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Reconstruction of the Coordinate Dependence of the Diagonal Form of the Dielectric Permittivity Tensor of a One-Dimensionally Inhomogeneous Medium

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Abstract—We demonstrate the possibility of a unique reconstruction of the coordinate dependence of all dielectric permittivity tensor components of an absorbing one-dimensionally inhomogeneous plate with any symmetry (except for 1, 2, and *m* classes) and a negligibly small spatial dispersion. This reconstruction can be performed, the zone of the strong frequency dispersion of the medium inclusive, provided that the reflection and transmission coefficients for the *p*- and *s*-polarized plane monochromatic waves are known in a certain range of incidence angles.

Key words: dielectric permittivity tensor, one-dimensionally inhomogeneous layer, light polarization, reflection coefficient, transmission coefficient, inverse problem.

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The overwhelming majority of methods [1–5] that have been created for the determination of the components of the dielectric permittivity tensor $\hat{\varepsilon}(\mathbf{r})$ of the one-dimensionally inhomogeneous media necessary for different practical applications for different reasons are not applicable in optics because of the large number of approximations used in them. The most widespread simplifications are: neglect of the absorption of the medium [1, 2], neglect of the frequency dispersion in a wide frequency range [2, 3] or usage of the simplest models of such dispersion in which only the distribution of the oscillator force is assumed to be inhomogeneous and the resonance frequencies are considered to be known and independent of coordinates [4], and the approximate solution of the appearing equations [1, 2]. Some methods for the solution of the inverse problems are not applicable, since as initial data they use quantities that cannot be directly measured in optics [5]. In addition, the considered one-dimensionally inhomogeneous media are usually assumed to be isotropic and their dielectric permittivity is assumed to be scalar [1–3, 5]. However, as is well known, the properties of the near-surface layer of any medium can noticeably differ from the properties of its bulk. Moreover, these differences appear even on an ideally clean surface because the atoms on the surface are in somewhat different conditions than those in the bulk. As a result, the symmetry properties of the bulk of the medium and those of the near-surface layer can also differ. The latter, in particular, can have no three-dimensional inversion center and symmetry planes parallel to the surface, i.e., it cannot be three-dimensionally isotropic. The plane boundary between the homogeneous media is a particular case of a one-dimensionally inhomogeneous system. Therefore, a one-dimensionally inhomogeneous medium, strictly speaking, cannot be isotropic as well. It cannot have a three-dimensional inversion center and symmetry planes perpendicular to the direction in which the inhomogeneous properties are revealed. As a result, of the 32 classes and seven limit groups that are possible for homogeneous media [6], in the case of one-dimensionally inhomogeneous media only ten classes (1, 2, *m*, *mm*2, 3, 4, 6, 3*m*, 4*mm*, 6*mm*) and two limit groups (∞, ∞m) occur. We will consider exactly these classes and groups. The symmetry of a definite one-dimensionally inhomogeneous plate depends on both the medium from which it is made and the orientation of its surfaces perpendicular to the direction of the inhomogeneity with respect to the crystallographic axes X_1, X_2, and X_3 [6] of this medium.

For the optical wavelength range, in [7] an algorithm for the unambiguous reconstruction of the spatial profile of the ε_{yy} component of the dielectric permittivity tensor of a one-dimensional absorbing plane plate that is inhomogeneous along the z-axis with a frequency dispersion of any type free from the approximations used in [1–5] was proposed. At $z = z_1$ and $z = z_2$ ($z_2 > z_1$) the plate bordered the homogeneous isotropic nonabsorbing media with the dielectric permittivity ε_0 and the medium forming the layer had a symmetry plane *xz* perpendicular to its surface (the symmetry class *m*). In [7] it was proven that the reconstruction of the $\varepsilon_{yy}(z)$ dependence at any frequency ω can be made according to the reflection and transmission coefficients of the *s*-polarized plane wave of the frequency ω, the incidence plane of which coincides with the symmetry plane of the medium measured in a certain range of incidence angles. If the medium of the plate has a higher symmetry class *mm*2, i.e., it has two symmetry planes *xz* and *yz*, then the components ε_{xx} and ε_{yy} of the dielectric permittivity tensor can be determined independently from the reflection and transmission coefficients of the *s*-polarized plane waves with the incidence planes *yz* and *xz*, respectively. It is shown below that in this case it is possible to uniquely reconstruct the component ε_{zz} of the dielectric permittivity tensor of the medium having a diagonal form [6] from the reflection and transmission coefficients of the *p*-polarized plane waves with the incidence planes *yz* or *xz*, which are also known in a certain range of incidence angles. A similar result exists for all other symmetry classes (except for classes 1, 2, and *m*) and limit groups possible in a one-dimensionally inhomogeneous media. However, in these classes (3, 4, 6, 3*m*, 4*mm*, 6*mm*, ∞, ∞m) $\varepsilon_{xx} = \varepsilon_{yy}$ [6] and to determine two independent components of the dielectric permittivity tensor it is sufficient to use two (*p*- and *s*-polarized) waves with an arbitrary orientation of the incidence planes.

