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Specific Features of Excitations of "Flow"-Type Self-Oscillations in a Two-Component Active Medium of a Fast-Flow Gas Laser

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Abstract—The mechanism of "flow"-type self-oscillations in a two-component active medium of a flow laser with an unstable resonator is studied. It is shown that this mechanism is associated with the excitation of edge self-oscillatory in-phase perturbations of the medium components. These flow perturbations with low damping reach the optical axis of the resonator and result in an instability.

Key words: fast-flow laser, unstable cavity, active medium, self-oscillations.

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INTRODUCTION

The mechanism and characteristics of flow-type self-oscillations in flow lasers [1–7] differ considerably from those of relaxation oscillations [6, 8]. Thus, while in the case of relaxation oscillations the feedback created by the medium flux is nonresonance, flow-type oscillations are excited at resonance frequencies determined by the time of flight τ_f of the active medium through the resonator. The specific feature of flow-type oscillations is the spatial modulation of the medium perturbations formed at the edge of the mirrors at the flow input into the resonator ("edge perturbation"). Along with such edge flow-type oscillations, internal flow-type oscillations formed at the field gradients inside the resonator also exist [6].

Detailed analysis of the mechanism of flow-type oscillations in an unstable resonator and their interaction with other types of self-oscillations was performed in [6] for the case of a one-component active medium. The qualitative specific features of the mechanism of flow-type oscillations in two-component media, such as a CO_2–N_2 mixture, are studied in this paper.

NUMERICAL MODEL

Calculations were performed based on the same theoretical model and the same equations that were used in the first part of paper [8] devoted to relaxation oscillations. Omitting the model description, we present the equations for complex amplitudes of small perturbation modes; the numerical and analytical solutions of these equations were studied in [8]:

$$d\bar{g}_2/dx = -(\bar{\Gamma} + \gamma_2 + W_s + \gamma_{23})\bar{g}_2 + \gamma_{32}\bar{g}_3 - P_s\bar{w}, \quad (1)$$

$$d\bar{g}_3/dx = -(\bar{\Gamma} + \gamma_3 + \gamma_{32})\bar{g}_3 + \gamma_{23}\bar{g}_2, \quad (2)$$

$$(x-1)(d\bar{w}/dx) = \bar{g}_2 - \bar{\Gamma}\tau_c\bar{w}. \quad (3)$$

Here, $\bar{g}_2$ and $\bar{g}_3$ are, respectively, the perturbations of the amplification coefficient of the laser-active component and the population of the level of energy-carrying component multiplied by the cross section of the optical transition normalized to the resonator losses; $\bar{w}$ is the field perturbation in the resonator normalized to the stationary field intensity W_s; $P_s = G_{2s}W_s$ is the generated power; G_{2s} is the stationary amplification coefficient; γ_2, γ_3, γ_{23}, γ_{32} are the relaxation and exchange constants normalized to τ_f^{-1}; τ_c is the field damping time in the unstable resonator; and $\bar{\Gamma} = \Gamma + i\Omega$ is the complex increment of self-oscillations. Unlike [8], in (1)–(3) and the formulas below, the coordinate x is counted from the flow input into the resonator; in this case, the position of the optical axis corresponds to $x = 1$.

At the flow input into the resonator ($x = 0$) the following boundary conditions are satisfied:

$$\bar{g}_2(0) = 0, \quad \bar{g}_3(0) = 0. \quad (4)$$

On the optical axis of the unstable resonator (3) yields

$$\bar{g}_2(1)/\bar{w}(1) = \bar{\Gamma}\tau_c. \quad (5)$$

In most calculations the following approximate conditions of the quasi-stationary resonator model were used instead of (5):

$$\bar{g}_2(1) = 0, \quad \Phi_2(1) = 0, \tag{6}$$

where Φ_2 is the phase difference of amplification oscillations and the field. In the region $\tau_c < 10^{-3}$ covering all types of flow lasers, distortions of solutions are formed in a narrow interval near the optical axis of the resonator. In the limit $\tau_c \longrightarrow 0$ solutions become exact in the entire opened interval $0 < x \le 1$.

