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Numerical Investigation of Optical Heartbeats with External Driving Forces

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Abstract—The role of harmonic and random external forces in a phenomenological nonlinear model of optical heartbeats is investigated. External forces trigger damped oscillations at the natural frequency of the system and higher harmonics. The numerical results are compared with experimental ones.

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INTRODUCTION

Effects related to an induced thermal lens can occur in experiments on laser light scattering of complex organic and biological molecules at a relatively weak absorption of optical radiation. One such effect, known as an optical heartbeat, occurs due to the oscillations of a thermal lens. This effect was observed for the first time by Jakeman et al. [1] and was independently rediscovered by Anthore et al. [2]. The phenomenon has been interpreted in several works (see, for example [3]). When a laser beam horizontally propagates through the central part of a cuvette filled with an absorbing liquid sample, a thermal lens is generated due to the heating of the liquid. The heated liquid moves towards the surface, and Benard–Marangoni convection takes place. In particular, the flows at the surface are directed from the central part to the walls and the upward flow provides the delivery of cold liquid to the beam waist. Thus, the temperature of a liquid at the beam waist and, hence, the thermal lens, are varied, so that the oscillations of the transmitted and scattered intensity can be observed.

The experimental dependences of the parameters of the light-scattering signal (e.g., the signal decay rate and the oscillation frequencies) on the characteristics of the liquid under study can be used in the study of biological molecules [4]. We note the extensive applications of thermal-lens spectroscopy (see, for example, [5–8]).

A phenomenological nonlinear model was developed in [9] to describe optical heartbeats:

$$\begin{cases} \dot{r} = u, \\ \dot{u} = (R'-1)u - R'^{1.40}(\sinh u - u) - \left(\dfrac{2\pi R'}{R'+1}\right)^2 \sinh r, \\ \dot{s} = \left(1 - \dfrac{1+s}{(1+r)^2}\right)y^{0.70} - \sqrt{|R-1||u|}(1+s^{5001}), \\ R' = R(1+r)(1+s). \end{cases} \quad (1)$$

Here, r is the ratio of the deformation of a free surface (at the vertical axis oriented positively upwards) to the distance between the laser beam and the free surface, u is the vertical velocity of the surface, s is the dimensionless temperature of the solution at the axis of the laser beam given by $s = (T - T_{st})/(T_{st} - T_a)$ (T is the absolute temperature at the axis of the laser beam, T_{st} is the steady-state temperature at the axis of the laser beam, T_a is the ambient temperature above the surface), $R = t_1/t_2$ (t_1 is the time of the heat transfer from the laser beam to the surface, t_2 is the time of the heat dissipation along the surface due to the Marangoni effect), and y is the ratio of the laser power to the laser power needed for the spontaneous oscillations at distance d_{max} (d_{max} is the maximum distance between the laser beam and the surface at which spontaneous oscillations can occur). The one-dimensional problem is solved, so the distribution of deformations and velocities over the surface is disregarded and r and u are interpreted as variables that correspond to the point at the center of the cuvette above the center of the beam waist. Note that in the equations dimensionless time results from the normalization by t_1.

It was shown that stable oscillations can take place at $R > 1$ [9]. At $R < 1$, the system behaves as a damped oscillator with an equilibrium point at $r(0) = 0$, $u(0) = 0$, and $s(0) = 0$. It is noteworthy that the oscillation frequency strongly depends on R.

Optical heartbeats can be experimentally observed at a certain combination of the parameters of the sample and laser excitation. The optical signal can be free of oscillations over a relatively long time interval, and then the oscillations can be triggered by a random perturbation.

It was demonstrated in [4] that the mixing or heating of a sample can induce optical heartbeats. Note that the external force was not included in the theoretical analysis of the effect in [9].

Noise is usually considered to be a limiting factor for dynamic systems, due to the degradation of the output signal of the system. However, there are many examples that demonstrate that noise can lead to more order in the dynamics of nonlinear systems. Among them we cite an increase in the lifetime of a metastable state (noise enhanced stability) [10–12], the synchronization with a weak periodic input signal (stochastic resonance) [13, 14], the formation of convective structures in spatially extended systems (noise sustained structures) [15], and the coherence resonance phenomenon in a noise-driven autonomous excitable oscillator [16, 17]. The role of noise in systems with limit cycles and self-sustained oscillations is often not trivial, because it can give rise to noise-induced transitions and deformation of limit cycles and sustained oscillations that depend on the parameter values of the system, the initial conditions, and the intensity of external noise. The occurrence of noise-ordered behavior is of primary importance in all physical and biological complex systems that are open systems interacting with the environment through periodic and random external driving forces.

