathbf{M}\frac{\delta W}{\delta \mathbf{M}}\right]_i = 0. \end{aligned} \tag{3}$$

It is obvious that Eq. (3) cannot be satisfied for the arbitrary tensor γ_{ij}.

Let us consider the case that will be used in the next paragraph while studying collective spin features of plasma-like systems with the anisotropic gyromagnetic ratio. Let the free energy of a system look like the Zeeman energy:

$$W = -(\mathbf{MH}). \tag{4}$$

$$\begin{aligned} \mathbf{M} &= \mathbf{M}_0 + \mathbf{M}_1, \\ \mathbf{H} &= \mathbf{H}_0 + \mathbf{H}_1, \end{aligned} \tag{5}$$

where the unperturbed values of the magnetization $\mathbf{M}_0$ and the magnetic field $\mathbf{H}_0$ are directed along the OZ axis. Substituting Eq. (3) for Eq. (4) and linearizing it with account for Eq. (5) one obtains that the energy will be conserved if a tensorial connection between the mechanical and magnetic moments of a system will have (in a given coordinate system) the following form:

$$M = \begin{pmatrix} \gamma_{11} & \gamma_{12} & 0 \\ \gamma_{21} & \gamma_{22} & 0 \\ 0 & 0 & \gamma_{33} \end{pmatrix} L. \tag{6}$$

As well, while considering a nondissipative plasma medium all the tensors must be hermitian: $\gamma_{ij} = \gamma_{ji}^*$. It is seen that demanding energy conservation, provided there is a linear law relation between vectors **L** and **M**, leads to the necessity to choose a coordinate system in which the gyromagnetic tensor has the form given in Eq. (6) where there is only one pair of nonzero complex conjugate cross components. If that is impossible to do, one should use the equation of motion for a total mechanical moment instead of the equation for a magnetic moment precession, which renders the problem more cumbersome and inconvenient from a mathematical perspective.

Within the framework of hydrodynamics theory let us calculate the tensor of dielectric permeability of anisotropic magnetoactive plasma in linear approximation. In the Born–Oppenheimer approximation a system of self-consistent hydrodynamics equations with account for spin looks like the ones given in [4, 5].

Let us pick a coordinate system so that the following conditions be satisfied:

$$\mathbf{H}_0 = \{0, 0, H_0\}, \quad \mathbf{M}_0 = \{0, 0, M_0\},$$
$$\mathbf{k} = \{k_x, 0, k_z\}.$$

In such a coordinate system, the tensorial connection γ_{ij} looks like that given in Eq. (6). Let us denote $\gamma_{11} = \gamma_{22} = \gamma$, $\gamma_{12} = \gamma_{21}^* = i\delta$. Then the dielectric permeability tensor has the following form (7)–(12):

$$\varepsilon_{xx} = 1 - \frac{\omega_p^2}{\omega^2 - \Omega^2} + \frac{k_z^2 c^2(\delta^2 - \gamma^2)}{(\omega - \delta\Omega)^2 - (\Omega\gamma)^2}\frac{\Omega_\mu \Omega}{\omega^2} - \frac{k_z^2 c^2 \delta}{(\omega - \delta\Omega)^2 - (\Omega\gamma)^2}\frac{\Omega_\mu}{\omega}, \quad (7)$$

$$\varepsilon_{xy} = \varepsilon_{yx}^* = \frac{i}{\omega}\frac{\omega_p^2 \Omega}{\omega^2 - \Omega^2} - i\frac{k_z^2 c^2 \gamma}{(\omega - \delta\Omega)^2 - (\Omega\gamma)^2}\frac{\Omega_\mu}{\omega} + i\frac{k_x^2 c^2 \delta}{\omega^2 - \Omega^2}\frac{\Omega_\mu}{\omega}, \quad (8)$$

$$\varepsilon_{yy} = 1 - \frac{\omega_p^2}{\omega^2 - \Omega^2} - \frac{2k_x^2 c^2}{\omega^2 - \Omega^2}\frac{\Omega_\mu \Omega}{\omega^2} + \frac{k_z^2 c^2\left(\delta^2 - \gamma^2 - \frac{\omega}{\Omega}\delta\right)}{(\omega - \delta\Omega)^2 - (\Omega\gamma)^2}\frac{\Omega_\mu \Omega}{\omega^2} - \frac{k_x^4 c^4}{\omega^2 - \Omega^2}\frac{\Omega_\mu^2}{\omega^2 \omega_p^2} - \frac{\Omega_\mu^2 k_x^2 c^4 k_z^2}{\omega^4 \omega_p^2}, \quad (9)$$

$$\varepsilon_{xz} = \varepsilon_{zx} = -\frac{k_z k_x c^2\left(\delta^2 - \gamma^2 - \frac{\omega}{\Omega}\delta\right)}{(\omega - \delta\Omega)^2 - (\Omega\gamma)^2}\frac{\Omega_\mu \Omega}{\omega^2}, \quad (10)$$