In the analytical model for the weakly inhomogeneous system the solutions of equations satisfying boundary conditions (6) are written in the form [6–8]

$$\begin{aligned} \bar{g}_2(x) &= \bar{g}_{2e}(x) + \bar{g}_{2u}(x), \\ \bar{g}_3(x) &= \bar{g}_{3e}(x) + \bar{g}_{3u}(x), \end{aligned} \tag{7}$$

where $\bar{g}_{2e}(x)$ and $\bar{g}_{3e}(x)$ are the quasi-homogeneous partial solutions; and $\bar{g}_{2u}(x)$, $\tilde{g}_{3u}(x)$ are the solutions of homogeneous system (1), (2) describing spatial oscillations of populations. The following relations were used for $\bar{g}_{2e}$ and $\bar{g}_{3e}$ [8]:

$$\bar{g}_{2e}(x) = -P_s\bar{w}[\bar{\Gamma} + \chi_{23} - \gamma_{23}\gamma_{32}/(\bar{\Gamma} + \chi_{32})]^{-1}, \tag{8}$$

$$\bar{g}_{3e}(x) = \gamma_{23}\bar{g}_{2e}/(\bar{\Gamma} + \chi_{32}), \tag{9}$$

where

$$\begin{aligned} \chi_{23} &= \gamma_{23} + \gamma_2 + W_s + (1/P_s)(dP_s/dx), \\ \chi_{32} &= \gamma_{32} + \gamma_3 + (1/P_s)(dP_s/dx). \end{aligned} \tag{10}$$

The functions $\bar{g}_{2u}(x)$ and $\bar{g}_{3u}(x)$ inside the small intervals $x_k \le x < x_{k+1}$ were approximated by the local solutions of the system of equations with constant coefficients,

$$\vec{g}_u(x) = \begin{pmatrix} \bar{g}_{2u}(x) \\ \bar{g}_{3u}(x) \end{pmatrix} = \begin{pmatrix} \bar{g}^{(1)}_{2u}(x_k) \\ \bar{g}^{(1)}_{3u}(x_k) \end{pmatrix} e^{\lambda_1(x_k)(x-x_k)} + \begin{pmatrix} \bar{g}^{(2)}_{2u}(x_k) \\ \bar{g}^{(2)}_{3u}(x_k) \end{pmatrix} e^{\lambda_2(x_k)(x-x_k)}. \tag{11}$$

In (11) the "vector" of the mixture perturbations $\vec{g}_u(x)$ is represented in the form of the superposition of the mixture eigenmodes, which are the eigenvectors of the matrix of coefficients of homogeneous system (1), (2). In this case, $\bar{g}^{(1)}_{3u}(x_k) = p_1(x_k)\bar{g}^{(1)}_{2u}(x_k)$, $\bar{g}^{(2)}_{2u}(x_k) = p_2(x_k)\bar{g}^{(2)}_{2u}(x_k)$, where $p_i(x_k)$, $\lambda_i(x_k)$ $(i = 1, 2)$ are expressed in terms of the coefficients of the system (1, 2) [9]. Along with vector $\vec{g}_u(x)$ (11), it is convenient to introduce the "vector of local solutions" $g_{2u}(x) = \begin{pmatrix} \bar{g}^{(1)}_{2u}(x) \\ \bar{g}^{(2)}_{2u}(x) \end{pmatrix}$ describing the solutions for the active component. Inside the intervals these vectors are connected as

$$g_{2u}(x) = \hat{P}g_u(x), \quad \vec{g}_u(x) = \hat{P}^{-1}g_{2u}(x), \tag{12}$$

where

$$\hat{P} = \frac{1}{p_1 - p_2}\begin{pmatrix} -p_2 & 1 \\ p_1 & -1 \end{pmatrix}, \quad \hat{P}^{-1} = \begin{pmatrix} 1 & 1 \\ p_1 & p_2 \end{pmatrix}. \tag{13}$$

The variation of the vector $g_{2u}(x)$ on the interval $\Delta x = x_{k+1} - x_k$ is determined by the transformation $g_{2u}(x_{k+1}) = t_{k+1}g_{2u}(x_k)$, where

$$\begin{aligned} \hat{T}_{k+1} &= \hat{P}(x_{k+1})\hat{P}^{-1}(x_k)\hat{\Lambda}(x_k), \\ \hat{\Lambda}(x_k) &= \begin{pmatrix} e^{\lambda_1(x_k)x} & 0 \\ 0 & e^{\lambda_2(x_k)x} \end{pmatrix}. \end{aligned} \tag{14}$$