The purpose of this work is the numerical investigation of optical heartbeats using the model given by Eqs. (1) in the presence of external driving forces of two types (periodic and random) and the qualitative comparison of the calculated results with the experimental data.

MODEL

For the simulation of optical heartbeats stimulated by external action, two terms were added to the second equation of system (1), which becomes the following stochastic differential equation:

$$\dot{u} = (R' - 1)u - R'^{1.40}(\sinh u - u)$$
$$-\left(\frac{2\pi R'}{R' + 1}\right)^2 \sinh r + a\sin(2\pi\nu t) + \sqrt{\varepsilon}\xi(t),$$

where $a\sin(2\pi\nu t)$ is a harmonic external force and $\xi(t)$ corresponds to a random force. We assume that the random force is a white Gaussian noise with the conventional statistical properties $\langle\xi(t)\rangle = 0$ and $\langle\xi(t)\xi(t')\rangle = \delta(t - t')$. The modified system was integrated using numerical techniques based on the Euler method.

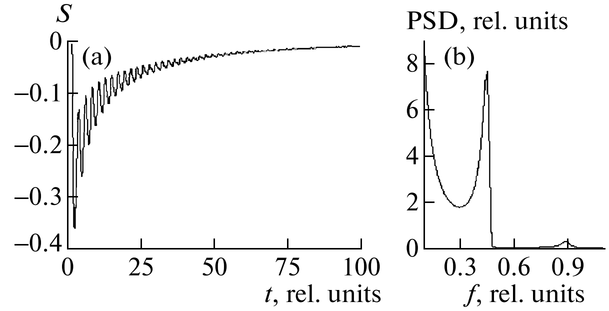

Fig. 1. (a) Time dependence of the dimensionless temperature at the axis of the laser beam and (b) the corresponding PSD at $R = 0.9$ and $y = 1$ under the initial conditions $r(0) = 0.2$, $u(0) = 0$, and $s(0) = 0$.

The experimentally observed oscillations of the light-scattering signals were related to the oscillations of the thermal lens. Therefore, we present the calculated data using the temperature variable s.

Below, we employ the dimensionless time determined for the model and the dimensionless frequency (inverse dimensionless time), since the analysis is restricted to a qualitative comparison of the experimental and calculated results. The time and frequency units are the same in all plots below.

RESULTS AND DISCUSSION

Figures 1a and 1b show the time dependence of the calculated temperature variable at the axis of the laser beam and its power spectral density (PSD), respectively, under the conditions for damping ($y = 1$ and $R = 0.9$) in the absence of external action. The initial conditions are $r(0) = 0.2$, $u(0) = 0$, and $s(0) = 0$. The frequency of the damped oscillations is 0.47. Note that the spectrum contains the second harmonic of the fundamental frequency. In all graphs, the PSDs are plotted in identical relative units, so that the corresponding curves can be quantitatively compared.

Figure 2 demonstrates the time dependence of the calculated temperature variable at the axis of the laser beam and the PSD for the same parameters that were used in the previous case, in the presence of a harmonic external force with frequency $\nu = 3$. It is seen that the amplitude of the oscillations at the eigenfrequency of the system decreases with time and only oscillations at multiple frequencies of the external force survive in the spectrum. Note also the presence of spectral peaks at the frequencies that represent combinations of the eigenfrequency and the external-force frequency. This result is in agreement with the experimental data (Fig. 3). In the course of the measurements, the sample was stirred at the time interval 65–120 s with a piece of a plastic wire attached to an electric motor working at a constant rate. The spec-

