ISSN 0027-1349, Moscow University Physics Bulletin, 2009, Vol. 64, No. 2, pp. 221–222. © Allerton Press, Inc., 2009.
Original Russian Text © Yu.G. Pavlenko, 2009, published in Vestnik Moskovskogo Universiteta. Fizika, 2009, No. 2, pp. 110–111.

Image Subtraction by Polarized Light

Yu. G. Pavlenko

Kolmogorov School, Moscow State University, Moscow, 121357 Russia

Received May 21, 2008

Abstract—It is shown that holographic images can be subtracted by substitution of a mirror for a phase plate.

DOI: 10.3103/S0027134909020258

INTRODUCTION

In most studies on holography, a "scalar" form of electromagnetic field is used, despite the fact that the electric-field strength is a vector. Taking into account the polarization of a wave, one can consider new consequences of the theory. One of them is related to the change in polarization upon reflection from the interface between two media. At wave incidence on a metal plate, polarization of the wave changes. If a planar wave with polarization vector a_i propagates in the direction of unit vector n_i, then after the reflection we have $a_f = -a_i + 2p_i(pa_i)$ and $n_f = n_i - 2p(pn_i)$, where p is the unit vector perpendicular to the plate. In the case of a wave with the polarization vector perpendicular to the plane of incidence, $pa_i = 0$, we have $a_f = -a_i$. This effect can be used for implementation of image subtraction upon holographic recording of several images.

1. SOLUTION OF THE MAXWELL EQUATIONS IN THE PARAXIAL APPROXIMATION

In the case of a monochromatic field, $E(t, x) = \mathrm{Re}E_\omega(x)\exp(-i\omega t)$, the Maxwell equations yield the complex amplitude equation:

$$\mathrm{rotrot}E_\omega - k^2 E_\omega = 0, \qquad (1)$$

where $k = \omega/c$. We will search for the solution of Equation (1) in the form of a quasi-planar wave in the vicinity of the z axis. In the paraxial approximation, it is convenient to denote the radius vector of the point as $x = (x_n, z)$, where $n = 1$ or 2.

Introducing the Green function, one can write the integral equation that relates the electric-field strength at some point with the strength values at the closed surface [1, 2]. Represent the solution of Equation (1) as $E_\omega(x) = e(x_n, z)\exp(ikz)$. Let the field strength $e(x_n, z') = u(x_n, z')\exp(-ikz')$ be known in the plane $z = z'$. Using the Kirchhoff's approximation, we obtain the solution of Equation (1) in the following form:

$$\theta(z-z')e_m(x) = \int d^2x' G_{ms}[x_n - x'_n, z - z'] \times u_s(x'_n, z')\exp(-ikz'). \qquad (2)$$

Here $\theta(z - z')$ is the Heaviside function and $G_{ms} = gP_{ms}(n')$, where g is the Green function of the Leontovich parabolic equation

$$2ik\partial_z g + \partial_{nn} g = 2ik\delta^{(3)}(x - x'),$$

$$g[x_n - x'_n, z - z'] = [2\pi i(z-z')/k]^{-1}\exp[ik(x_n - x'_n)^2/2(z-z')].$$

The polarization tensor is $P_{ms}(\mathbf{n}') = \delta_{ms}(\mathbf{n}'\mathbf{N}) - N_m n'_s$, where $n' = (x - x')/(z - z')$, $\mathbf{N} = (n' + p)/2$, and $\mathbf{p} = (0, 0, 1)$.

Note that the relation $n'_m P_{ms}(x') = 0$ is valid and the invariance condition for the Frenel transformation is

$$\int d^2x' g[x_n - x'_n, z - z']g[x'_n - x_{0n}, z' - z_0] = g[x_n - x_{0n}, z - z_0]. \qquad (3)$$

In the limit $z \longrightarrow z'$ relation (2) becomes an identity as $up = 0$:

$$g[x_n - x'_n, z - z'] \longrightarrow \delta^{(2)}(x_n - x'_n), \quad n' \longrightarrow p. \qquad (4)$$

2. OBJECT "SUBTRACTION"

Formation of a Hologram

Let us place a transparent photo layer in the plane $z = 0$. The amplitude of a reference linearly polarized wave is $\mathbf{E}(\mathbf{x}) = \mathbf{a}\exp(-ikz)$, where $a = (0, a, 0)$ and $a = a^*$. The wave intensity $J_0 = c\varepsilon_0 a^2/2$. The electric-field strength for the wave scattered by objects B located in the plane $z = -d_1$ is determined by Frenel transformation (2):

$$b_m(x) = \int d^2x_{0n} G_{ms}[x_n - x_{0n}, z + d_1] \times a_s f_b(x_{0n})\exp(ikd_1), \quad z \geq -d_1, \qquad (5)$$

where $f_b(x_n)$ is the function that characterizes the response of objects B.

In the hologram plane $z = 0$ the field strength amplitude is $e^{(1)} = a + b(x_1, x_2, 0)$. The mean intensity value is $J^{(1)}(x_1, x_2) = (c\varepsilon_0/2)[a^2 + bb^* + (ab^* + a^*b)]$.