At the point $x = x_n$ we have $g_{2u}(x_n) = \hat{T}_n\hat{T}_{n-1}\ldots\hat{T}_1 g_{2u}(0)$. In the limit $\Delta x \longrightarrow 0$, $n \longrightarrow \infty$ we have

$$g_{2u}(x) = \hat{T}(x)g_{2u}(0). \tag{15}$$

The matrix $\hat{T}(x)$ is connected with the fundamental solution matrix $\hat{M}(x)$ [9] as

$$\hat{T}(x) = \hat{P}(x)\hat{M}(x)\hat{P}^{-1}(0). \tag{16}$$

In the case of the weakly inhomogeneous system where the quantities $\lambda_i(x)$ and $p_i(x)$ are the "slow variables" and the following condition is satisfied: $|dp_i/dx| \ll p_1 - p_2$, we have

$$\begin{aligned} T_{11}(x) &= \left(1 - \int_0^x \frac{dp_1/dx}{p_1 - p_2}dx\right)\exp\left(\int_0^x \lambda_1 dx\right); \\ T_{12}(x) &= -\int_0^x \frac{dp_2/dx}{p_1 - p_2}\exp\left(\int_0^{x'} \lambda_2 dx'' + \int_{x'}^x \lambda_1 dx''\right)dx', \end{aligned} \tag{17a}$$

$$\begin{aligned} T_{22}(x) &= \left(1 + \int_0^x \frac{dp_2/dx}{p_1 - p_2}dx\right)\exp\left(\int_0^x \lambda_2 dx\right); \\ T_{21}(x) &= \int_0^x \frac{dp_1/dx}{p_1 - p_2}\exp\left(\int_0^{x'} \lambda_1 dx'' + \int_{x'}^x \lambda_2 dx''\right)dx'. \end{aligned} \tag{17b}$$

For homogeneous system (1), (2), introducing the damping coefficients for spatial oscillations $k_{1,2} = -\lambda_{1,2} - \bar{\Gamma}$, we find

$$k_{1,2} = \gamma_3 + \hat{\gamma}_\Sigma/2 \mp (1/2)\sqrt{\hat{\gamma}_\Sigma^2 - 4\gamma_{32}\hat{W}_s}, \quad p_{1,2} = (\hat{\gamma}_{32} + W_s - k_{1,2})/(\gamma_{32}). \tag{18}$$

Here, the following notation is introduced: $\hat{\gamma}_{ik} = \gamma_{ik} + \gamma_i$, $\gamma_\Sigma = \gamma_{23} + \gamma_{32}$, $\hat{\gamma}_\Sigma = \gamma_\Sigma + \hat{W}_s$, $\hat{W}_s = W_s + \gamma_2 - \gamma_3$. It follows from (18) that $p_1 > 0$ and $p_2 < 0$ are real numbers. Thus, in the first solution with $\lambda = \lambda_1$ the component oscillations are in-phase, and in the second case with $\lambda = \lambda_2$, they are anti-phase. In the typical case $\gamma_{ik} \gg W_s$, the following approximate relations are satisfied:

$$k_1 = \xi_2(W_s + \gamma_2) + \xi_3\gamma_3 - \xi_2\xi_3\hat{W}_s^2/\gamma_\Sigma, \quad p_1 = (\gamma_{23} + \xi_3\hat{W}_s)/\gamma_{32}, \tag{19}$$

$$k_2 = \gamma_\Sigma + \xi_3(W_s + \gamma_2) + \xi_2\gamma_3 + \xi_2\xi_3\hat{W}_s^2/\gamma_\Sigma, \quad p_2 = -1 + W_s/\gamma_\Sigma, \tag{20}$$

where ξ_2, ξ_3 are the relative component concentrations in the mixture. It can be seen from (19), (20) that the anti-phase perturbations damp much faster than in-phase perturbations. In the limiting case of the slow exchange $\gamma_{ik} \ll W_s$ we have

$$k_1 = \hat{\gamma}_{32} - \xi_2\xi_3\gamma_\Sigma^2/\hat{W}_s, \quad p_1 = (\hat{W}_s + \gamma_{23} - \gamma_{32})\gamma_{32}^{-1}, \tag{21}$$

$$k_2 = W_s + \hat{\gamma}_{23} + \xi_2\xi_3\gamma_\Sigma^2/\hat{W}_s, \quad p_2 = -\gamma_{23}/\hat{W}_s. \tag{22}$$

