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Contemporary Climate Changes in the Southwest of the Valdai Hills: A Statistical Analysis of the Long-Term Dynamics of the Air Temperature

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Abstract—As the scientific community accepts the modern global climate changes, statistical analysis of a time series of hydrometeorological parameters becomes topical. A time series of air temperature was decomposed in this work; the decomposition allows one to distinguish regular, seasonal, and random components and to assess their statistical significance and adequacy to observation results. On the basis of a linear-regression model, a statistically significant increase in the annual average air temperature in the region under study was determined, both for the entire observation period and for separate months of the year.

Keywords: hydrometeorological parameters, statistical analysis, time series, regression model, seasonal decomposition, testing of a statistical hypothesis.

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INTRODUCTION

Studies of global physical processes in the atmosphere are based on data that are received from experiments on the measurement of environmental parameters. Estimation of trends in hydrometeorological parameters has become especially urgent in recent years due to climate changes, which are mainly shown in an increase in the global average air temperature [1]. However, climate changes are of a regional nature; therefore, representative estimation of the trends on the regional scale can be based exclusively on local observation data or data from a network of weather stations. The aim of this work is to ascertain and make a statistical estimate of the trends in the air temperature that were found on the basis of long-term observation data in a certain region, viz., the southwest of the Valdai Hills. Data from specially protected natural reservations (natural reserves and national parks) provide unique information about changes in the natural environment and climate [2–8].

1. A STATISTICAL ANALYSIS OF A TEMPERATURE TIME SERIES

A time series of the air temperature was statistically analyzed on the basis of weather-station data. The air temperature was recorded every 3 hours with following averaging to daily average values and is expressed in degrees Celsius (°C); the instrument error is 0.1°C.

Data from any weather station are an array of variable values that are measured in a strictly constant time period, i.e., they are a time (dynamic) series. Therefore, the air-temperature time series is considered further as an ordered sequence, N, of daily average temperature values, $X_1, X_1, \dots, X_N$ at time points, $t_1, t_1, \dots, t_N$, where N is the number of series levels (days).

A statistical analysis of time series assumes that the initial data includes determinate and random components. The determinate component usually consists of a trend that determines the main tendency of the time series and some regular oscillations about the trend, viz., cycles and periodic seasonal oscillations [9–11]. To represent a time series in the form of some composition of its components, these components are to be sequentially distinguished, that is, an initial time series is to be decomposed. A visual analysis of the data in Fig. 1 shows that the temperature time series includes regular (seasonal and trend) and irregular (random) components.

Since the amplitude of seasonal oscillations did not show a tendency toward an increase in the studied

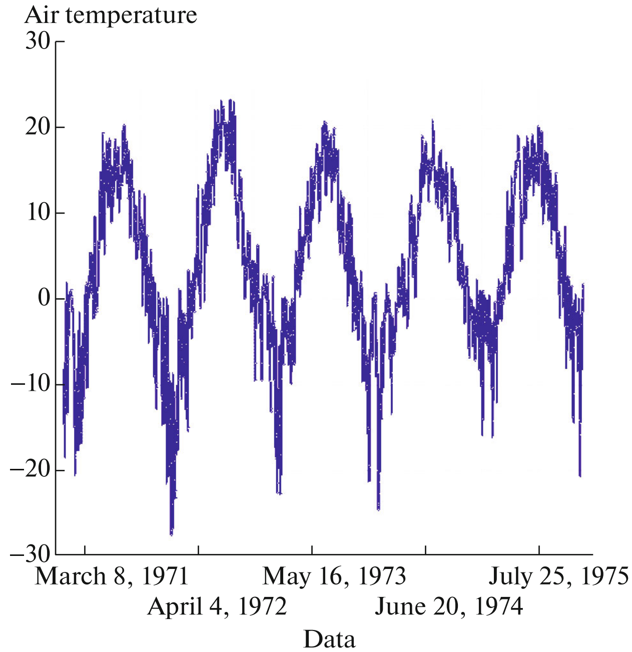

Fig. 1. A plot of the time series of air temperature in °C (1971–1975).

period (from 1971 to 2011), it is reasonable to construct an additive model of the time series [11]:

$$X_t = S_t + u_t + \varepsilon_t, \tag{1}$$

where S_t is the constant seasonal component, u_t is the trend that determines the main tendency of the time series, and ε_t is the irregular component, $t = 1, 2, \ldots, N$, where N is the number of series levels. The 30-year average temperature value at the given time point t is considered as its climate norm at each time point t [1]. Therefore, we assume the seasonal component, S_t, to be constant during 40 years of temperature and precipitation observations, since it is impossible to reveal significant variations in the seasonal time frames for this period.