Let a *p*-polarized plane wave with an incidence plane *yz* propagating in the positive or negative direction of the *z*-axis be incident at the angle α on the above plate with symmetry class *mm*2. The magnetic field strength in it is $H_p\mathbf{e}_x\exp[i(\omega t - k_y y - k_z(z - z_1))]$ + c.c. (at $z < z_1$) or $H_n\mathbf{e}_x\exp[i(\omega t - k_y y + k_z(z - z_2))]$ + c.c. (at $z > z_2$), respectively. Here $\mathbf{e}_x$ is the unit vector along the *x*-axis, ω is the wave frequency, $k_y = k_0\sin\alpha$ and $k_z = k_0\cos\alpha$ are the components of its wave vector, $k_0 = \omega\sqrt{\varepsilon_0}/c$, and *c* is the velocity of light in vacuum. Then the vector of the magnetic field strength in the plate can be written as $\mathbf{H}_\pm(y, z) = H_\pm(z)\mathbf{e}_x\exp[i(\omega t - k_y y)]$ + c.c., where $\mathbf{H}_+$ corresponds to the propagation of the wave in the positive and $\mathbf{H}_-$ in the negative direction of the *z*-axis, and the change of $H_\pm(z)$ is described by

$$\frac{d}{dz}\left(\frac{1}{\varepsilon_{yy}}\frac{dH_\pm}{dz}\right) + \left[\frac{\omega^2}{c^2} - \frac{\lambda}{\varepsilon_{zz}}\right]H_\pm = 0, \quad (1)$$

where $\lambda = k_y^2$. Further, we assume that $\varepsilon_{yy}(z) \neq 0$, $\varepsilon_{zz}(z) \neq 0$.

Due to the Maxwell boundary condition on the surface of the layer, we have:

$$\begin{gathered} H_+(z_1) = (1 + R_p)H_p, \\ (\varepsilon_{yy}^{-1}dH_+/dz)\big|_{z=z_1} = -ik_z\varepsilon_0^{-1}(1 - R_p)H_p, \\ H_+(z_2) = T_pH_p, \\ (\varepsilon_{yy}^{-1}dH_+/dz)\big|_{z=z_2} = -ik_z\varepsilon_0^{-1}T_pH_p, \end{gathered} \quad (2)$$

$$\begin{gathered} H_-(z_2) = (1 + R_n)H_n, \\ (\varepsilon_{yy}^{-1}dH_-/dz)\big|_{z=z_2} = ik_z\varepsilon_0^{-1}(1 - R_n)H_n, \\ H_-(z_1) = T_nH_n, \\ (\varepsilon_{yy}^{-1}dH_-/dz)\big|_{z=z_1} = ik_z\varepsilon_0^{-1}T_nH_n. \end{gathered} \quad (3)$$

Here $R_{p,n}$ are the amplitude reflection coefficients of the waves incident in the positive and negative directions of the *z*-axis, respectively, and $T_{p,n}$ are the amplitude transmission coefficients of the corresponding waves (all coefficients are calculated from the amplitude of the magnetic field).