Here, the solutions are "split", the in-phase solution with $p_1 \gg 1$ describes the medium with preferential perturbation of the inactive component ($g_{3u} \gg g_{2u}$), and the anti-phase perturbation ($p_2 \longrightarrow 0$), with that of the active component $g_{3u} \ll g_{2u}$).

Relations (19)–(22) allow one to obtain rather simple expressions for the matrix elements T_{ik}. In the case of strong exchange, the off-diagonal elements T_{12} and T_{21} responsible for "mixing" of solutions turn out to be small, and accounting for these elements yields only small corrections. Let us give the expressions for the diagonal elements:

$$T_{11}(x) = \{1 + (\xi_3/\gamma_\Sigma)[W_s(0) - W_s(x)]\} \times \exp\left(-\bar{\Gamma}x - \hat{\gamma}x - \xi_2\int_0^x W_s dx\right), \tag{23}$$

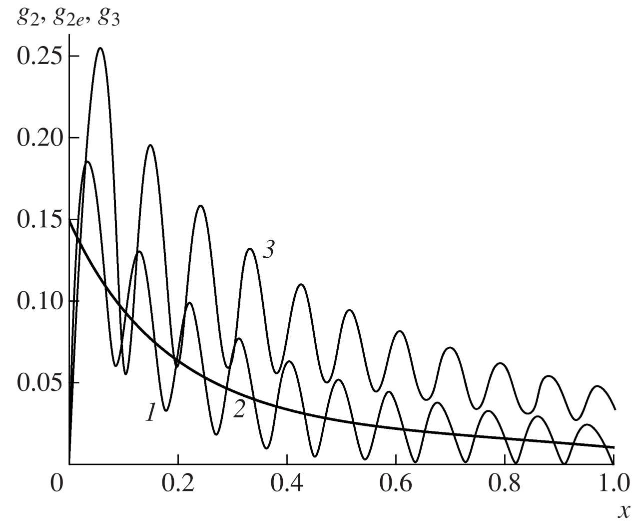

Fig. 1. Perturbation mode amplitudes (*1*) g_2, (*3*) g_3, and (*2*) "slow" solution g_{2e} of flow-type oscillations of order $m = 11$ in the mixture $\xi_2 : \xi_3 = 1 : 3$. Calculation conditions: matched system, $\gamma_2 = 0.2$, $\gamma_3 = 1$, $\Gamma = 0.5$, $\gamma_{32} = 20$.

$$T_{22}(x) = \{1 - (\xi_2/\gamma_\Sigma)[W_s(0) - W_s(x)]\} \times \exp\left(-\bar{\Gamma}x - (\gamma_\Sigma + \hat{\gamma}')x - \xi_3\int_0^x W_s dx\right). \tag{24}$$

Here, $\hat{\gamma} = \xi_2\gamma_2 + \xi_3\gamma_3$, $\hat{\gamma}' = \xi_3\gamma_2 + \xi_2\gamma_3$ are the effective relaxation constants. The magnitude of the increment of flow-type oscillations depends on the damping rate of edge oscillations in the resonator and the initial amplitudes of the perturbation modes $\bar{g}_{2u}^{(1)}(0)$ and $g_{2u}^{(2)}(0)$, which are determined from boundary condition (4). It is convenient to characterize their values by the complex coefficients

$$\bar{\chi}_1^0 = -\bar{g}_{2u}^{(1)}(0)/\bar{g}_{2e}(0), \quad \bar{\chi}_2^0 = -\bar{g}_{2u}^{(2)}(0)/\bar{g}_{2e}(0), \quad (\bar{\chi}_1^0 + \chi_2^0 = 1). \tag{25}$$

Using transformations (13) and relation (9) we find

$$\bar{\chi}_1^0 = \frac{-p_2(0) + \gamma_{23}/(\bar{\Gamma} + \chi_{32}(0))}{p_1(0) - p_2(0)}, \quad \bar{\chi}_2^0 = \frac{p_1^0 - \gamma_{23}/(\bar{\Gamma} + \chi_{32}(0))}{p_1(0) - p_2(0)}. \tag{25a}$$

