ISSN 0027-1349, Moscow University Physics Bulletin, 2010, Vol. 65, No. 3, pp. 223–226. © Allerton Press, Inc., 2010.
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Diffraction of a Gaussian Light Beam at the Right-Angle Edges of a Plate and the Interference Pattern in the Shadow Region

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Received January 20, 2010

Abstract—A circular Gaussian beam of monochromatic laser radiation is normally incident to the plane of a right-angle thin metal plate and diffracts at the edges of the plate angle. It is revealed that two main fragments of a composite diffracted wave with cylindrical wave fronts whose axes are mutually orthogonal occur and interfere with each other in the shadow region behind the plate. This is found from the characteristic interference pattern on a flat screen behind the plate. This interference pattern consists of intensive curvilinear, wedge-shaped bands that are located symmetrically in pairs with respect to the bisector of the direct central angle of the circular shadow sector.

Key words: edge diffraction, interference in the shadow region.

DOI: 10.3103/S0027134910030148

It is known [1] that in the case of a monochromatic light field in a vacuum with wavelength λ, in the paraxial approximation performed on the basis of the parabolic equation method

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+2ik\frac{\partial}{\partial z}\right)A = 0$$

one can obtain the mathematical description of an extensive variety of the forms of the law of the distribution of the complex amplitude A in narrow structurally stable laser beams propagating with wave number $k = 2\pi/\lambda$ along the z-axis of the xyz. Cartesian coordinate system. In a laser experiment, it is reasonable to choose the form of the law of the distribution of the amplitude A corresponding to the achievement of the aim of the relevant study [2]. In [3], the fundamental circular Gaussian mode of the linearly polarized monochromatic laser beam turned out to be the most suitable one. In the laser beam a sharp flat blade of a safety razor shaded a circular sector with the central angle $\alpha = 180°$. It is interesting to study the behavior of the diffracted light field in the paraxial spatial region when the diffraction obstacle shades the circular sector of the laser beam with another value of the central angle, for example, $\alpha = 90°$.

This work is aimed at the experimental study of the structure of the intensity distribution of the monochromatic light field in the region of the geometric shadow behind a right-angle nontransparent flat metal plate, at the edge of which a normally incident laser circular Gaussian beam diffracts.

The principle optical scheme of the experiment using a UIG-22M holographic measuring device with an argon laser of the LGN-503 type is shown in Fig. 1. Laser *1* continuously generates a circular Gaussian beam of monochromatic radiation with wavelength $\lambda = 514.5$ nm and with an effective radius of 1.25 mm. In the plane of the figure, the axis *2* of the laser beam coincides with the z-axis of the right-handed Cartesian coordinate system xyz. The x-axis lies in the plane of the figure and the y-axis is perpendicular to it. In real three-dimensional space the x-axis is oriented vertically upwards, and the y- and z-axes are oriented horizontally, parallel to the plane of the table of the holographic setup. The oscillations of the electric vector of the linearly polarized laser wave occur along the x-axis.

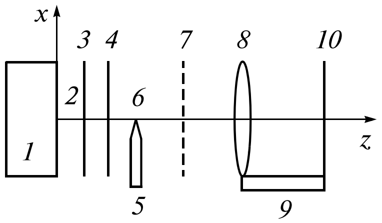

Fig. 1. Principle optical scheme of the experiment: *1*, laser; *2*, axis of the circular Gaussian beam of the laser radiation; *3*, set of light filters; *4*, mechanical shutter; *5*, cross-section of the steel plate of the narrow blade of the safety razor; *6*, wedge-shaped sharp edge of the blade; *7*, removable photographic frame of the holographic setup; *8*, thin collective lens at the front end of the movable optical rail; *9*, movable optical rail on the table of the holographic setup; and *10*, photographic frame at the back end of the movable optical rail.

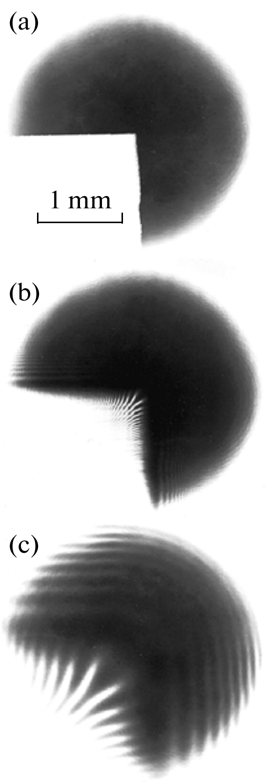

Fig. 2. Examples of the reversed image of the distribution of the light field in three planes behind the diffraction obstacle when moving away from the obstacle at different distances L: a) $L = 0$ cm, b) $L= 3$ cm, and c) $L = 30$ cm.

