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Method of Specification of the Initial Conditions for Numerical Tsunami Modeling

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Abstract—A new method for the specification of initial conditions in the tsunami problem is suggested. The initial elevation of a water surface is determined from the solution of the three-dimensional problem in the framework of potential theory with account for both the horizontal and vertical components of the bottom deformations and the distributions of depths in the vicinity of the source. The tsunamigenic Central Kuril Islands earthquakes in 2006 and 2007 are taken as examples to demonstrate the efficiency of the new method.

Key words: tsunami waves, bottom earthquake, permanent bottom deformations, tsunami generation.

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INTRODUCTION

Numerical models of tsunami waves, as a rule, are developed on the basis of the theory of long waves, which operates by the vertical coordinate-averaged equations of hydrodynamics [1–4]. The process of tsunami formation by an underwater earthquake is regarded as instantaneous. Thus, the system of equations in the theory of long waves is solved by taking the initial conditions into consideration, represented by some displacement of the free water surface from an equilibrium position (the initial elevation) at a zero field of flow velocity. The initial elevation is assumed to be equal to vertical permanent bottom deformations, resulting from an underwater earthquake. The bottom deformations are calculated on the basis of earthquake source parameters, using analytic solutions of the stationary problem of the theory of elasticity (using formulas by Y. Okada) [5].

The traditional approach of specification of the initial conditions has wide application in numerical modeling of real events, so it adequately displays the main mechanism of tsunami generation (water displacement by bottom deformation). In the present work the modified approach of specification of the initial conditions, which more reasonably describes the tsunami generation process is proposed and justified. The modified approach is compared with the traditional one using the example of a tsunami induced by earthquakes (November 15, 2006 and January 13, 2007), occurring near the Central Kuril Islands.

The imperfection of the traditional method of specification of the initial conditions is due to two causes. First of all, at the moment of the completion of the bottom deformation, the water surface deviation $\xi(x, y, \tau)$ from an equilibrium position and vertical permanent deformation $\eta_z(x, y, \tau)$ are not equal, even in the case of horizontal flat bottom and impulse movement ($\tau \ll L(gH)^{-1/2}$, where τ is movement duration, L is horizontal extension of the source, and g is intensity of gravity). Secondly, in case of an inclined bottom, the horizontal components of deformation also lead to water displacement, and, accordingly, make a contribution to the initial elevation.

As it follows from the analytic solution of the problem of tsunami generation in the constant depth basin by small vertical deformations that the spatial spectrum of the water surface displacement is modulated by the rapidly damped function $1/\mathrm{ch}\,kH$, where k is wave number [3]. Accordingly, the bottom movements cannot create disturbances with a wave length $\lambda < H$ on the surface. For near-surface earthquakes, and especially in the cases where a fault is exposed on the bottom surface, the spatial distribution of the bottom deformations can be characterized by horizontal nonuniformities with scales less then the oceanic depth. Thus, the direct translation of the bottom deformations on the water surface leads to artificial saturation of the tsunami spectrum with short-wave components, which do not exist in reality. In the numerical models, for an adequate description of the short-wave components it will be necessary to decrease space–time intervals, which leads to an increase in the calculation time. The resonant response of the shelf and small scale bays to short-wave components makes it difficult to interpret the calculation data, and, in some cases, they stimulate the development of the disequilibrium of numerical schemes. It must be emphasized once more that all these problems are associated with spectral components, which are not typical for a real tsunami. These components are artificially introduced into a model by the oversimplified method of specification of the initial conditions.

K. Kajiura [6] pointed out the necessity to take into account a "smoothing effect," especially in the cases where the oceanic depth is comparable to the horizontal extension of the source. At the present time, there are only two works where authors attempted to take this effect into account under real tsunami calculations. In [7], the initial elevation is calculated using an analytic formula obtained for the case of horizontal flat bottom. Use of this formula for a variable depth basin is not obviously correct. As established in [8], the initial elevation was calculated on the basis of a three-dimensional Laplace equation, but the mathematic model used for its calculation was not described.