We remember that for a rather wide class of $\varepsilon_{yy}(z)$ and $\varepsilon_{zz}(z)$ functions (for example, piecewise continuous functions [8, 9]) Eq. (1) has continuous solutions, which are continuously differentiable in the continuity region of the $\varepsilon_{yy}(z)$ function. Let the $\varphi_{1,2}(z, \lambda)$ functions be such solutions (1) with the boundary conditions:

$$\begin{gathered} \varphi_1(z_1, \lambda) = 1, \quad d\varphi_1(z, \lambda)/dz\big|_{z=z_1} = 0, \\ \varphi_2(z_1, \lambda) = 0, \quad d\varphi_2(z, \lambda)/dz\big|_{z=z_1} = 1, \end{gathered} \quad (4)$$

with $\varepsilon_{yy}^{-1}d\varphi_{1,2}(z, \lambda)/dz$ continuous everywhere, including the break point $\varepsilon_{yy}(z)$, and continuously differentiable in the continuity region $\varepsilon_{zz}(z)$. Then $\varphi_{1,2}(z, \lambda)$ at any λ form a fundamental system of solutions (1). Their Wronski determinant due to (1) and (4) varies according to the law:

$$\begin{gathered} W \equiv \varphi_1(z, \lambda)d\varphi_2(z, \lambda)/dz \\ - \varphi_2(z, \lambda)d\varphi_1(z, \lambda)/dz = \varepsilon_{yy}(z)/\varepsilon_{yy}(z_1), \end{gathered} \quad (5)$$

and solutions (1) with the boundary conditions (2) or (3) can be presented as

$$H_\pm(z, \lambda) = [C_{1\pm}\varphi_1(z, \lambda) + C_{2\pm}\varphi_2(z, \lambda)]H_{p,n}. \quad (6)$$

It follows from (2), (3) and (6) taking into account (4) that the $C_{1\pm,2\pm}$ constants, values of the $\varphi_{1,2}$ functions and their derivatives in the point $z = z_2$ are related to the $R_{p,n}$ and $T_{p,n}$ coefficients as:

$$\begin{gathered} C_{1+} = 1 + R_p, \quad C_{1-} = T_n, \\ C_{2+} = -ik_z\varepsilon_1(1 - R_p)/\varepsilon_0, \quad C_{2-} = ik_z\varepsilon_1T_n/\varepsilon_0, \\ (1 + R_p)\Psi_1 - ik_z\varepsilon_1(1 - R_p)\Psi_2/\varepsilon_0 = T_p, \\ \varepsilon_0(1 + R_p)\Psi_{1z} - ik_z\varepsilon_1(1 - R_p)\Psi_{2z} = -ik_z\varepsilon_2T_p, \\ T_n\Psi_1 + ik_z\varepsilon_1T_n\Psi_2/\varepsilon_0 = 1 + R_n, \\ \varepsilon_0T_n\Psi_{1z} + ik_z\varepsilon_1T_n\Psi_{2z} = ik_z\varepsilon_2(1 - R_n). \end{gathered} \quad (7)$$

Here $\Psi_{1,2}(\lambda) = \varphi_{1,2}(z_2, \lambda)$, $\Psi_{1z,2z}(\lambda) = d\varphi_{1,2}(z, \lambda)/dz\big|_{z=z_2}$, and $\varepsilon_{1,2} = \varepsilon_{yy}(z_{1,2})$.

One can see from the last two formulas in (7) that T_n can be zero only if $\lambda = k_0^2$ ($k_z = 0$). By expressing $\Psi_{1,2}$ and $\Psi_{1z,2z}$ via $R_{p,n}$ and $T_{p,n}$ from relations (7) (at $\lambda \neq k_0^2$) and substituting the obtained expressions in (5), we get $T_p = T_n \equiv T$. Taking into account the last equality, from (7) we also have

$$ik_z\varepsilon_2(1-R_{p,n})/\varepsilon_0 T$$
$$= \Psi_{1z} + ik_z\varepsilon_{2,1}\Psi_{1,2z}/\varepsilon_0 \equiv f_{1,2}(k_z), \tag{8}$$
$$(1+R_n)/T = \Psi_1 + ik_z\varepsilon_1\Psi_2/\varepsilon_0 \equiv f_3(k_z).$$

At each fixed $z \in [z_1, z_2]$, $\varphi_{1,2}(z, \lambda)$ are unique analytical functions of λ without singular points in the finite part of the plane, i.e., the integer functions [8, 9]. Consequently, $\Psi_{1,2}$ and $\Psi_{1z,2z}$ are also integer functions of λ, and thus $k_z^2 = k_0^2 - \lambda$. The latter means that they are even integer functions of k_z, and $f_{1,2,3}$ due to the definition (8) are integer functions of k_z. Using the parity of the $\Psi_{1,2}$ and $\Psi_{1z,2z}$ functions with respect to k_z, from (8) one can obtain the following relations:

$$\Psi_1(\lambda) = \varepsilon_0[f_1(k_z) - f_1(-k_z)]/2ik_z\varepsilon_2,$$
$$\Psi_{1z}(\lambda) = [f_1(k_z) + f_1(-k_z)]/2, \tag{9}$$
$$\Psi_{1,1z}(\lambda) = [f_{3,2}(k_z) + f_{3,2}(-k_z)]/2,$$

where $\lambda \equiv k_y^2 = k_0^2 - k_z^2$.