Then introducing $T'_{ik}(x) = \exp(\bar{\Gamma}x)T_{ik}(x)$, we find from boundary condition (6) on the axis

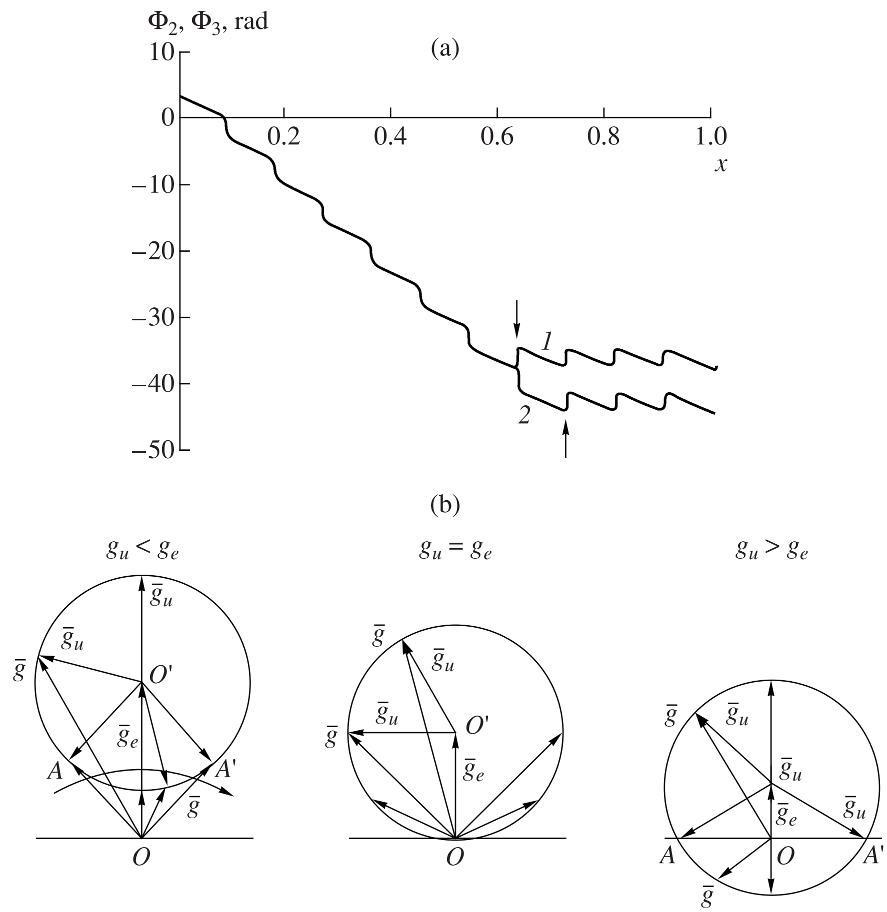

Fig. 2. (a) Phases of perturbation modes of the components (*1*) Φ_2 and (*2*) Φ_3. Arrows show phase bifurcations; (b) vector diagrams of addition $\bar{g}_e$ and $\bar{g}_u$ for different amplitude ratios.

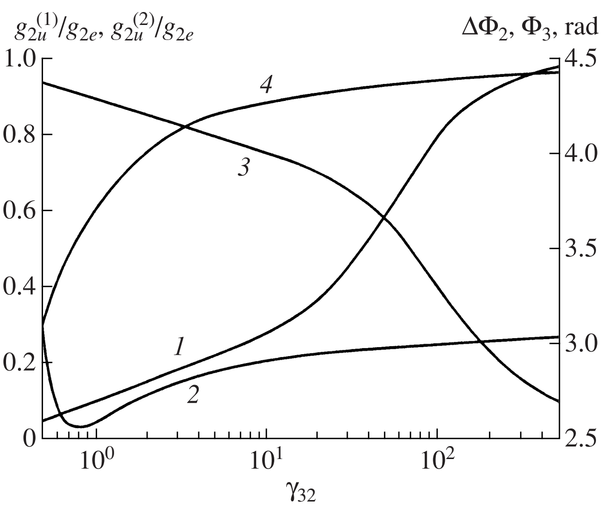

Fig. 3. Amplitudes of (*1*) in-phase $g_{2u}^{(1)}$ and $\bar{g}_{2u}^{(2)}$ (*3*) antiphase solutions normalized to g_{2e}; and phase shifts (*2*) $\Delta\Phi_2 = \Phi_{2u}^{(1)} - \Phi_{2e}$, (*4*) $\Delta\Phi_3 = \Phi_{2u}^{(2)} - \Phi_{2e}$ at the input of the resonator for different exchange rates.