A logical development of the method of specification of the initial conditions is a calculation of the initial elevation $\xi(x, y, \tau)$, which follows from solution of the three-dimensional task, concerning gravitational waves in a liquid, accounting for all three components of a bottom deformation vector $\eta = (\eta_x, \eta_y, \eta_z)$ and depth distribution $H(x, y)$ in the area of the source. This problem is solved as follows:

$$\Delta F = 0, \tag{1}$$

$$F_{tt} = -gF_z, \quad z = 0, \tag{2}$$

$$\frac{\partial F}{\partial n} = \left(\frac{\partial \eta}{\partial t}, n\right), \quad z = -H(x, y), \tag{3}$$

where F is a flow velocity potential, n is the normal to the bottom surface. The desired initial elevation is calculated using the next formula: $\xi(x, y, \tau) = -g^{-1}F_t(x, y, 0, \tau)$. If the process of the bottom deformation is long-term ($\tau \sim L(gH)^{-1/2}$), then in the initial conditions one should use the initial distribution of flow velocities together with the initial elevation in the tasks of spreading

$$\mathbf{V}(x, y, \tau) = \frac{1}{H(x, y)} \int\limits_{-H(x, y)}^{0} dz \nabla F(x, y, z, \tau).$$

In the case where the bottom deformation can be considered as instantaneous, the evolution task (1)–(3) easily reduces to more simple, statical one:

$$\Delta \hat{F} = 0, \tag{4}$$

$$\hat{F} = 0, \quad z = 0, \tag{5}$$

$$\frac{\partial \hat{F}}{\partial n} = (\eta, n), \quad z = -H(x, y), \tag{6}$$

where $\hat{F} = \int\limits_0^\tau F dt$.

The initial elevation is determined from the task (4)–(6) solution, according to the formula:

$$\xi(x, y) = \hat{F}_z(x, y, 0).$$

In November 2006 and January 2007, two strong earthquakes took place to the east of Simushir Island (Central Kurily Islands) with epicenters in the area of the Kuril–Kamchatka Trench (November 15, 2006, $M_w = 8.3$ and January 13, 2007, $M_w = 8.1$, respectively). Both these seismic events induced tsunami waves, recorded over the entire Pacific Ocean and registered by many coastal and deep-ocean tide gauges [8, 9]. According to the data recorded by remote tide gauges the tsunami of November 2006 was characterized by waves of higher amplitude as compared to the tsunami of January 2007. The coast of the Central Kuril Islands, closest to the tsunami sources is not equipped with tide gauges, and, additionally, this is an uninhabited and difficult to access region. During the expedition survey of this area, carried out in the summer of 2007 the height of wave uprushes up to 20 m was observed [10]. It is not known for certain which event was responsible for such wave heights.

For calculation of permanent bottom deformations induced by the earthquake on November 15, 2006 (January 13, 2007) the data of displacement structure in digital format, available at the server of the U.S. Geological Survey (USGS) have been used. The fault plane is represented by a 400 × 137.5 km (200 × 35 km) rectangle, elongated along the horizontal. The orientation of the fault plane is defined by the strike angle 220.23° (42.36°) and dip angle 14.89° (57.88°). The upper edge of the fault plane was located under the bottom surface at a depth of 5.9 km (4.6 km); the lower one, at a depth of 40.6 km (33.4 km). The fault plane is divided into 220 (175) rectangular elementary subfaults with size of 20 × 12.5 km (8 × 5 km). For every subfault the displacement vector has been determined. The displacement value along the fault reached 8.9 m (20.3 m).

The vector field of the bottom deformation η, included in the formula (6), has been calculated as a superposition of contributions from elementary subareas with use of Y. Okada's formulas. The calculation results are shown in Fig. 1. The amplitude of vertical bottom deformation for both events was as follows: elevation is 2.7 m and 1.9 m, subsiding is 0.6 m and 7.7 m, respectively. The horizontal bottom deformation was 3.8 m and 3.4 m, respectively. In both cases the strongest deformations were associated with the deep water and step slopes of the Kuril-Kamchatka Trench.

The initial elevation has been determined from the task (4)–(6), solved numerically by the relaxation method [11]. Under the calculations the space steps $\Delta x \approx \Delta y \approx 1000$ m, $\Delta z \approx 200$ m have been used. Bathymetry of the examined area was taken from the GEBCO Digital Atlas (GDA).