If the T and R_p coefficients (or T and R_n) are known in a certain interval of the incidence angles $\alpha^{(1)} \leq \alpha \leq \alpha^{(2)}$, then using (8), for the values $k_z \in [k_0\cos\alpha^{(2)}, k_0\cos\alpha^{(1)}]$ one can find $f_1(k_z)$ (or $f_{2,3}(k_z)$), which are integer functions. The last circumstance is sufficient for their unique analytical continuation to the entire complex plane k_z [9]. If one knows $f_1(k_z)$ (or $f_{2,3}(k_z)$), by means of (9) one can find $\Psi_1(\lambda)$ and $\Psi_{1z}(\lambda)$ for any λ.

As already noted, the $\varepsilon_{xx}(z)$ and $\varepsilon_{yy}(z)$ dependences can be uniquely reconstructed from the amplitude reflection and transmission coefficients of the *s*-polarized light waves [7]. Therefore, now we are interested in finding the third diagonal component of the dielectric permittivity tensor $\varepsilon_{zz}(z)$. Below we limit ourselves to the case where

$$\varepsilon_{yy}(z) = |\varepsilon_{yy}|\exp\{i\beta_y(z)\},$$
$$\varepsilon_{zz}(z) = |\varepsilon_{zz}|\exp\{i\beta_z(z)\}, \tag{10}$$
$$\beta_{y,z}(z) \in (-\pi; 0],$$

i.e., the values of the piecewise analytical functions $\varepsilon_{yy}(z)$ and $\varepsilon_{zz}(z)$ are located in the lower open half of the complex plane and at the positive part of the real axis, which is added so that our proof also holds for the often used model of the ideal nonabsorbing medium with positive dielectric permittivity. The case $\beta_{y,z}(z) \in [0; \pi)$ is considered similarly. Depending on the choice of the type of the temporal multiplier ($\exp(i\omega t)$ or $\exp(-i\omega t)$) one of these two cases ($\beta_{y,z}(z) \in (-\pi; 0]$ or $\beta_{y,z}(z) \in [0; \pi)$) corresponds to the dielectric permittivity of a real stationary media, in which there is always absorption.

Using the main ideas of the method of reference models [10], we prove that when (10) holds, the complex function $\varepsilon_{zz}(z)$ for a layer of given thickness with the given $\varepsilon_{yy}(z)$ dependence is uniquely defined by the $\Psi_{1,1z}(\lambda)$ functions. We assume the opposite. Let two different profiles $\bar{\varepsilon}_{zz}(z)$ and $\varepsilon_{zz}(z)$ exist for which $\bar{\varphi}_1(z, \lambda)$ and $\varphi_1(z, \lambda)$ functions, which are solutions of equations

$$\frac{d}{dz}\left(\frac{1}{\varepsilon_{yy}}\frac{d\bar{\varphi}_1}{dz}\right) + \left[\frac{\omega^2}{c^2} - \frac{\lambda}{\bar{\varepsilon}_{zz}}\right]\bar{\varphi}_1 = 0, \tag{11}$$

$$\frac{d}{dz}\left(\frac{1}{\varepsilon_{yy}}\frac{d\varphi_1}{dz}\right) + \left[\frac{\omega^2}{c^2} - \frac{\lambda}{\varepsilon_{zz}}\right]\varphi_1 = 0 \tag{12}$$

with boundary condition

$$\bar{\varphi}_1(z_1, \lambda) = 1, \quad d\bar{\varphi}_1(z, \lambda)/dz|_{z=z_1} = 0,$$
$$\varphi_1(z_1, \lambda) = 1, \quad d\varphi_1(z, \lambda)/dz|_{z=z_1} = 0, \tag{13}$$

meet equalities

$$\bar{\varphi}_1(z_2, \lambda) = \varphi_1(z_2, \lambda) \equiv \Psi_1(\lambda),$$
$$d\bar{\varphi}_1(z, \lambda)/dz|_{z=z_2} = d\varphi_1(z, \lambda)/dz|_{z=z_2} \equiv \Psi_{1z}(\lambda). \tag{14}$$