$$\varepsilon_{yz} = \varepsilon_{zy}^* = -i\frac{k_x k_z c^2}{\omega^2}\frac{\Omega_\mu}{\omega} - i\frac{k_z k_x c^2 \gamma}{(\omega - \delta\Omega)^2 - (\Omega\gamma)^2}\frac{\Omega_\mu}{\omega}, \quad (11)$$

$$\varepsilon_{zz} = 1 - \frac{\omega_p^2}{\omega^2} - \frac{k_x^2 c^2\left(\delta^2 - \gamma^2 - \frac{\omega}{\Omega}\delta\right)}{(\omega - \delta\Omega)^2 - (\Omega\gamma)^2}\frac{\Omega_\mu \Omega}{\omega^2}. \quad (12)$$

In the system of equations (7–12) the following notations are used: $\omega_p^2 = \frac{4\pi e^2 n}{m}$ is the square of Langmuir frequency, $\Omega = \frac{eH_0}{mc}$ is the cyclotron frequency, and $\Omega_\mu = \frac{4\pi e I_0}{mc}$ is the characteristic frequency connected with self-magnetic moment.

Let us consider the waves propagating both along the external magnetic field **H** and in a transverse direction to the latter.

In Figs. 1a and 1b, the dispersion relations for the waves propagating along and transverse to the orientation of external magnetic field are shown, respectively, for the particular case $\Omega = 0.5\omega_p$, $\Omega_\mu = 0.05\omega_p$, $\gamma = 1.1$, $\delta = 0.5$ (high-density plasma medium). Accounting for the anisotropy of the medium leads to a substantial change in the shape of two lower branches with respect to the homogeneous case. Furthermore, taking into account the tensorial nature of the gyromagnetic ratio leads to the convergence of the lower modes with an increasing δ parameter (the degree of medium anisotropy) while approaching the γ parameter. For the values $\delta > \gamma$ one of those modes completely disappears. The above relations between frequencies are chosen for illustrative purposes. They are realizable only in high-density plasma; at low density the characteristic frequency Ω_μ is a few orders less than the cyclotron frequency.

Thus, the tensorial connection proposed in the paper in fact makes it possible to use the equation of the magnetic moment precession instead of the equation of motion for the total mechanical moment to describe phenomena in highly anisotropic media within the hydrodynamics approximation for an anisotropic magnetoactive plasma. The parameters of a tensorial connection between the magnetic and mechanical moments of electrons affect the character of plasma

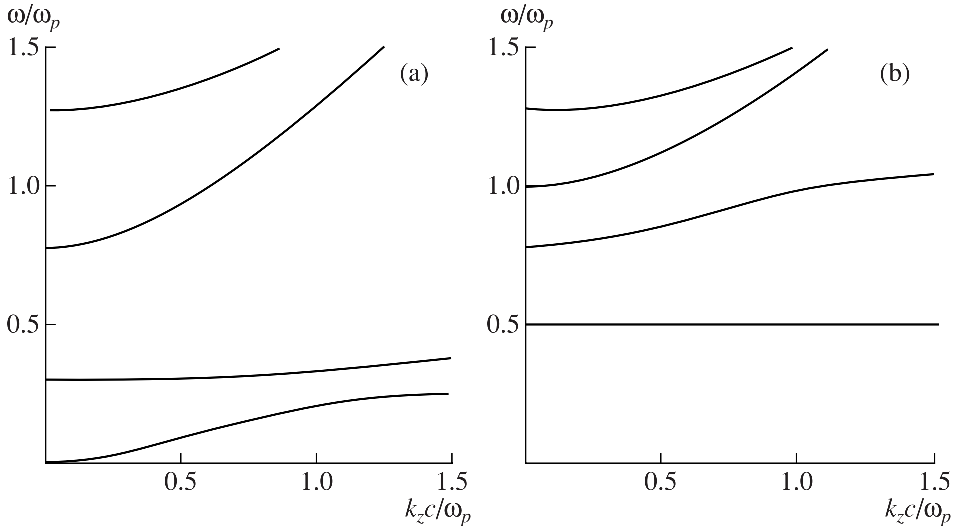

Fig. 1. (a) Dispersion curves for the waves propagating in magnetoactive anisotropic plasma along the orientation of the external magnetic field for the case $\Omega = 0.5\omega_p$, $\Omega_\mu = 0.05\omega_p$, $\gamma = 1.1$, $\delta = 0.5$. (b) Dispersion curves for the waves propagating in magnetoactive anisotropic plasma along the orientation of the external magnetic field for the case $\Omega = 0.5\omega_p$, $\Omega_\mu = 0.05\omega_p$, $\gamma = 1.1$, $\delta = 0.1$.

modes, which in the case of high-density plasma leads to the merging of the lower branches of the dispersion relations obtained.

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