The intensity of the laser radiation beam is reduced to the required level by a set of light filters *3*. The mechanical shutter *4* controls the duration of the illumination time of the nontransparent diffraction obstacle *5* with a weakened beam. The narrow blade of a safety razor serves as a diffraction obstacle. The blade is of the form of a rectangular steel band 37 mm long, 7 mm wide, and 0.1 mm thick. Several end-to-end holes are perforated in this band. One long edge of the steel band of the blade is wedge-shaped. In our experiment, the plane of the metal plate of the blade is set parallel to the coordinate xy-plane so that the long sharp edge of the blade *6* is parallel to the x-axis. The short edge of the blade is parallel to the y-axis, and the top of the right angle of the plate *5* coincides with the axis *2* of the circular Gaussian beam. In the laser beam, the blade shades only one circular sector with the central angle $\alpha = 90°$. The remaining part of the laser beam freely passes the blade and is normally incident on the plane of the input window of the movable photographic frame *7*.

The photographic frame *7* can be gradually moved on the table of the holographic setup along the z-axis and moved away from the blade plane to an arbitrary distance L up to its maximum value $L = 200$ cm. One can see changes occurring in the diffraction pattern with the screen moved away from the plane of the blade on a flat screen, viz., a simple sheet of white paper in the input window of the photographic frame. The most interesting diffraction patterns are photo recorded by changing the sheet of the usual paper in the photographic frame to a sheet of photographic paper.

When it is necessary to study the fine structure of the diffraction pattern, an auxiliary optical system is placed on the table of the holographic setup instead of the frame *7*. It consists of a thin collective lens *8* with a focal distance *f* of 7 cm, which is fixed on the front end of a standard optical rail *9* 150 cm long by means of a rider, and the photographic frame *10* of the holographic setup with an 9 × 12 cm input window, which is fixed on the back end of the rail. In the plane of the input window of the photographic frame *10* an inverse 20-fold image of the intensity distribution of the light field from the front conjugate plane of the optical system at a distance of 3.5 mm in front of forward focal plane of the lens is formed. The optical rail *9* can be smoothly moved on the table of the holographic setup along the z-axis and the front conjugate plane of the optical system can be superposed with any plane of the light field we are interested in at the distance L behind the plane of the diffraction obstacle *5*. The sheet of photographic paper in the photographic frame *10* for the time $\tau = 10$ s is illuminated with laser radiation. After a standard procedure of developing the illuminated sheet, a reversed image of the distribution the light field appears on the sheet of photographic paper. Figure 2 shows the three most typical cases in the distribution of the light field in diffraction experiment as an example.

If the forward conjugate plane of the optical system is superposed with the plane of the diffraction obstacle ($L = 0$ cm), then, as seen in Fig. 2a, no interference patterns are observed inside the paraxial spatial region, the size of which is limited by the transverse size of the circular Gaussian beam. When the conjugate plane of the optical system is moved away from this initial position by the small distance $L > 0$, one can see the interference bands in both illuminated and shaded parts of the paraxial region. In the experiment, the optical system is consequently moved stepwise from the blade along the z-axis. The forward conjugate plane of the optical system was moved from its initial reference position ($L = 0$ cm) to its final position ($L = 30$ cm) with a step of $\Delta = 3$ cm between two consequent positions.

The experiment shows that visible qualitative changes occur rapidly in the diffraction patterns close to both edges of the right-angle diffraction obstacle *5* when the distance L is only several centimeters. Figure 2b shows the reversed image of the distribution light field at the distance $L = 3$ cm behind the obstacle. One can see on the photo that in the peripheral region of the illuminated paraxial spatial region a system of alternating black and white interference bands is formed. The black bands in the photo correspond to the places of the local maxima of the light field and white bands correspond to the local minima. The bands are parallel to the nearest direct segments, which are the sides of the circular sector of the geometric shadow behind the diffraction obstacle with the central angle $\alpha = 90°$.