We multiply (11) by $\varphi_1(z, \lambda)$ and (12) by $\bar{\varphi}_1(z, \lambda)$ and subtract the second expression from the first one. By integrating the obtained difference from z_1 to z_2, we obtain

$$-\lambda\int_{z_1}^{z_2}[\bar{\varepsilon}_{zz}^{-1} - \varepsilon_{zz}^{-1}]\varphi_1\bar{\varphi}_1 dz + \int_{z_1}^{z_2}\frac{d}{dz}(\varepsilon_{yy}^{-1}d\bar{\varphi}_1/dz)\varphi_1 dz$$
$$-\int_{z_1}^{z_2}\frac{d}{dz}(\varepsilon_{yy}^{-1}d\varphi_1/dz)\varphi_1 dz = 0. \tag{15}$$

We calculate the second and third integrals in (15) by parts at each continuity interval of the function $\varepsilon_{yy}(z)$, $\bar{\varepsilon}_{zz}(z)$ and $\varepsilon_{yy}(z)$, $\varepsilon_{zz}(z)$, respectively. Using the continuity of the $\bar{\varphi}_1(z, \lambda)$, $\varepsilon_{yy}^{-1}d\bar{\varphi}_1/dz$, $\varphi_1(z, \lambda)$, and $\varepsilon_{yy}^{-1}d\varphi_1/dz$ functions at the $[z_1, z_2]$ segment, including the break points of the components of the dielectric permittivity tensor, the boundary conditions (13), and the equalities (14), we show that the second and third integrals in (15) are equal. Therefore, at $\lambda \neq 0$

$$J(k_y) \equiv \int_{z_1}^{z_2}\Delta(z)\varphi_1(z, \lambda)\bar{\varphi}_1(z, \lambda)dz = 0, \tag{16}$$

where $\Delta(z) = \bar{\varepsilon}_{zz}^{-1} - \varepsilon_{zz}^{-1}$. Following [9], we call the $F(z)$ function of the real variable a piecewise analytical one at the $[z_1, z_2]$ segment, if the latter can be divided into such $Z_i \equiv [z^{(i)}, z^{(i+1)}]$ segments, where $i = 0, 1, \ldots, N$, $z^{(0)} = z_1$ and $z^{(N+1)} = z_2$, which in each of them $F(z)$ has all derivatives (at the ends of the segment, all one-way derivatives), and the residue term in the Taylor formula at $n \longrightarrow \infty$ tends to zero. As is well known, in this case the Taylor series converges uniformly to the $F(z)$ function in each of the Z_i segments [9]. Further we assume that such a partition is performed and in each Z_i segment the $\varepsilon_{yy}(z)$, $\varepsilon_{zz}(z)$, and $\bar{\varepsilon}_{zz}(z)$ functions are analytical. In this case, the $\Delta(z)$ function is also analytical in each Z_i. Our aim is to prove that it follows from equality (16) that $\Delta(z) \equiv 0$ in all Z_i.

We assume that in some Z_i segments the $\Delta(z)$ function is not identically zero and m is the maximum number of such a segment. Then there exist such finite $n \geq 0$ and $\gamma \neq 0$ that

$$\Delta(z) = (z - z^{(m+1)})^n[\gamma + s(z)]/n!, \qquad (17)$$

where $z \in Z_m$ and $s(z)$ is the analytical function in Z_m, moreover $s(z^{(m+1)}) = 0$. In this case, the integral (16) can be written as:

$$J = \frac{1}{n!}\int\limits_{Z_m}(z - z^{(m+1)})^n[\gamma + s(z)]\varphi_1(z, \lambda)\bar{\varphi}_1(z, \lambda)dz + \sum_{i=0}^{m-1}\int\limits_{Z_i}\Delta(z)\varphi_1(z, \lambda)\bar{\varphi}_1(z, \lambda)dz \qquad (18)$$

(if $m = 0$, then the second summand in formula (18) is absent).

Taking into account the piecewise continuity of the nonzero $\varepsilon_{yy}(z)$, $\varepsilon_{zz}(z)$ and $\bar{\varepsilon}_{zz}(z)$ functions, it follows from conditions (10) that there exists such positive A that for any $z \in [z_1, z_2]$ the inequalities

$$\mathrm{Re}\{[\varepsilon_{yy}(z)/\varepsilon_{zz}(z)]^{1/2}\} \geq A > 0, \quad \mathrm{Re}\{[\varepsilon_{yy}(z)/\bar{\varepsilon}_{zz}(z)]^{1/2}\} \geq A > 0 \qquad (19)$$

hold.

