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CONDENSED MATTER PHYSICS

Switching between the Stable States of a Long Josephson φ Junction

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Abstract—This paper describes current transport through a long Josephson junction with an alternating critical—current density. This alternating density can be achieved in experiments by incorporating a magnetic layer to the weak link in a special manner. The prospects for the practical use of such structures are related to the possibility of obtaining bistable Josephson elements on their basis. Joint analysis of both current—phase relations and dynamic characteristics made it possible to optimize the operation mode for a fast superconducting memory cell based on bistable contact and to assess the energy dissipation for the read and write operations.

Keywords: superconductivity, fast single-quantum logic, magnetism, Josephson effect, Josephson junction, Josephson memory.

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INTRODUCTION

Josephson structures with magnetic layers (F) in the weak link region are the main candidates for the role of the key elements of fast energy-effective memory that is compatible with chains of superconductive (S) electronics [1–9]. Among multiple concepts of the performance of the mentioned key elements the proposals that require the use of distributed magnetic contacts with a spatially inhomogeneous density of the critical—current stand out. The use in the weak bond of (1) a magnetic layer with variable thickness (Fig. 1a) [10] or (2) a normal "sublayer" (N) that occupies only a part of the total contact area (Fig. 1b) [11] or (3) a bilayer normal metal/ferromagnet (N/F) with an NF boundary that is oriented along the direction of the passage of the non-dissipative current (Fig. 1c) [12] provides the alternating-sign density of the critical—current under certain conditions. For certain relationships between two strongly different current channels and for quite large, i.e., comparable with the Josephson length, λ_j, sizes, the system as an integer behaves as a so-called φ junction, i.e., as a Josephson element, whose phase in the ground state in the absence of the transport current passing through it is $\pm\varphi$ ($0 < \varphi < \pi$) [13]. Two stable states of such an element (corresponding to a Josephson phase of $+\varphi$ and $-\varphi$) can be used as the 0 and 1 elements of memory; the presence of two values of the critical—current (transition to the resistive state accompanied by the voltage pulse on the contact here depends on the "choice" of one of the stable states by the system) makes it possible to perform the read operation; the application of the magnetic field makes it possible to transfer the φ junction from one stable state to another by performing the write operation [14]. Here it is necessary to emphasize that the write operation can be performed with the use of relatively weak (to several oersted) fields during the characteristic Josephson time measured in picoseconds.

The simplest model for the qualitative description of the dynamic characteristics of a φ junction is an asymmetric two-contact interferometer with different signs of the critical current of Josephson junctions and rather high connecting inductance [15]. Important successes on the path to a more complete theoretical picture of the physical phenomena in the system were achieved during the analysis of equations of the sine-Gordon type with a stepwise change of the critical—current density [16]. However, one definite drawback of these expressions for the form of the current—phase relation (CPR) is the assumption of the smallness of the variation of the Josephson phase of the structure in the direction that is perpendicular to the direction of the passage non-dissipative current during the derivation. In this work we propose an improved variant of models that were created on the basis of a parallel chain of Josephson junctions (Fig. 1d): some of the junctions are conventional 0 contacts, while some are

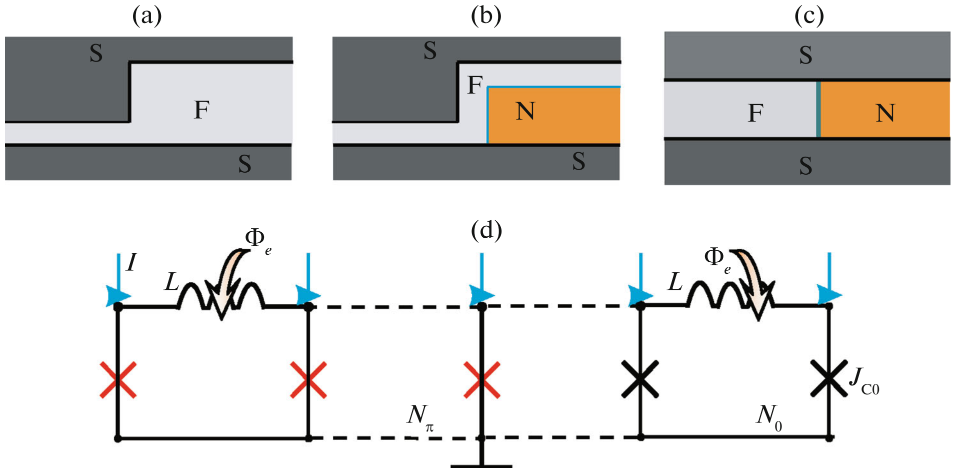

Fig.1. (a), (b), (c) Schematic images of long Josephson structures that contain superconducting (S) and magnetic (F) layers, as well as normal metal layers (N); (d) the principal scheme of this model on the basis of a parallel chain of Josephson 0 and π junctions.

