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Fraunhofer Diffraction on a Polygon and Calculation of Binary Mask Images

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Abstract—A simple analytical formula for the spectrum of spatial harmonic components of a diffracted field is derived. It is used for construction of an effective algorithm of image calculation in photolithography.

Key words: photolithography, mask, Fraunhofer diffraction.

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INTRODUCTION

The spatial spectrum of the Fraunhofer diffraction by plane object is given by the Fourier transform of the object's transmission function $F(x^0, y^0)$ [1]:

$$S_F(\nu_x, \nu_y) = \iint\limits_A F(x^0, y^0) e^{-2\pi i(\nu_x x^0 + \nu_y y^0)} dx^0 dy^0. \quad (1)$$

Here, A denotes the plane in which the transmission function is different from zero.

If the function S_F is known, then there is an opportunity to solve the problem of image detection, when the image is produced by an optical system and coherent or incoherent lighting is used. Thus, the intensity distribution $I(x^i, y^i)$ in the image plane is given by Gopkin's formula, as the lighting is coherent [1, 2]:

$$\begin{aligned} I(x^i, y^i) = \int\int\int\int_{-\infty}^{+\infty} & K(x^i, y^i, x^0, y^0) K^*(x^i, y^i, x^{0'}, y^{0'}) \\ & \times F(x^0, y^0) F^*(x^{0'}, y^{0'}) \\ & \times B(x^0, y^0, x^{0'}, y^{0'}) dx^0 dy^0 dx^{0'} dy^{0'}, \end{aligned} \quad (2)$$

where integration with respect to the (x^0, y^0) and $(x^{0'}, y^{0'})$ points is taken in the plane of the object, $K(x^i, y^i, x^0, y^0)$ is the function of coherent impulse response of the projection system, and $B(x^0, y^0, x^{0'}, y^{0'})$ is the cross-intensity function for the light that illuminates the object. It is often convenient for formula (2) to be rearranged in the spectral form [3] for numerical calculations:

$$\begin{aligned} I(x^i, y^i) = \int\int_{-\infty}^{+\infty} & S_B(\nu'_x, \nu'_y) \\ \times \left| F^{-1}\{ S_F(\nu_x - \nu'_x, \nu_y - \nu'_y) S_K(\nu_x, \nu_y) \} \right|^2 & d\nu'_x d\nu'_y, \end{aligned} \quad (3)$$

where F^{-1} is the inverse Fourier transform and (ν'_x, ν'_y) and (ν_x, ν_y) are the components of spatial frequency in the object and image planes, respectively. The magnitudes $S_B(\nu'_x, \nu'_y)$, $S_F(\nu_x - \nu'_x, \nu_y - \nu'_y)$, and $S_K(\nu_x, \nu_y)$ are the spectra of the cross-intensity, transmission, and coherent impulse response functions.

When numerical simulation of the imaging is performed, the S_B and S_K functions are fixed and generally expressed transparently; thus, the speed and accuracy of the intensity calculations are mainly bounded by how the spectrum of the transmission function S_F is defined.

The function S_F is found more easily for the masks, which are used in optical lithography because the transmission function $F(x^0, y^0)$ is generally given by a function that is piecewise constant within the polygonal domains:

$$F(x^0, y^0) = \begin{cases} e^{i\varphi}, & (x^0, y^0 \in A) \\ 0, & \text{in other points;} \end{cases} \quad (4)$$

and the assignment of a polygon is reduced to the enumeration of its vertex coordinates in a standard format, for example, GDSII [4]. If the spectrum is known for the light field that is diffracted by the polygonal opening A, then the calculations with the use of (3) should present no problems.

When the binary mask is a rectangular opening (Fig. 1a), the function S_F is well known [1, 5]:

$$\begin{aligned} S_F^{\square}(\nu_x, \nu_y) = -\frac{e^{i\varphi}}{(2\pi)^2 \nu_x \nu_y} \{ & e^{-2\pi i(\nu_x x_1 + \nu_y y_1)} \\ - e^{-2\pi i(\nu_x x_2 + \nu_y y_1)} + & e^{-2\pi i(\nu_x x_2 + \nu_y y_2)} - e^{-2\pi i(\nu_x x_1 + \nu_y y_2)} \}. \end{aligned} \quad (5)$$