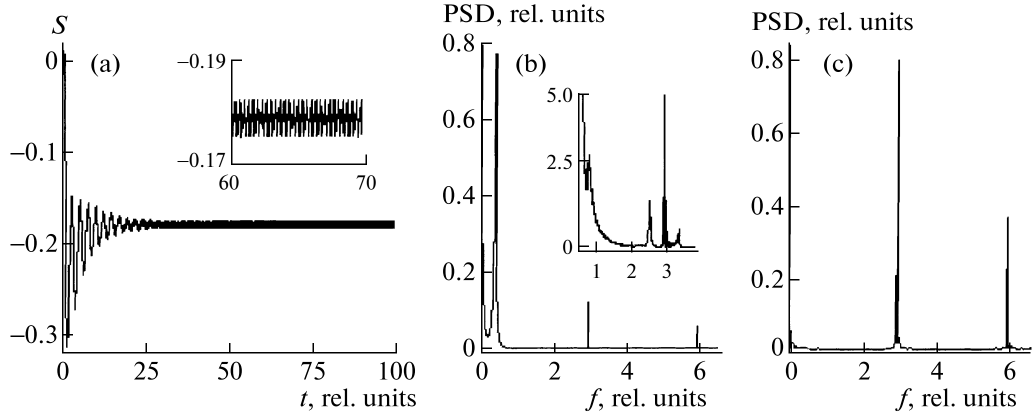

Fig. 2. (a) Time dependence of the dimensionless temperature at the axis of the laser beam at $R = 0.9$ and $y = 1$ under the initial conditions $r(0) = 0$, $u(0) = 0$, and $s(0) = 0$ in the presence of the harmonic force ($a = 0.1$ and $\nu = 3$) and PSDs calculated for the time intervals (b) of 2–30 s and (c) of 80–100 s. For the inset in panel (b), the PSD is scaled up by a factor of 1000.

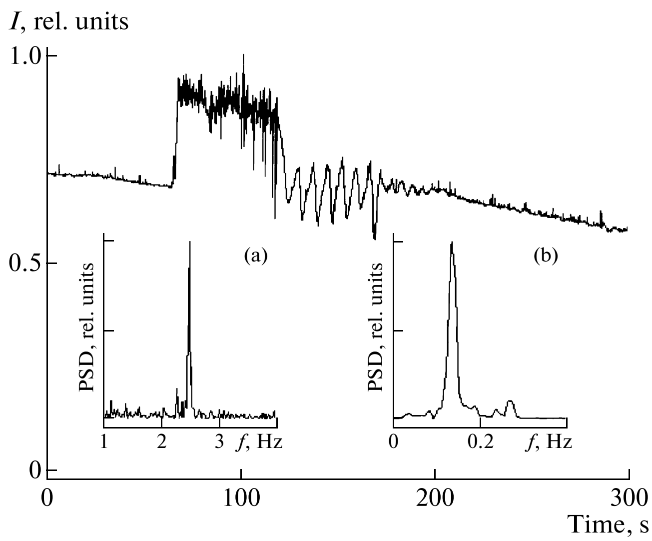

Fig. 3. Plot of the fluorescence intensity of a Rhodamine 6G solution in dimethylformamide (concentration, 4 μM and laser power, 500 mW) vs. time upon stirring at interval of 65–120 s and the PSDs calculated for (a) a time interval corresponding to the stirring and (b) the time interval of 125–200 s.

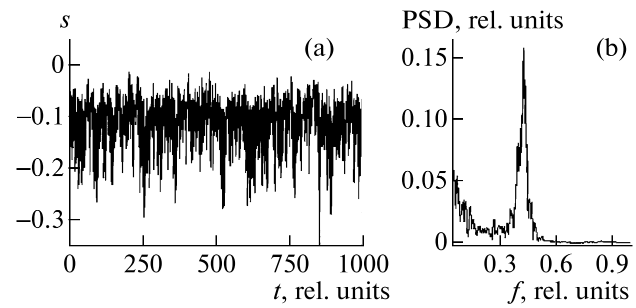

Fig. 4. (a) Time dependence of the dimensionless temperature at the axis of the laser beam obtained for $R = 0.9$ and $y = 1$ under the initial conditions $r(0) = 0$, $u(0) = 0$, and $s(0) = 0$ in the presence of the external random force ($\varepsilon = 0.01$) and (b) the corresponding PSD.

trum of the optical signal calculated for the interval of mixing exhibits a developed peak at the external-force frequency (Fig. 3a).