Figure 2b shows also the pattern of interference bands that appeared inside the region of the geometric shadow. The black interference bands located far from the top of the right angle of the shadow region look like short direct segments. They are perpendicular to the nearest direct line, which is a side of the circular sector of the geometric shadow. As the top of the direct central angle of the shadow sector is approached, the black interference bands are lengthened and curved. Only the middle black band of the system has the form of a symmetrical wedge-shaped figure, the axis of which is superposed with the line of the bisector of the central angle of the shadow sector. The other black bands of the interference patterns close to it are in the form of twisted wedges. The latter are not symmetrical and are located in pairs that are symmetrical with respect to the bisectors of the central right angle of the shadow sector. It was established that in this interference pattern the local spatial frequency of the alternation of the black and white bands measured along both mutually perpendicular boundaries of the geometric shadow quickly increases as it moves from the top of the central angle of the shadow sector to its periphery.

The complete set of the features of the structure of the interference patterns (in both the illuminated and the shaded parts of the paraxial spatial region) we described convinces us that the axes of the cylindrical fronts of the two fragments of a single complex diffraction wave propagating onward behind the plane of the diffraction obstacle coincide with the edge of the angle of the diffraction obstacle.

Upon the following stepwise increase in the distance *L*, the effect of the accumulation of the intensity of the light field diffracted in the region of the shadow sector and the considerable change of the local spatial frequency of the alternation of the black and white interference bands inside the shadow sector was experimentally found. It was established from the average of a series of ten photos that with the increase in the distance *L*, the area of the reversed image of the light field recorded on the photos as the middle black wedge and that of its nearest neighbors, which have the form of twisted wedges, consecutively increase. The increase in the sizes of the wedge-shaped bands of the interference patterns with the increase in the distance *L* indicates an increase in the penetration depth of the diffracted electromagnetic field in the shadow sector. As an example, Fig. 2 shows the reversed image of the distribution of the light field in the paraxial spatial region at $L = 30$ cm.

The dominant element in this interference pattern in the shadow region is the middle interference band, which has the form of a symmetrical wedge-like figure. This band always exists, even at very large distances *L* when it practically loses all its curvilinear neighbors, viz., the other participants of the process of the formation of the interference patterns in the shadow sector. Such a behavior of the curvilinear bands of the interference pattern, which are symmetrical on the whole, is due to the simultaneous reduction of the curvature value of the cylindrical fronts of these two fragments of the complex diffraction edge wave, which occurs with an increase in the distance *L*.

The experimental discovery of the symmetry of the interference pattern and the effect of the accumulation of the intensity diffracted light field inside the paraxial region of the geometric shadow is a direct confirmation of the fact that the mechanism of the diffraction dissipation beyond the electromagnetic radiation does not depend on whether the wave that is normally incident on the direct edge of the obstacle is *E*- or *H*-polarized. Both interfering fragments of the single wave of the diffracted field in our experiment are completely equal. Their penetration depth in the shadow region increases equally upon increase in the value of the movement away (at the distance *L*) from the plane of the diffraction obstacle. Such a behavior of the fragments of the complex diffracted laser wave agrees with the conclusions that were theoretically obtained as far back as 1896 in the classical work of Sommerfeld [4]. In the strict Sommerfeld theory, in one case, the *E*-polarized plane monochromatic wave diffracts on the edge of an ideal conducting nontransparent half plane, and in the other case, the H-polarized wave diffracts. The conditions used for performing our experiment were, however, more complex than those considered in the Sommerfeld theory. The results of our laser experiment, in which the incident electromagnetic wave of the laser beam is considered to be *E*-polarized on the one edge of the right-angle obstacle and *H*-polarized on the other edge, need an independent theoretical discussion from the point of view of current ideas based on the possibility of using the parabolic equation method in the paraxial approximation.

Conclusions that are important for practical applications were obtained in the physical theory of diffraction of Ufimtsev, in which the diffraction of the electromagnetic wave at the wedge was studied by means of the parabolic equation method [5, chapter 5]. Ufimtsev theoretically proved that the energy of a diffracted electromagnetic field reaches the surrounding space from the border of the light and shadow that exists behind any diffraction obstacle. Such penetration of the energy of a diffracted electromagnetic field is a general rule for a diffraction obstacle of an arbitrary form made from a material that both ideally reflects and absolutely absorbs the electromagnetic radiation incident on the obstacle. The widening of the modern ideas about diffraction is connected with the shadow contour theorems proved by Ufimtsev. The further the receiver located behind the diffraction obstacle is moved away at the distance L from the boundaries of the obstacle contour upon the incidence of the electromagnetic radiation on it, the deeper is the penetration of the diffracted electromagnetic field in the region of the geometric shadow behind the obstacle. This statement of the theory, which agrees well with the results of our laser experiment, is of definite interest for several important practical applications.

ACKNOWLEDGEMENTS

We are grateful to Prof. A. M. Saletskii for help in the experimental study.

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