The calculation results of the initial elevation are shown in Figs. 2 and 3. For the 2006 event the maxi-

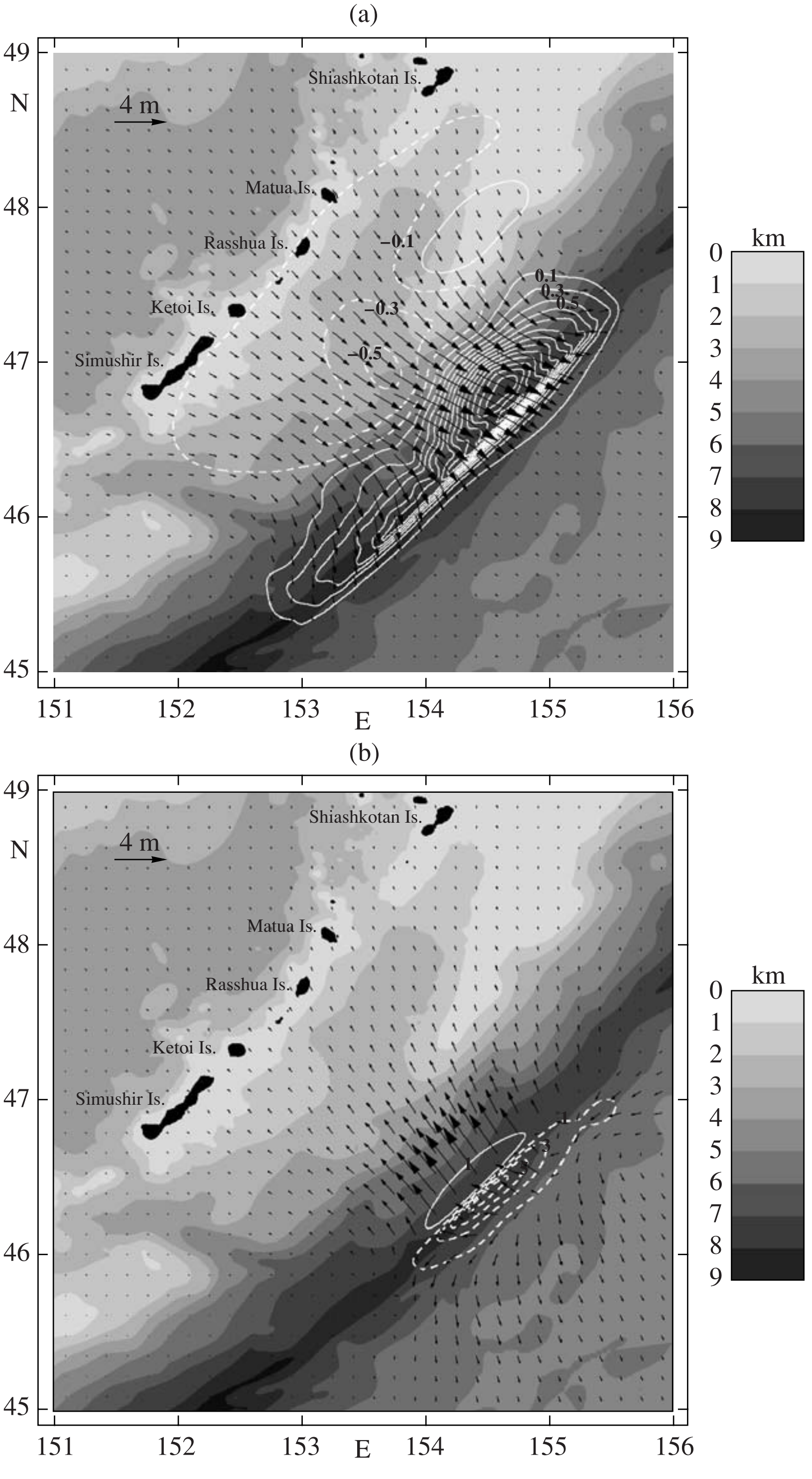

Fig. 1. Permanent bottom deformations in the tsunami source of November 2006 (November 15, 2006) (a) and January 2007 (January 13.2007) (b). The vertical deformations are shown by white contours (solid line is elevation; dashed line, subsiding). Figures near isolines are deformation values (m). Black arrows are vectors of horizontal deformation. The depth distribution is shown by gradual gray tone. Isobath interval is one km.