We propose to carry out the analysis of the dynamics of the temperature time series using smoothing with the simple moving average method and the following construction of a regression model of the time series [12, 13]. The simple moving-average method [12, 14] allows the transformation of an initial series of temperatures or precipitation X_t, $t = 1, \ldots, N$ (1) into a series of moving averages $\hat{X}_i$ of the studied parameter:

$$\hat{X}_i = \frac{\sum_{t=i-p}^{i+p} X_t}{K}, \quad p = \frac{K-1}{2}, \tag{2}$$
$$t = 1, \ldots, N, \quad i = p+1, \ldots, N-p,$$

where K is the smoothing interval equal to the seasonal period, $K = 365$. The arithmetic average is calculated serially for K first values, then for K values beginning from the second one, and so on. Thus, the smoothing interval "slides" along the time series with a unit step and the resulted average value, $\hat{X}_i$, relates to the middle of the chosen interval [12, 15]. The moving average series is shorter than the initial series by a seasonal period, K, since the first and last $K/2$ terms of the initial series are excluded. The smoothing procedure provides results that are more resistant to abnormal values.

After smoothing the initial temperature series, one can pass to the classical seasonal decomposition of the studied time series. Averaging the difference between the levels of the initial and smoothed series over the entire observation period, we find the seasonal component [12]:

$$S_t = \frac{\sum_{j=0}^{J-1} [X_{k+Kj} - \hat{X}_{k+Kj}]}{J}, \quad t = k + Kj, \tag{3}$$
$$k = 1, \ldots, K, \quad j = 0, \ldots, J-1,$$

where J is the number of the seasonal periods (years), $t = 1, 2, \ldots, N$, $J = 41$, and $K = 365$.

2. REGRESSION ANALYSIS OF THE MAIN TREND IN THE DEVELOPMENT OF THE TEMPERATURE TIME SERIES. ESTIMATION OF THE STATISTICAL SIGNIFICANCE AND ADEQUACY OF THE CONSTRUCTED REGRESSION MODEL

After substituting seasonal component S_t (3) from initial time series (1), $t = 1, 2, \ldots, N$, the linear regression equation can be derived, which characterizes the time dependence of the series levels $y_t = X_t - S_t$:

$$y_t = \beta_0 + \beta_{1t} + \varepsilon_t, \tag{4}$$

where β_0 and β_1 are the parameters of the linear regression equation, ε_t is the random component, and $t = 1, 2, \ldots, N$.

Let us note that if the decomposition of time series (1) is successful and the linear model is adequate to observation results, then the random parameter ε_t is normally distributed ($\varepsilon_t \sim N(0, \sigma^2)$, and ε_t, $t = 1, 2, \ldots, N$ values are noncorrelated). It should be also taken into account that the construction of the linear-regression model is reasonable only in the case of a sufficiently strong correlation between the series levels, $y_t = X_t - S_t$, and the time, t, which is estimated by the empirical Pearson's linear-correlation coefficient r_u [12, 15, 16]:

$$r_u = \frac{Q_{ty}}{\sqrt{Q_t Q_y}}, \tag{5}$$

where $Q_t = \sum_{t=1}^{N}(t_i - \bar{t})^2$, $Q_y = \sum_{i=1}^{N}(y_i - \bar{y})^2$, $\bar{t} = \frac{\sum_{i=1}^{N} t_i}{N}$, $\bar{y} = \frac{\sum_{i=1}^{N} y_i}{N}$, $Q_{ty} = \sum_{i=1}^{N}(t_i - \bar{t})(y_i - \bar{y})$, The closer $r_u \in [-1; 1]$ is to unity, the more pronounced the linear dependence of y_t on t is. In practice, construction of regression model (4) is reasonable at $|r_u| > 0.3$.