$$\bar{\Gamma} = \ln(\bar{g}_{2e}(0)/\bar{g}_{2e}(1)) + \ln[(\hat{T}'_{11} + T'_{21})\bar{\chi}_1^0 + (T'_{12} + T'_{22})\bar{\chi}_2^0], \tag{26}$$

where $T'_{ik} = T'_{ik}(1)$.

In the particular case of an infinitely fast exchange when the mode mixing is absent, we obtain for the increment of the flow-type oscillations, in agreement with [3]

$$\Gamma_\infty \approx \ln(G_s(0)W_s(0)/W_s(1)) - \hat{\gamma} - \xi_2\int_0^1 W_s(x)dx. \tag{27}$$

It can be easily shown that in this case the two-component medium turns out to be similar to the one-component medium with $q' = \xi_2(q_2 + q_3)$, $\gamma' = \hat{\gamma}$, $W'_s = \xi_2 W_s$.

The applied approach based on the decomposition of the edge perturbation of the medium in the mixture modes can be generalized to the case of multicomponent media with several active components.

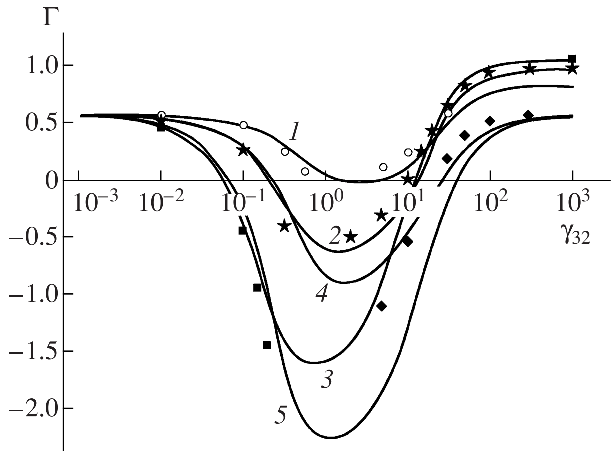

Fig. 4. Increments of flow-type oscillations mode ($m = 5$) in the mixtures (*1*) 1 : 1, (*2*) 1 : 3, (*3*) 1 : 10 for matched systems and different exchange rates. The increments of flow-type oscillations in the mixtures (*4*) 1 : 1 and (*5*) 1 : 3 in the absence of matching. Points show calculation using the analytical model.

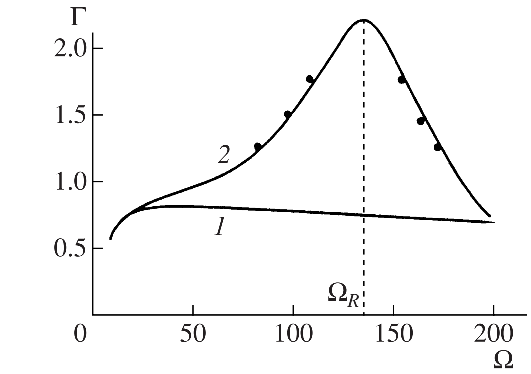

Fig. 5. Influence of relaxation resonance on increments of flow-type oscillations. Increments as functions of self-oscillation frequency (*1*) in the absence of relaxation resonance ($\tau_c \longrightarrow 0$, $\Omega_R \longrightarrow \infty$), (*2*) in the presence of relaxation resonance ($\Omega_R = 137$). The mixture 1 : 1, $\gamma_{23} - \gamma_{32} = 300$. Points show calculation using the analytical model.