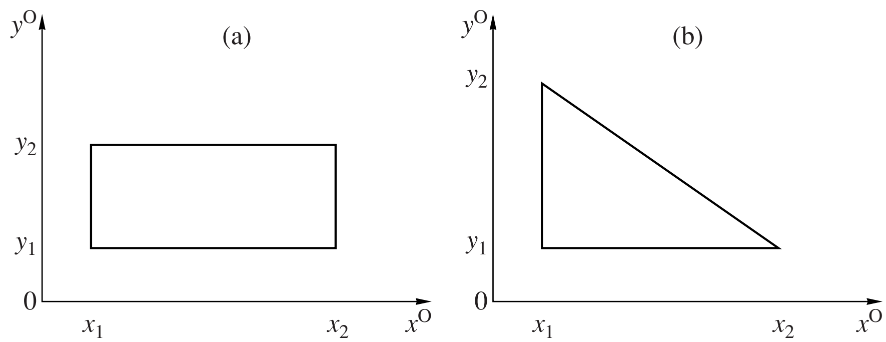

Fig. 1. (a) Rectangular and (b) triangular openings and their vertex coordinates;

Formula (5) demonstrates that the spectrum of the response to a rectangle might be presented as a sum of components, each containing only the coordinates of one of the rectangular vertices.

There is no difficulty in proving that the spectrum of the transmission function for a triangular opening might also be presented in a similar form. The calculation of (1) for the right-angled triangle in Fig. 1b results in

$$S_F^{\Delta}(\nu_x, \nu_y) = -\frac{e^{i\varphi}}{(2\pi)^2}\left[\frac{e^{-2\pi i(\nu_x x_1 + \nu_y y_1)}}{\nu_x \nu_y} - \frac{e^{-2\pi i(\nu_x x_2 + \nu_y y_1)}}{\nu_x(\nu_y + k^{-1}\nu_x)} - k^{-1}\frac{e^{-2\pi i(\nu_x x_1 + \nu_y y_2)}}{\nu_y(\nu_y + k^{-1}\nu_x)}\right], \quad (6)$$

where $k = -\dfrac{y_2 - y_1}{x_2 - x_1}$. However, notice that it is necessary to recalculate the factors of (6) for a vertex disposition different from that of the considered triangle.

Let us now derive a formula for the spectrum of the transmission function for an arbitrary polygon, so that the formula would generalize equalities (5) and (6). Notice that as (4) takes place, the integration element in (1) might be interpreted as the product $e_z \mathrm{rot}\, a$, where $a = \dfrac{1}{4\pi i}\left(\dfrac{e_x}{\nu_y} - \dfrac{e_y}{\nu_x}\right)e^{-2\pi i(\nu_x x^0 + \nu_y y^0)}$ and e_x, e_y, and e_z are the unit vectors of the rectangular coordinate system. We transform the integral over the opening area A (1) into an integral along the opening boundary, using the Stokes formula for that purpose [6]: $\iint_A n\, \mathrm{rot}\, a dA = \oint_C a dl$. Then formula (1), with (4) taken into account, assumes the form

$$S_F(\nu_x, \nu_y) = \frac{e^{i\varphi}}{4\pi i}\oint_C \left(\frac{e_x}{\nu_y} - \frac{e_y}{\nu_x}\right)e^{-2\pi i(\nu_x x^0 + \nu_y y^0)} dl. \quad (7)$$

Here, the contour C is an arbitrary closed line without self-intersections. The line is traversed in the anticlockwise direction. If the boundary of the area A is a polygon with N vertices ($r_1, r_2, \ldots, r_{N-1}, r_N$), then the transmission function spectrum might be represented as the sum of two integrals over the connecting links:

$$S_F = \frac{e^{i\varphi}}{4\pi i}\sum_{n=1}^{N} L(r_n, r_{n-1}), \quad (8)$$

where $r_0 = r_N$. Having accomplished the elementary transformations (7), we derive the expression $L(r_n, r_{n-1})$ for the segment given by the equation $(y^0 - y_{n-1}) = k_n(x^0 - x_{n-1})$, where the angular factor is $k_n = (y_n - y_{n-1})/(x_n - x_{n-1})$:

$$L(r_n, r_{n-1}) = -\frac{1}{2\pi i(\nu_x + \nu_y k_n)}\left(\frac{1}{\nu_y} - \frac{k_n}{\nu_x}\right) \times \left(e^{-2\pi i(\nu_x x_n + \nu_y y_n)} - e^{-2\pi i(\nu_x x_{n-1} + \nu_y y_{n-1})}\right). \quad (9)$$

Thus, formula (8), with (9) taken into account, provides an analytical solution to the problem of the Fraunhofer diffraction by an opening whose boundary is an arbitrary polygon. This result might also be presented in the following manner, similar to the (5) and (6) form:

$$S_F(\nu_x, \nu_y) = \frac{e^{i\varphi}}{8\pi^2 \nu_x \nu_y}\sum_{n=1}^{N} e^{-2\pi i(\nu_x x_n + \nu_y y_n)}(f_n - f_{n+1}), \quad (10)$$

where $f_n = (\nu_x - k_n\nu_y)/(\nu_x + k_n\nu_y)$, $f_{N+1} = f_1$. If $k_n \longrightarrow \infty$ tends to infinity (vertical segments), then the passage to the limit in (9) and (10) is not difficult to perform.