To solve the main problems of the linear regression analysis of the time series $y_t = X_t - S_t$, it is necessary to assess the parameters β_0 and β_1 in Eq. (4) and verify their statistical significance and the adequacy of the constructed regression model to the observation results. Parameters $\tilde{\beta}_0$ and $\tilde{\beta}_1$, which are found by the least-squares method (LSM) [11, 14–17], are usually considered as assessments of the parameters of linear regression (4), since LSM estimates are unbiased, have a minimal variance in the class of unbiased linear assessments, and coincide with assessments of the maximum likelihood under the above-mentioned properties of the random component ε_t. In this case, the function $y = \tilde{\beta}_0 + \tilde{\beta}_1 t$ defines the empirical (sample) regression of the time series, which allows calculation of the regression values of the series levels $\tilde{y}_t$:

$$\tilde{y}_t = \tilde{\beta}_0 + \tilde{\beta}_1 t, \quad t = 1, 2, \ldots, N. \tag{6}$$

Linear-regression model (4) is considered insignificant if $\beta_1 = 0$, since there is no linear dependence of the series levels y_i on time t in Eq. (4) in this case. Therefore, to analyze the significance of linear-regression model (4), it is necessary to verify the null hypothesis $H_0 : \beta_1 = 0$ at the alternative $H_1 : \beta_1 \neq 0$ at the significance level α [17–20]. For this, the sample test statistics, F_s, for verification of the null hypothesis are defined:

$$F_s = \frac{\sum_{t=1}^{N}(\tilde{y}_t - \bar{y})^2}{S^2}, \tag{7}$$

where $S^2 = \frac{\sum_{t=1}^{N}(y_t - \tilde{y}_t)^2}{N-2}$ is the residual variance and $\bar{y} = \frac{\sum_{t=1}^{N} y_t}{N}$, $\tilde{y}_t$ are the values of the series levels that are calculated from regression equation (6), $t = 1, 2, \ldots, N$ If the hypothesis $\beta_1 = 0$ is true, then the sample statistics F_s have the Fisher distribution [11]: $F_s \sim F(1, N-1)$. Therefore, the hypothesis H_0 is accepted at the specified significance level α, if $F_s < F_{1-\alpha}(1, N-2)$, where $F_{1-\alpha}(1, N-2)$ is the reciprocal Fisher distribution of the order $(1-\alpha)$, and regression model (4) is insignificant. If $F_s > F_{1-\alpha}(1, N-2)$, the hypothesis $H_0 : \beta_1 = 0$ is rejected, and the regression model is considered statistically significant.

Again, the null hypothesis, $H_0' : \beta_1 = 0$, is verified at the alternative $H_1' : \beta_1 \neq 0$ at the significance level α'. The sample test statistics are considered:

$$t_s = \frac{\tilde{\beta}_0}{\sqrt{D\tilde{\beta}_0}}, \tag{8}$$

where $D\tilde{\beta}_0$ is the variance of the assessment $\tilde{\beta}_0$, the hypothesis H_0' is accepted at the significance level α, if $|t_s| < t_{1-\frac{\alpha'}{2}}(N-2)$, where $t_{1-\frac{\alpha'}{2}}(N-2)$ is the reciprocal Student's distribution of the order $1 - \frac{\alpha'}{2}$ [11, 18]. In the opposite case, H_0' is rejected at the significance level α', and the alternative $H_1' : \beta_1 \neq 0$ is accepted. If both the null hypothesis H_0 and H_0' are rejected after the verification, then the linear-regression equation is considered to be statistically significant in both parameters.

The determination coefficient, R^2, is an important characteristic of the linear-regression model. It shows the contribution of trend (6) in the variance, Dy, of the initial process:

$$R^2 = \frac{\sum_{i=1}^{N}(\tilde{y}_i - \bar{y})^2}{\sum_{i=1}^{N}(y_i - \bar{y})^2} = 1 - \frac{\sum_{i=1}^{N}(y_i - \tilde{y}_i)^2}{\sum_{i=1}^{N}(y_i - \bar{y})^2}, \tag{9}$$

since according to the main dispersion analysis identity $\sum_{i=1}^{N}(y_i - \bar{y})^2 = \sum_{i=1}^{N}(\tilde{y}_i - \bar{y})^2 + \sum_{i=1}^{N}(y_i - \tilde{y}_i)^2$. Let Q_e be the residual sum of squares

$$Q_e = \sum_{i=1}^{N} e_i^2, \quad e_i = y_i - \tilde{y}_i. \tag{10}$$

If the regression model of the series is linear, as in our case, then $\sqrt{R^2} = |r|$, where r is the empirical assessment of the empirical Pearson's linear-correlation coefficient (5).