NUMERICAL RESULTS AND DISCUSSION

Figures 1 and 2a show the spatial structure of the modes. The analysis of the curves shows that except for the narrow region near the point $x = 0$ they correspond to the in-phase mode of edge perturbation. The shift of maximums of the curves in Fig. 1 is due to the phase difference of oscillations, $\Delta\Phi = \Phi_{2e}(x) - \Phi_{3e}(x)$. The initial amplitudes and phases of edge perturbation modes at the input of the resonator are shown in Fig. 3. It can be seen that for the data in Fig. 1 at the flow input into the resonator the anti-phase mode is preferential, but according to (20), it damps fast at $x \approx 10^{-2}$. The perturbation phases Φ_2 and Φ_3 (with respect to the phase of field oscillations) demonstrate different behavior depending on the ratio of the amplitudes g_u and g_e (Fig. 2a). If $g_u > g_e$, the phase monotonically decreases with increasing x. This takes place for curve *3* in Fig. 1, where the amplitude g_{3u} considerably exceeds g_{3e} (by approximately a factor of 4). If, on the contrary, $g_u < g_e$, the phase variation has the character of saw-tooth oscillations [6, 7]. This case corresponds to curve *1* in Fig. 1. At the point at which $g_u = g_e$, phase bifurcation takes place. This is elucidated by the vector diagrams of summing perturbations shown in Fig. 2b. For simplification of the diagram it is assumed that the vector $\bar{g}_e$ is directed along the imaginary axis. If x changes, the end of the vector $\bar{g}_u$ moves along the circle with the center at the point O' coinciding with the end of the vector $\bar{g}_e$. The end of the total vector $\bar{g}(x) = \bar{g}_e(x) + \bar{g}_u(x)$ also moves along this circle. If $g_u = g_e$, the phase jumps by π at the point O.

Figure 4 shows the dependences of the increments of flow-type oscillations on the exchange rate for the set of mixtures. The calculations were performed for the “matched” medium with the specially chosen pumping profile [8] and in the absence of matching. In the latter case, the component pumping rates were assumed to be proportional to their concentrations, and the resonator loss value, to ξ_2. The parameters of the system were chosen in such a way that in the absence of exchange the increment of the flow-type oscillations was positive.

In the region of weak exchange $\gamma_{ik} \ll W_s, \Omega$, when the backward perturbation flux $\bar{J}_{32} = \gamma_{32}\bar{g}_3$ does not play a noticeable role [8] the action of the forward flux $\bar{J}_{23} = \gamma_{23}\bar{g}_2$ leads to the increase in the relaxation rate. Therefore, according to (22), we have $\Gamma = \Gamma_0 - \gamma_{23}$, where Γ_0 is the increment in the absence of exchange. In this region, the anti-phase edge perturbation mode is preferentially excited.

In the intermediate region of incomplete exchange $W_s < \gamma_{ik} \leq \Omega$ the contribution of the slowly damping in-phase edge perturbation mode slowly increases. As a result, the increment stops to decrease and starts to grow, and the instability threshold is reached. In the region of strong exchange, $\gamma_{ik} \gg \Omega$ the increments take limiting values Γ_∞ (27). For matched mixtures with the same $W_s(x)$, $G_{2x}(x)$, the values of Γ_∞, according to (27), turn out to be higher for “poor” mixtures ($\xi_2 < \xi_3$). It was assumed in calculations without matching that $\gamma_2 = \gamma_3$; therefore, for all mixtures the values of Γ_∞ were the same.

Figure 5 illustrates the influence of the relaxation resonance on the increments of flow-type oscillations. Similar to the case of the one-component medium [6] the region of interaction of flow and relaxation oscillations turns out to be wide enough.

CONCLUSIONS

Calculations show that, in a similar manner to the case of relaxation oscillations [8], the perturbation exchange between the components of the active medium of a flow laser results in the qualitative change in the picture of flow-type self-oscillatory instability. The appearance of a rather wide region of stable stationary generation in the range of exchange rates $W_s < \gamma_{23} < \Omega$, in which flow-type oscillations are efficiently suppressed, is characteristic for mixtures. Instabilities are formed in the region of higher exchange rates $\gamma_{32} \geq \Omega$ where the contribution into the perturbation of the inphase oscillation mode of the mixture becomes determining. In CO_2–N_2 mixtures, instability can arise for partial CO_2 pressures of the order of several Torr.

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