Since the intensity of scattered light is much less than the intensity of the reference wave, further we will omit the terms proportional to the squared amplitudes of the scattered wave, so the latter expression will be written as

$$J^{(1)}(x_1, x_2) \approx (c\varepsilon_0/2)[a^2 + (ab^* + a^*b)].$$

Now, we place objects C in the plane $z = -d_2$. At the second exposure, we divide the reference beam into two beams, one of which lights objects B and C and the other is incident at the photo layer after reflection from the mirror. Let polarization vector a is directed perpendicular to the plane of incidence on the mirror. Then, after reflection from the mirror, the electric-field strength vector in the hologram plane is

$$e^{(2)} = -a + b(x_1, x_2, 0) + c(x_1, x_2, 0),$$

$$c_m(x_1, x_2, 0) = \int d^2 x_{0n} G_{ms}[x_n - x_{0n}, d_2] \times a_s f_c(x_{0n}) \exp(ikd_2).$$

The intensity of the wave incident on the hologram is

$$J^{(2)}(x_1, x_2) = (c\varepsilon_0/2) \times [a^2 - (ab^* + a^*b + ca^* + ca^*)].$$

The total intensity after two exposures is

$$J^{(12)}(x_1, x_2) = J^{(1)} + J^{(2)} = (c\varepsilon_0/2)[2a^2 - (ca^* - ac^*)].$$

It implies that the information on objects B has been subtracted.

The amplitude transmission of the hologram within the linear part of the characteristic curve of the photoemulsion can be presented as $T = v_0 - v_1(J^{(12)} - J_0)$, where v_0 and v_1 are the constant coefficients [3, 4]. Omitting the squared amplitudes of scattered light, we obtain:

$$T(x_1, x_2) \approx v_0 - (c\varepsilon_0 v_1/2)[a^2 - (ca^* + ac^*)]. \quad (6)$$

Image Recovery

Let us point the planar wave $\mathbf{E} = a\exp(ikz)$ to the hologram. Assume that in (2) $z' = 0$ and substitute the amplitude transmittance $u_s = T(x_n)a_s$. As a result, we obtain the field amplitude beyond the hologram in the region $z \geq 0$:

$$e_m(x) = \int d^2 x' G_{ms}[x_n - x'_n, z] T(x'_n) a_s. \quad (7)$$

Considering (6), we obtain from (7) the sum of three terms:

$$e_m(x) = e_m^{(0)}(x) + e_m^{(im)}(x) + e_m^{(r)}(x). \quad (8)$$

The first term is the wave of constant amplitude:

$$e_m^{(0)}(x) = [v_0 - (c\varepsilon_0 v_1 a^2/2)] \times \int d^2 x' G_{ms}[x_n - x'_n, z] a_s = [v_0 - (c\varepsilon_0 v_1 a^2/2)] a_m, \quad (9)$$

or $E^{(0)} = (v_0 - v_1 J_0) a \exp(ikz)$.

The second term in (8) is

$$e_m^{(im)}(x) = (c\varepsilon_0 v_1/2) \int d^2 x' G_{ms}[x_n - x'_n, z] a_s \times \int d^2 x_{0n} G_{ki}[x'_n - x_{0n}, d_2] a_k a_{*i} f_c(x_{0n}) \exp(ikd_2). \quad (10)$$

Taking (3) into account, we obtain

$$e_m^{(im)}(x) = (c\varepsilon_0 v_1/2) \int d^2 x_{0n} G_{ms} \times [x_n - x_{0n}, z + d_2] a_s \Lambda f_c(x_{0n}) \exp(ikd_2), \quad (11)$$

where $\Lambda = P_{ik}(n_{20}) a_k a_{*i}$ and $n_{20} = [(x_n - x_{0n})/(z + d_2)]$ [1]. Since $\Lambda \approx aa^*$, amplitude $e^{(im)}$ corresponds to the wave forming the real image in the region $z > -d_2$ from "imaginary" objects C located in the plane $z = -d_2$: $E^{(im)}(x) = v_1 J_0 c(x) \exp(ikz)$.

The third term in (8) is

$$e_m^{(r)}(x) = (c\varepsilon_0 v_1/2) \int d^2 x' G_{ms}[x_n - x'_n, z] a_s \times \int d^2 x_{0n} G^*_{ki}[x'_n - x_{0n}, d_2] a_k a_i f^*_c(x_{0n}) \exp(-ikd_2) = v_1 J_0 \int d^2 x_{0n} G_{ms}[x_n - x_{0n}, z - d_2] \times a_s f^*_c(x_{0n}) \exp(-ikd_2).$$

Amplitude $e^{(r)}$ corresponds to the wave diverging in the region $z > d_2$ from "real" objects C located in the plane $z = d_2$.

CONCLUSIONS

The proposed scheme implements holographic subtraction of images by the change in polarization of the reference light beam upon reflection from the mirror. In contrast to the known scheme, a phase plate is not necessary here [3].

REFERENCES

1. M. B. Vinogradova, O. V. Rudenko, and A. P. Sukhorukov, *Theory of Waves* (Nauka, Moscow, 1979) [in Russian].
2. S. Solimeno, B. Krozin'yani, and P. Di Porto, *Diffraction and Waveguiding of Optical Radiation* (Mir, Moscow, 1989) [in Russian].
3. L. M. Soroko, *Principles of Holography and Coherent Optics* (Nauka, Moscow, 1971) [in Russian].
4. M. Miler, *Holography* (Mashinostroenie, Leningrad, 1979) [in Russian].