To estimate the adequacy of the linear-regression model, the residuals e_t, $t = 1, 2, \ldots, N$, in Eq. (10) should be analyzed. If model (6) is adequate, then Q_e (10) has the Pearson's distribution: $Q_e \sim \sigma^2\chi^2_{N-2}$ [17]. Since the number of observations, N, is quite large in the time series of meteorological parameters, $N = JK$, $J = 41$, $K = 365$, then the residuals e_t, $t = 1, 2, \ldots, N$ are distributed almost normally. Hence, to estimate the adequacy of model (6), the hypothesis of a normal distribution of the residuals, e_t, is to be verified on the basis of the Pearson's test for goodness of fit [9, 11]. In statistical packets, the results of verification of the hypothesis of the normal distribution of the sample

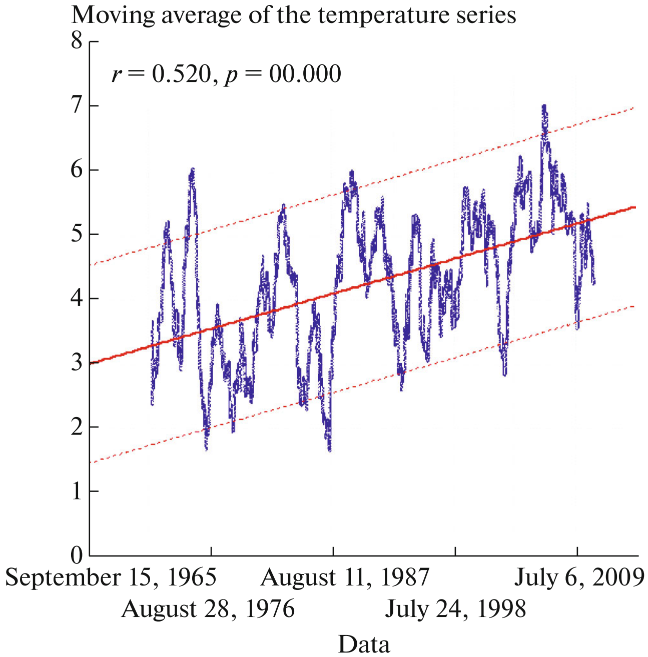

Fig. 2. A moving average, $\hat{X}_t$, of the air-temperature time series for 1971–2011 with a smoothing interval of 1 year, linear regression plot (straight line), and 0.9-confidence band of the regression equation (dashed lines).

data that are under study are shown in a special plot, viz., the so-called probability paper. This method is based on the fact that the empirical distribution function, $\hat{F}(x)$, which is constructed from the sample data, converges to the theoretical distribution function, $F(x)$, of a random variable observed at $N \to \infty$, $-\infty < x < \infty$. The plot of the function $F^{-1}(F(x))$, where $F^{-1}(\cdot)$ is a function that is inverse to $F(\cdot)$, is a straight line. If the residuals, e_t, are normally distributed, $N(0, \sigma^2)$, then the values of the function $F^{-1}(\hat{F}(x))$, where $\hat{F}(x)$ is the sample distribution function of the residuals e_t, are located near the straight line on the probability paper. Let us assume that the sample data obey the normal distribution law. If a function that is inverse to the distribution function of a normal random variable is selected as $F^{-1}(\cdot)$, then the plot of $F^{-1}(\hat{F}(x))$ is close to a straight line.

3. DECOMPOSITION OF THE TEMPERATURE TIME SERIES. THE RESULTS OF THE CALCULATION EXPERIMENT

3.1. The Trend Component of the Temperature Time Series

Statistical analysis of the time series of air temperature was carried out in this work using the STATISTICA software package. A fragment of the time series of the air temperature is shown in Fig. 1a. Time series (1) of the air temperature for 1971–2011 was smoothed using a simple moving-average method (2) with a smoothing interval of 1 year (Fig. 2, blue curve). The linear time dependence of the smoothed temperature series levels $\hat{X}_t$ allowed the empirical Pearson's correlation coefficient r_u (5) to be estimated as 0.52, which indicates a linear correlation dependence of the smoothed daily average air temperature, $\hat{X}_t$, on time.