Taking (10) into account, here and further it is assumed that for any real p, q, and k the equality $[\varepsilon^p_{yy}(z)\varepsilon^q_{zz}(z)]^{1/k} = |\varepsilon_{yy}|^{p/k}|\varepsilon_{zz}|^{q/k}\exp\{i(p\beta_y(z) + q\beta_z(z))/k\}$ holds. It follows from (19) that for all Z_i the expression $\mathrm{Re}\{k_y[\varepsilon_{yy}(z)/\varepsilon_{zz}(z)]^{1/2}\}$ does not change the sign at positive k_y. Therefore, for any $k_y > 0$ large enough, in each Z_i segment there exist two continuously differentiable linearly independent solutions $F^{(i)}_{\pm}(z, k_y)$ of Eq. (1), presented as [11]:

$$F^{(i)}_{\pm} = [\varepsilon_{yy}(z)\varepsilon_{zz}(z)]^{1/4}\exp[\pm k_y Q_i(z)][1 + o(1)], \quad dF^{(i)}_{\pm}/dz = \pm k_y[\varepsilon^3_{yy}(z)/\varepsilon_{zz}(z)]^{1/4} \times \exp[\pm k_y Q_i(z)][1 + o(1)], \qquad (20)$$

where $Q_i(z) = \int_{z^{(i)}}^{z}[\varepsilon_{yy}(\nu)/\varepsilon_{zz}(\nu)]^{1/2}d\nu$. Here and further $o(1)$ denotes the k_y and possibly z functions, the definite type of which is not important for us and which uniformly (if the dependence on z is present) tend to zero at $k_y \longrightarrow \infty$.

In each Z_i segment the $\varphi_1(z, \lambda)$ function is a linear combination of $F^{(i)}_{\pm}(z, k_y)$:

$$\varphi_1(z, \lambda) = C^{(i)}_{+}(k_y)F^{(i)}_{+}(z, k_y) + C^{(i)}_{-}(k_y)F^{(i)}_{-}(z, k_y), \qquad (21)$$

where $C^{(0)}_{\pm}$ are determined from the boundary conditions (13), and $C^{(i)}_{\pm}$ $(i \geq 1)$ from the continuity conditions of the $\varphi_1(z, \lambda)$ and $\varepsilon^{-1}_{yy}d\varphi_1(z, \lambda)/dz$ functions at the point $z = z^{(i)}$. At $1 \leq i \leq N$, the $C^{(i)}_{\pm}$ coefficients meet the recurrent relations:

$$C^{(i)}_{\pm} = A^{(i)}_{\pm}C^{(i-1)}_{+}\exp[k_y Q_{i-1}(z^{(i)})][1 + o(1)] + A^{(i)}_{\mp}C^{(i-1)}_{-}\exp[-k_y Q_{i-1}(z^{(i)})][1 + o(1)]. \qquad (22)$$

In formula (22)

$$C^{(0)}_{\pm} = [1 + o(1)]/2[\varepsilon_{zz}(z_1)\varepsilon_{yy}(z_1)]^{-1/4},$$
$$A^{(i)}_{\pm} = \frac{\varepsilon^{1/2}_{zz}(z^{(i)} - 0)\varepsilon^{1/2}_{yy}(z^{(i)} - 0) \pm \varepsilon^{1/2}_{zz}(z^{(i)} + 0)\varepsilon^{1/2}_{yy}(z^{(i)} + 0)}{2\varepsilon^{1/4}_{zz}(z^{(i)} - 0)\varepsilon^{1/4}_{yy}(z^{(i)} - 0)\varepsilon^{1/4}_{zz}(z^{(i)} + 0)\varepsilon^{1/4}_{yy}(z^{(i)} + 0)}. \qquad (23)$$

We substitute the relations (20)–(22) and analogous expressions written for the $\bar{\varphi}_1(z, \lambda)$ functions into formula (18). As a result, we obtain the following estimate of the second summand in the right-hand part of (18):

$$\sum_{i=0}^{m-1}\int\limits_{Z_i}\Delta(z)\varphi_1(z, \lambda)\bar{\varphi}_1(z, \lambda)dz = o(\exp\{k_y Q_{s,m}\}).$$