The effect of the random force (noise action) is illustrated in Fig. 4. Note the developed spectral peak at the eigenfrequency of the system.

To simulate the excitation of optical heartbeats, we performed calculations using the external forces exerted on the sample during the time interval 0–20 s. Figures 5 and 6 show the results calculated for the harmonic and random external forces, respectively. It is seen that, in each case, the external force induces damped oscillations at the eigenfrequency and higher harmonics. This result is also in qualitative agreement with the experimental data (Fig. 3b). In the experiments with solutions of laser dyes in several solvents the samples were stirred or heated over 2–3 min. Optical heartbeats were observed after the termination of the external action. For the experimental measurements of the elastic scattering and fluorescence, the eigenfrequency of the intensity oscillations was 0.13 Hz and the oscillations were observed at an interval with a duration of about 80 s.

As was mentioned, optical heartbeats can be triggered at an arbitrary moment in an experiment on laser light scattering due to fluctuations (e.g., temperature fluctuations). In the framework of the model, the absence of oscillations corresponds to the combination of parameters that belongs to the damped regime. If the parameters are close to the threshold values, the fluctuations of the temperature or surface coordinate can lead to optical heartbeats. The calculated data qualitatively prove this assumption.

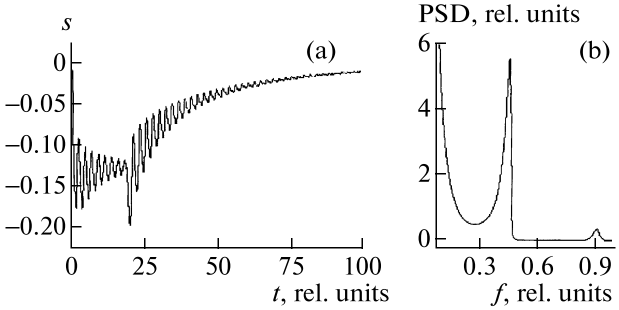

Fig. 5. (a) Time dependence of the dimensionless temperature at the axis of the laser beam obtained for $R = 0.9$ and $y = 1$ under the initial conditions $r(0) = 0$, $u(0) = 0$, and $s(0) = 0$ in the presence of the harmonic external force ($a = 5$ and $\nu = 3$) exerted over the time interval 0–20 and (b) PSD calculated for the time interval of 20–100.

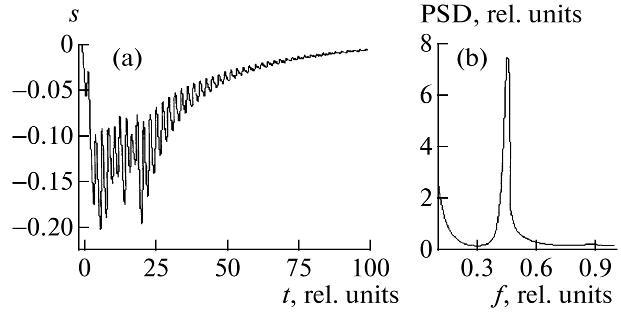

Fig. 6. (a) Time dependence of the dimensionless temperature at the axis of the laser beam obtained for $R = 0.9$ and $y = 1$ under the initial conditions $r(0) = 0$, $u(9) = 0$, and $s(0) = 0$ in the presence of the external random force ($\varepsilon = 0.01$) exerted over the time interval 0–20 and (b) PSD calculated for the time interval of 20–100.

CONCLUSIONS

The model calculations show that the spectrum of the temperature variable contains a spectral component at the eigenfrequency of the system in the presence of harmonic or random external action. Adding external forces to the existing theoretical model provides a qualitative description of the induced experimental effects observed in optical heartbeat measurements. In particular, the results calculated for the harmonic external force are in agreement with the experimental data on the excitation of optical heartbeats by stirring at a constant rate. The results calculated for the noise excitation can be used to interpret spontaneously emerging optical heartbeats: the excitation can be due to uncontrolled vibrations of the experimental setup leading to the random oscillations of the surface of liquid in the cuvette. The temperature fluctuations in the sample can also trigger optical heartbeats.

In further studies, we plan to supplement the qualitative analysis with a quantitative comparison of the calculated and experimental results aiming at the determination of the parameters of solutions (including solutions of biologically important molecules).

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