Linear-regression equation (4), (6) was plotted (Fig. 2, the straight line), where the levels of the smoothed series, $\hat{X}_t$, are used as the y_t values. The following estimates of the linear-regression model were found from the regression analysis of the temperature series: $\beta_0 = -0.272$ and $\beta_1 = 0.000137$ Verification of the null hypothesis, H_1: $\beta_1 = 0$, of the insignificance of the linear-regression model at the alternative, H_1 : $\beta_1 \neq 0$, at the significance level $\alpha = 0.01$ showed a vanishing minimum significance level, p, during observation of $\hat{X}_t$. The value $p = P(F(1, N-1) > F_s)$ is called the p level, where $F(1, N-1)$ is a random parameter with the Fisher distribution and F_s is the sample Fisher statistics. Since $\alpha > p$, the $H_0 : \beta_1 = 0$ hypothesis should be rejected. Verifying the null hypothesis, $H_0' : \beta_1 = 0$, at the alternative, $H_1' : \beta_1 \neq 0$, at the significance level $\alpha' = 0.01$ on the basis of sample test statistics, t_s (8), we find that the minimum significance level p also is close to zero during observations of $\hat{X}_t$. Thus, the $H_0' : \beta_1 = 0$ hypothesis should be rejected and both parameters of the linear regression should be considered significant.

The determination coefficient, $R_u^2 = 0.27$, (9) shows the significant (>25%) contribution of trend (6) in the variance, $D\hat{X}$, of the levels of the smoothed series. Hence, linear-regression model (4) is statistically significant at a significance level lower than 0.01; the conclusion can be drawn that a general tendency toward an increase in the air temperature occurred for this period.

To estimate the adequacy of the linear-regression model that was constructed of the smoothed temperature series $\hat{X}_t$, residuals e_t, $t = 1, 2, \ldots, N$ (10) were estimated. The verification of the hypothesis of the normal distribution of the residuals e_t with the use of the Pearson's test for goodness of fit (Fig. 3, left) shows that the histogram of the residuals is close to the frequency curve of the normal distribution. The closeness of the empirical distribution of the residuals, e_t, to the theoretical normal distribution is also shown in the probability paper (Fig. 3, right): the values of the $F^{-1}(\hat{F}(x))$ function are near a straight line.