Here $Q_{s,m} = \sum_{i=0}^{m}[Q_i(z^{(i+1)}) + \bar{Q}_i(z^{(i+1)})]$, $\bar{Q}_i(z) = \int_{z^{(i)}}^{z}[\varepsilon_{yy}(\nu)/\bar{\varepsilon}_{zz}(\nu)]^{1/2}d\nu$, $z \in Z_i$ and it is taken into account that due to (19) $\mathrm{Re}\{Q_i(z)\}$ and $\mathrm{Re}\{\bar{Q}_i(z)\}$ are nonnegative functions monotonously increasing in Z_i. To calculate the first summand in formula (18) we use (19) and the theorem formulated and proved in the Supplement which generalizes lemma 1.5.1 from [10] to our case. As a result, we obtain that

$$Jk_y^{n+1}\exp\{-k_yQ_{s,m}\}$$
$$= (-1)^n\gamma\left[\frac{\varepsilon_{zz}(z^{(m+1)}-0)\bar{\varepsilon}_{zz}(z^{(m+1)}-0)}{\varepsilon_{yy}(z^{(m+1)}-0)}\right]^{\frac{n+1}{2}}$$
$$\times\frac{\varepsilon_{yy}^{1/2}(z^{(m+1)}-0)\bar{\varepsilon}_{zz}^{1/4}(z^{(m+1)}-0)\varepsilon_{zz}^{1/4}(z^{(m+1)}-0)}{[\bar{\varepsilon}_{zz}^{1/2}(z^{(m+1)}-0)+\varepsilon_{zz}^{1/2}(z^{(m+1)}-0)]^{n+1}} \quad (24)$$
$$\times\prod_{i=0}^{m}A_+^{(i)}\bar{A}_+^{(i)}[1+o(1)].$$

In (24) $A_+^{(0)} \equiv C_+^{(0)}$ $\bar{A}_+^{(0)} \equiv \bar{C}_+^{(0)}$, and the expressions for $\bar{C}_+^{(0)}$ and $\bar{A}_+^{(i)}$ are obtained from (23) by substituting $\varepsilon_{zz}(z)$ by $\bar{\varepsilon}_{zz}(z)$.

Due to (16) $J(k_y \neq 0) = 0$, therefore, the left-hand part in (24) is zero. Since as follows from (10) and (23), all $A_+^{(i)}$ and $\bar{A}_+^{(i)}$ are nonzero, then at $k_y \longrightarrow \infty$ the equality (24) holds only if $\gamma = 0$, which contradicts the initial assumption (17). Thus, $\Delta(z) \equiv 0$ at $z \in [z_1, z_2]$.

Thus, we proved that the knowledge of the amplitude transmission coefficient $T = T_p = T_n$ and one of the reflection coefficients (R_p or R_n) in a certain interval of the incidence angles of the p-polarized plane monochromatic waves, as well as the $\varepsilon_{yy}(z)$ dependence is sufficient for the unique reconstruction of the profile of the other component $\varepsilon_{zz}(z)$ of the dielectric permittivity tensor of the studied absorbing plate at a fixed frequency. Proof is performed for the piecewise-analytical functions $\varepsilon_{yy}(z)$ and $\varepsilon_{zz}(z)$, i.e., it includes the practically important case of the stepwise change of the dielectric properties inside a plate formed of inhomogeneous layers made of different media. Due to the proven uniqueness of the reconstruction, finding the $\varepsilon_{zz}(z)$ function can be reduced to searching for a unique zero minimum of a specific functional, for example, analogous to that proposed in [7, 12] for the reconstruction of the $\varepsilon_{yy}(z)$ component.

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APPENDIX

Theorem. *Let in the* $[a, b]$ *segment the* $r(z, \rho)$ *function be presented as* $r(z, \rho) = r_0(z)[1 + \zeta(z, \rho)]$, *where* $r_0(z) = R(z)(z - b)^n[\gamma + s(z)]/n!$ *and the* $\zeta(z, \rho)$ *function at all* $\rho \geq \rho_0 > 0$ *is uniformly limited in* $[a; b]$ *and tends to zero when the parameter* ρ *tends to infinity, i.e.,* $|\zeta(z, \rho)| \leq \eta(\rho) = o(1)$. *The* $R(z)$ *and* $s(z)$ *functions are analytical and* $s(b) = 0$.