2. EXAMPLE OF NUMERICAL MODELING

To consider a case using the derived relations, let us check the results of calculating the intensity distribution for light that went through the mask, which is presented in Fig. 2a.

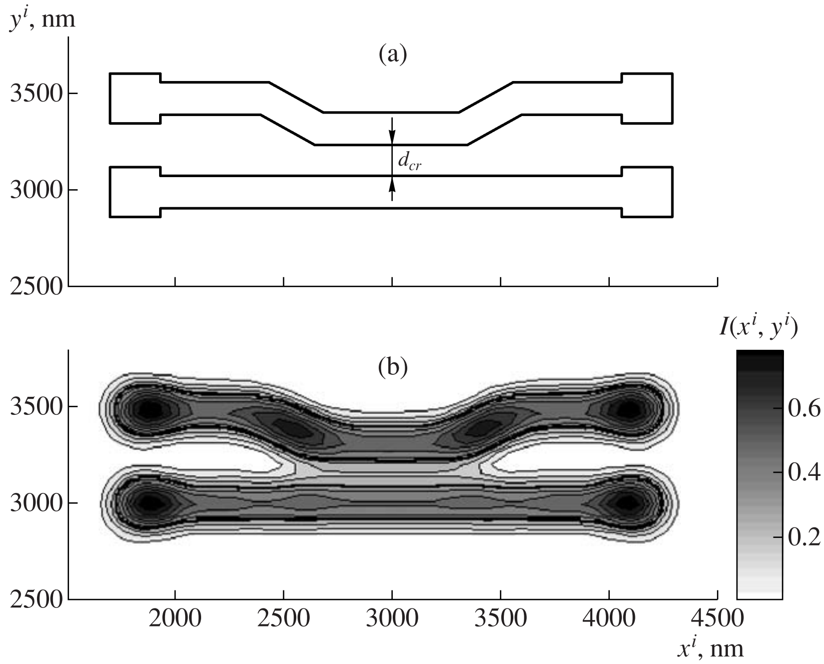

Fig. 2. (a) Test mask with critical size d_{cr} = 160 nm. (b) Calculated intensity distribution in the image plane. Circular illumination source λ = 248, coherence parameter σ = 0.5, and numerical aperture NA = 0.6.

Let us present the spectrum of the coherent impulse response $S_k(\nu_x, \nu_y)$ as follows, not taking the aberrations of the projection system into account [1, 5]:

$$S_K(\nu_x, \nu_y) = \begin{cases} 1, \text{ if } \nu_x^2 + \nu_y^2 \le \nu_{\max}^2 \\ 0, \text{ if } \nu_x^2 + \nu_y^2 > \nu_{\max}^2, \end{cases}$$

where $\nu_{\max}$ is the maximal frequency within the spectrum of spatial harmonics in an image plane; that frequency is determined by the light operating wavelength λ and the numerical aperture NA of the projection system: $\nu_{\max} = \mathrm{NA}/\lambda$.

It was supposed in the calculations that the mask was illuminated by an area source according the Koehler concept [2, 5] as follows:

$$S_B(\nu'_x, \nu'_y) = \begin{cases} 1, \text{ if } \nu_x'^2 + \nu_y'^2 \le (\nu_{\max}\sigma)^2 \\ 0, \text{ if } \nu_x'^2 + \nu_y'^2 > (\nu_{\max}\sigma)^2, \end{cases}$$

where σ is the coherence parameter.

The results of image calculations are presented in Fig. 2b. The threshold level for the intensity I_{th} is indicated in boldface; the intensity is given by $I_{\text{th}} = I_{\max} d^{-1}$, where d is the factor of photoresistance contrast [4], which is assumed to have a value of three. A reduction in calculation time by approximately one half in comparison with the approach based on decomposition of the mask configuration into elementary parts (rectangles and triangles) was accomplished. The total gain provided by the use of the formula (10) in performing simulations turns out to be rather significant, if we consider the complexity of present-day lithography masks.

Thus, this paper presents an analytical solution to the problem of the Fraunhofer diffraction by an opening, with the boundary being an arbitrary closed polygonal line. It is demonstrated that the usage of the solution allows us to increase the image calculation speed. This is remarkable, as numerical simulation is very widely used for making mask corrections in optical lithography [4].

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