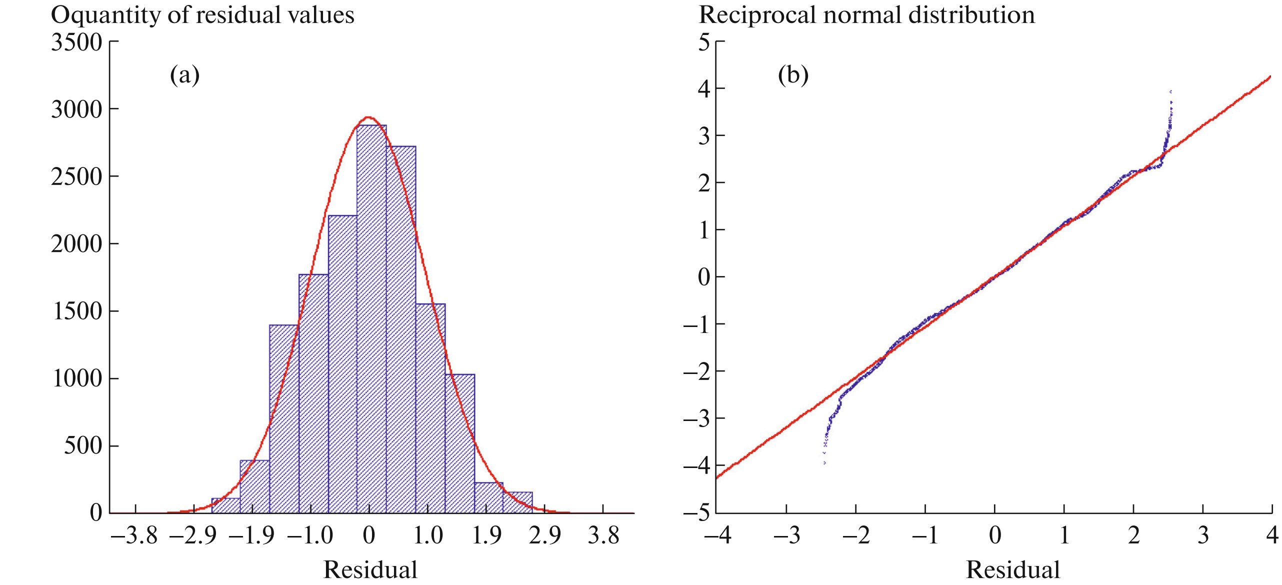

Fig. 3. A histogram of the residuals, e_t, of the moving averages, $\hat{X}_t$, of the temperature series (left). The closeness of the empirical distribution of the residuals, e_t, of the temperature series to the theoretical normal distribution (right).

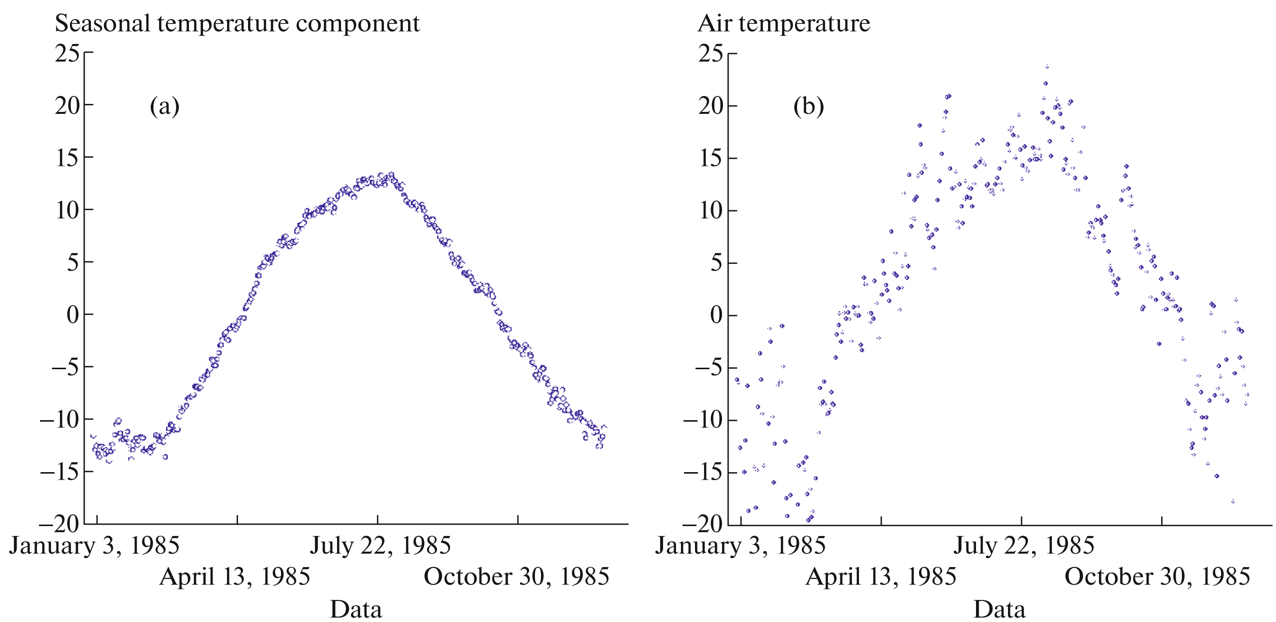

Fig. 4. The seasonal component, S_t, of the temperature series (left). The air temperature time series, X_t, in 1985 (right).