Then we present the integral $I(\rho) = \int_a^b r(z, \rho)G(z, \rho)dz$, *where* $G(z, \rho) = \exp\{\rho\int_a^z q(\nu)d\nu\}$ *and* $q(z)$ *is an analytical function, the real part of which is larger than zero, at large* ρ *values such as*

$$I(\rho) = G(b, \rho)\rho^{-(n+1)}[(-1)^n\gamma R(b)/q^{n+1}(b) + o(1)].$$

Proof. We present the integral $I(\rho)$ as follows

$$I = \int_a^b r_0(z)G(z, \rho)dz$$
$$+ \int_a^b \zeta(z, \rho)r_0(z)G(z, \rho)dz \equiv I_0 + \bar{I}.$$

By calculating the integral I_0 by parts, we have

$$I_0 = \frac{1}{\rho}\int_a^b \frac{r_0(z)}{q(z)}\frac{dG(z, \rho)}{dz}dz$$
$$= \frac{1}{\rho}\left[G(b, \rho)\frac{r_0(b)}{q(b)} - \frac{r_0(a)}{q(a)}\right] + I_1, \quad (A.1)$$

where $I_1 = -\rho^{-1}\int_a^b r_1(z)G(z, \rho)dz$. Here and further at $j \geq 1$ the $r_j(z)$ function is found from the recurrent relation $r_j(z) = \frac{d}{dz}[r_{j-1}(z)/q(z)]$. According to the condition of the theorem $q(z) \neq 0$ and, consequently, all $r_j(z)$ are analytical. Therefore, such integration by parts can be repeated again and again. As a result, we obtain

$$I_0 = G(b, \rho)\sum_{l=0}^{n}\frac{(-1)^l}{\rho^{l+1}}\left[\frac{r_l(b)}{q(b)} - \frac{r_l(a)}{q(a)G(b, \rho)}\right] + I_{n+1}, \quad (A.2)$$

where $I_{n+1} = (-1)^{n+1}\rho^{-(n+1)}\int_a^b r_{n+1}(z)G(z, \rho)dz$. According to the condition of the theorem, the $q(z)$ function is continuous in the $[a; b]$ segment and is nonzero at any z. Consequently, $1/q(z)$ and $r_j(z)$ are limited in this segment for any $j \geq 1$. Due to the conditions of the theorem $|G(z, \rho)|$ monotonously increases with increase in z. Therefore, by integrating I_{n+1} by parts analogous to (A.1) and using the properties of the modulus of sum and modulus of integral, we obtain

$$|I_{n+1}| = o(|G(b, \rho)|\rho^{-(n+1)}). \quad (A.3)$$

Taking into account the form of the $r_0(z)$ function determined in the conditions of the theorem and using the rules of differentiation, it is easy to find the $r_l(b) = 0$ at $l = 0, 1, \ldots, n-1$ and $r_n(b) = \gamma R(b)/q^n(b)$. Therefore, from (A.2) and (A.3) we have

$$I_0 = G(b, \rho)\rho^{-(n+1)}$$
$$\times[(-1)^n\gamma R(b)/q^{n+1}(b) + o(1)]. \quad (A.4)$$

We estimate now the integral $\bar{I}$. According to the condition of the theorem, $R(z)$ and $s(z)$ functions are limited and the $\zeta(z, \rho)$ function is uniformly limited in the $[a; b]$ segment by the $\eta(\rho)$ function. Consequently, $|\bar{I}| \le C\eta(\rho)\bar{I}_0$, where C is a constant independent of ρ and

$$\bar{I}_0 = \int_a^b \frac{(b-z)^n}{n!} \exp\left(\rho \int_a^z \mathrm{Re}\{q(\nu)\} d\nu\right) dz.$$

But the integral $\bar{I}_0$ is obtained from the integral I_0 if in the latter it is assumed that $R(z) \equiv (-1)^n$, $s(z) \equiv 0$, $\gamma = 1$, and $\mathrm{Im}\{q(z)\} \equiv 0$. Consequently, one can use for it the relation (A.4) which takes the form

$$\bar{I}_0 = |G(b, \rho)| \rho^{-(n+1)} [\mathrm{Re}\{q^{-(n+1)}(b)\} + o(1)].$$

Then, since $\eta(\rho) = o(1)$, $\bar{I} = G(b, \rho)\rho^{-(n+1)} o(1)$, which in combination with (A.4) proves the theorem.

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