mum values of the water surface deviation (elevation is 2.5 m; downwelling is 0.5 m) do not differ significantly from the corresponding characteristics of the vertical deformation. However, for the 2007 event, the deviation amplitude of the water surface (uplifting is 0.8 m; subsiding is 4.4 m) was two times less than that of the vertical bottom deformation. The observed significant difference occurs due to the fact that in the second case,

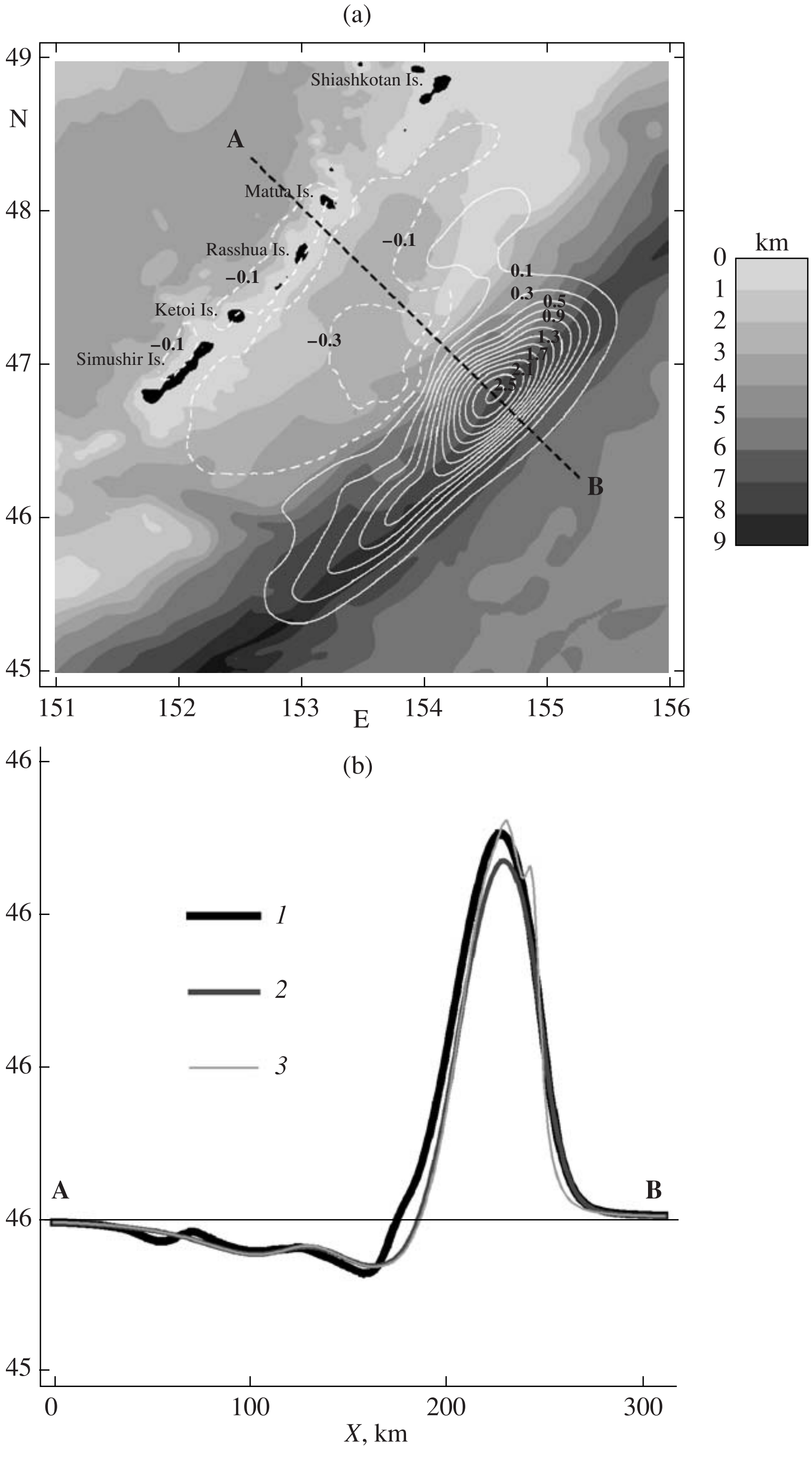

Fig. 2. Initial elevation in the tsunami source of November 2006 (November 15, 2006). (a) Solid white lines is elevation; dashed lines, subsiding. Figures near isolines are values of the water surface deviation (m). (b) Profile of the initial elevation is along black dashed line AB: (*1*) calculation with account of all three components of the bottom deformation vector, (*2*) calculation with account of only vertical component of the bottom deformation vector, (*3*) vertical bottom deformation.

the width of the bottom deformation area is comparable with oceanic depth.

Peculiarities of the "fine structure" of the initial elevation are clearly shown at the profiles presented in Fig. 2b and 3b. These profiles represent the displacement of the water surface along dashed lines, shown in Fig. 2a and 3a. The profiles have been calculated taking into account all three components of the bottom deforma-