3.2. The Seasonal Component of the Temperature Series

The most important step of the decomposition of the time series is distinguishing the seasonal component S_t (3). The seasonal component of the air-temperature series X_t (1) is shown in Fig. 4 (left). The corresponding part of the initial time series X_t for 1985 is shown in Fig. 4 (right) for comparison. The zero point at the horizontal axis corresponds to January 1, 1985. The dispersion analysis of the seasonal component, S_t, has shown that it contributes much more in the variance of the initial time series, X_t, than the linear trend component: the determination coefficient $R_s^2 = 0.78$ for the seasonal component to $R_u^2 = 0.27$ for the linear trend component. Thus, as is expected, the seasonal component is the most significant component in the air-temperature time series. In practice, the seasonal component is variable; therefore, it contributes differ-

The significance of the linear trend of the temperature for different months

Temperature increase	Model significance, $\alpha < 0.01$	Determination coefficient	Correlation coefficient	10-year trend, °C
All months	significant	0.27	0.52	0.5
January	significant	0.15	0.39	1.13
February	insignificant			
March	significant	0.04	0.21	0.36
April	significant	0.17	0.41	0.67
May	significant	0.02	0.16	0.27
June	significant	0.05	0.22	0.34
July	significant	0.23	0.48	0.74
August	significant	0.28	0.53	0.69
September	significant	0.11	0.33	0.51
October	significant	0.11	0.33	0.54
November	insignificant			
December	insignificant			

ently to temperature time series for different years, e.g., $R_s^2 = 0.60$ in 1985, $R_s^2 = 0.91$ in 1993, and $R_s^2 = 0.73$ in 2003. For the same reason, the Pearson's correlation coefficient r_y is lower when constructing the linear-regression model of temperature series $y_t = X_t - S_t$ (4) without the constant seasonal component, S_t, than the Pearson's correlation coefficient, r_u, of the regression equation for the moving average series $\hat{X}_t$: $r_y = 0.12$ and $r_u = 0.52$.

Despite the low value of the correlation coefficient, r_y, the linear-regression model of the series, $y_t = X_t - S_t$, is significant at the significance level $\alpha \geqslant 10^{-16}$; hence, a general tendency toward an increase in the daily average air temperature for the studied period is confirmed.

3.3. Analysis of the Linear Regression Model of the Air Temperature Time Series for Different Months

Another aim of this work is the study of the air-temperature dynamics in different months of the year separately. Let us consider 12 air temperature time series, $X_{t1}, X_{t2}, \ldots, X_{t12}$, where X_{ti}, $i = 1, \ldots, 12$, is the temperature time series for the ith month that is found from the initial series X_t, $X_{ti} = \sum_{j=1}^{j=41} X_{ti}^j$, where X_{ti}^j is the fragment of the series X_t, which includes the temperature values of the ith month of the jth year. The table shows the results of the regression analysis of the temperature time series for different months of the year.

The regression model of the smoothed series, X_{ti}, $i = 1, \ldots, 12$, allows the following conclusion: the linear-regression model is significant for the series $\hat{X}_{ti}$, $i = 1, 4, 7, 8, 9, 10$, i.e., for January, April, July, August, September, and October. For these months, the $H_0: \beta_1 = 0$ hypothesis is rejected at the significance level $\alpha = 0.01$ and the linear Pearson's correlation coefficient exceeds 0.33, which indicates the beginning of a tendency toward an increase in the air temperature for 40 years in these months of the year. Analyzing the temperature time series, we can see the strongest warming in January (see the table).

CONCLUSIONS

A time series of the daily average air temperature has been decomposed using analytical smoothing with the simple moving-average method; linear-regression models of the smoothed time series for the time period 1971–2011 were constructed; the seasonal and irregular components were distinguished.

The linear trend of the temperature increase was distinguished; its significance (at a level of no more than 0.01) and adequacy to observation data were ascertained. Dispersion analysis shows that the seasonal component contributes much more to the variance of the initial time series than the linear-trend component and is more significant in the air temperature time series. This linear trend of the temperature series 0.50°C/10 years agrees with ROSHYDROMET data (0.48°C/10 years, European part of Russia, 1976–2006).

The linear-regression models for the temperature series were constructed for individual months. Their significance and contributions in the variance of the initial temperature time series were analyzed. A trend toward an increase in the air temperature for 40 years has been revealed for the 1st, 4th, 7th, 8th, 9th, and 10th months of the year.

The analysis of the temperature time series and the linear-regression model indicate a statistically significant increase in the air temperature by 1.6°C for 40 years in the region at a measurement error of 0.1°C.

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