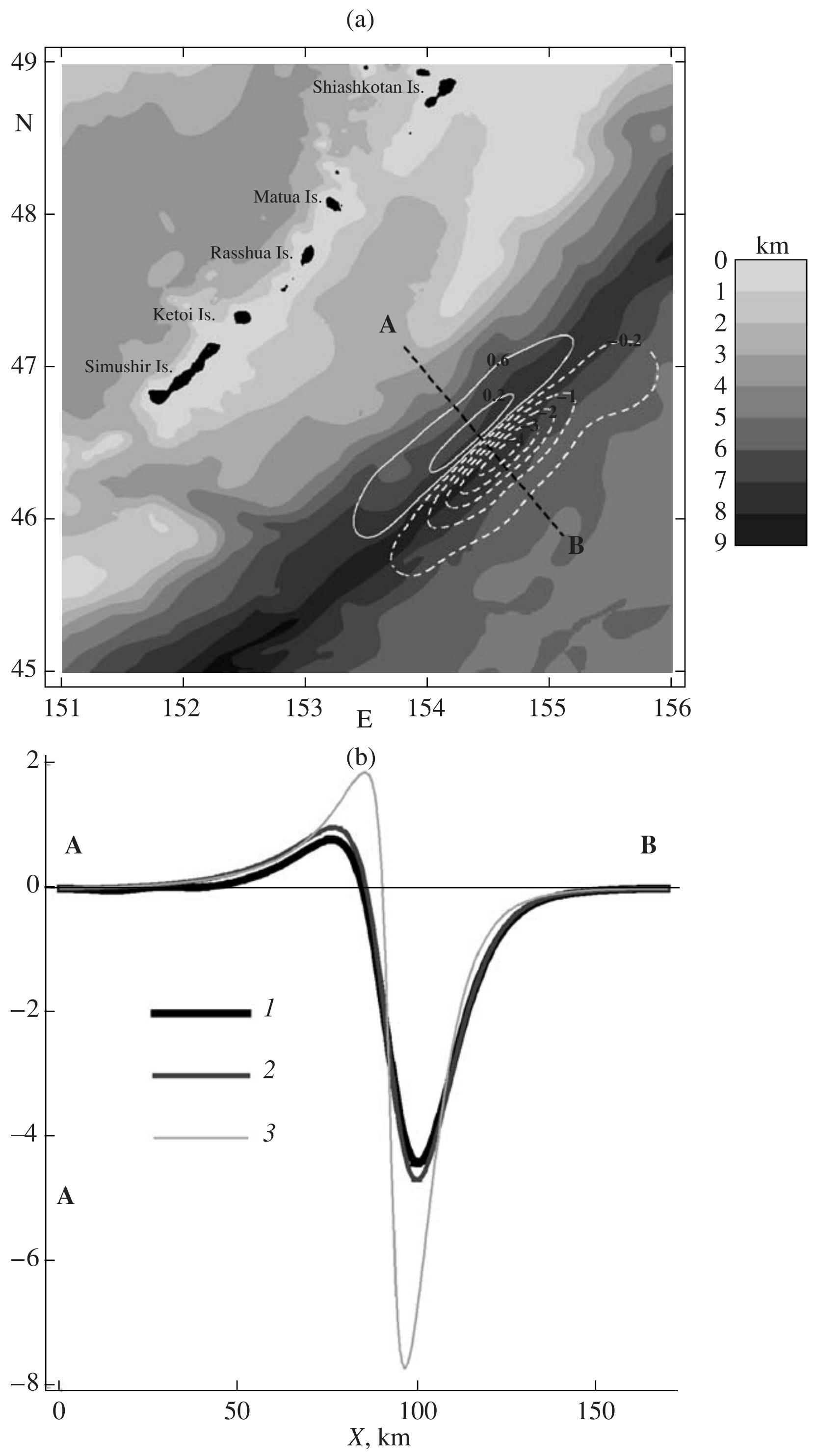

Fig. 3. Same as Fig. 2 but for tsunami of January 2007 (January 13, 2007).

tions (*1*), and the only vertical component (*2*). For comparison, the profile of the vertical bottom deformation is also presented (*3*).

As shown in Fig. 2b, a double-peaked “fine structure,” which is peculiar to the vertical bottom deformation in the peak region, is not manifested at the water surface displacement. In addition, as follows from the difference between curves (1) and (2), the horizontal deformation or displacement of the northern-west slope of the Trench towards the south-east in fact makes a significant contribution to the initial elevation.

The water volume, displaced by the bottom deformation V_0 and the potential energy of the initial elevation W_0: a – initial elevation, calculated with account of all three components of the bottom deformation vector; b – initial elevation, calculated with account of only vertical component of the bottom deformation vector; c – initial elevation, equivalent to vertical bottom deformation

	a		b		c	
Date	V_0, km^3	W_0, 10^{14} J	V_0, km^3	W_0, 10^{14} J	V_0, km^3	W_0, 10^{14} J
November 15, 2006	8.97	1.00	6.09	0.85	6.08	0.97
January 13, 2007	–6.37	1.23	–5.20	1.36	–5.20	2.36

A significant difference between the vertical bottom deformation and the water surface displacement can be observed in Fig. 3b. This means that the traditional approach for the tsunami of January 2007 (that is, translation of permanent bottom deformations to the water surface) leads to double amplitude overestimation. The proximity of curves (1) and (2) testifies that in this case the horizontal deformation plays an insignificant role.

The demonstrative characteristics of the tsunami source are the water volume, displaced by the bottom deformation and the potential energy of the initial elevation. These characteristics have been calculated using the following formulas: $V_0 = \iint \xi(x, y, \tau)dxdy$, $W_0 = \frac{\rho g}{2}\iint \xi^2(x, y, \tau)dxdy$, where ρ is the water density.

All calculations have been made taking into account all three components of the bottom deformation vector, the only vertical component, and, additionally, by traditional methods, in the case of $\xi = \eta_z$. All results are given in table.

As shown from Table 1, the contribution of the horizontal component of the bottom deformation into the displaced volume exceeds 30% for the 2006 event and approximately comes up to 20% for the 2007 event. This significant contribution is associated with the vastness of the area subjected to the horizontal deformation. According to the absolute value of the displaced water volume, the first event was more powerful. It is interesting that the estimation of energy leads to another result, in which the tsunami of January 2007 was more powerful. The energy value is sufficiently sensitive to the calculation method, which is most noticeable in the results of the 2007 event study. It should illustrate that the traditional method overestimates the energy value by about twofold.

We note finally that the initial elevation calculated by the method proposed certainly smoothes the small-scale spatial uniformities of the bottom deformation, but it includes components with a wave length $\lambda > {\sim}H$. These waves, is contrast with long ones ($\lambda \gg H$) are subjected to phase dispersion, and, accordingly, their transoceanic propagation cannot be described in adequate way by the theory of long waves. The fact that the second high-amplitude and high-energy event (January 13, 2007), recorded by the remote tide gauges was relatively weak can be explained by energy "smearing" in the space due to phase dispersion.

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