π contacts with a negative critical–current that have found great use in modern superconductive electronics [17], which makes it possible to take the quasicontinuous variation of the Josephson phase along the spatially inhomogeneous structure into account. The write and read operations were optimized on the basis of the modified model of the magnetic Josephson junction with a spatially inhomogeneous region of the weak link.

1. SIMULATION OF THE DYNAMICS OF A DISTRIBUTED JOSEPHSON JUNCTION

We use the "resistively shunted junction model" for the description of the concentrated Josephson elements assuming that the total current through the junction consists of three components: (1) the current that passes without energy dissipation; (2) the resistive component of the current; and (3) a capacitive term that occurs due to the formation of an effective capacitor by superconducting electrodes (S) [18]:

$$I = I_c \sin\phi + \frac{\hbar}{2e_0 R_N}\frac{\partial\phi}{\partial t} + \frac{\hbar C}{2e_0}\frac{d^2\phi}{dt^2}, \qquad (1)$$

where I_c is the critical current, i.e., the maximum non-dissipative current that can pass through the contact, R_N is the normal resistance of the junction, C is the capacity of the contact, $\hbar$ is the Planck constant, e_0 is the electron charge, and t is time. The form of the non-dissipative current term, $I_c \sin\varphi$ reflects the electroneutrality of the system as a whole and the 2π periodicity of the phase of the order parameter (the stationary Josephson effect). The voltage on the Josephson junction is proportional to the time derivative of the Josephson $V = \frac{\hbar}{2e_0}\frac{\partial\phi}{\partial t}$ phase (the non-stationary Josephson effect), from which the form of the second V/R_N and third $C\frac{\partial V}{\partial t}$ term in (1) follows.

We simulate the distributed contact as a parallel chain of Josephson junctions that are united by a small inductance $L(\lambda^{-1} \equiv l = 2\pi L I_c/\Phi_0)$, Φ_0 is the magnetic flux quantum). Then for elements of the distributed contact we write a system of discrete sine-Gordon equations:

$$\begin{aligned}
\frac{\partial^2\phi_1}{\partial t^2} &= i + \left(\frac{\phi_2 - \phi_1 - \varphi_e}{l}\right) - j_c \sin\phi_1 - \alpha\frac{\partial\phi_1}{\partial t},\\
\frac{\partial^2\phi_k}{\partial t^2} &= i + \left(\frac{\phi_{k+1} - \phi_k - \varphi_{k-1}}{l}\right) - j_c \sin\phi_k - \alpha\frac{\partial\phi_k}{\partial t}, \qquad (2)\\
\frac{\partial^2\phi_N}{\partial t^2} &= i - \left(\frac{\phi_N - \phi_{N-1} - \varphi_e}{l}\right) - j_c \sin\phi_N - \alpha\frac{\partial\phi_N}{\partial t},
\end{aligned}$$

where $j_c \begin{cases} j_{c,\pi}, k \le N_\pi \\ j_{c,0}, k > N_\pi \end{cases}$, N and N_π are the total number of elements and the number of π contacts, respectively, φ_e is the external dimensionless flux, and α is the dimensionless decay in the system. The number of contacts may be associated with the sizes of the real system using the relationship $l = (d/(N-1))^2$, where d is the linear size of the junction expressed in Josephson lengths λ_j and the unit current density is accepted as the critical–current density in the 0 region $j_{c,0}$.

To build the CPR of the distributed contact, it is necessary for each value of the power–supply current, i, that does not exceed the critical current of the structure to find the value of the established average phase $\psi = \frac{1}{N}\sum_{k=1}^{N}\phi_k$, where ϕ_k is the established phase on the k-th junction that was obtained from system of equations (2) using the well-known Runge–Kutta

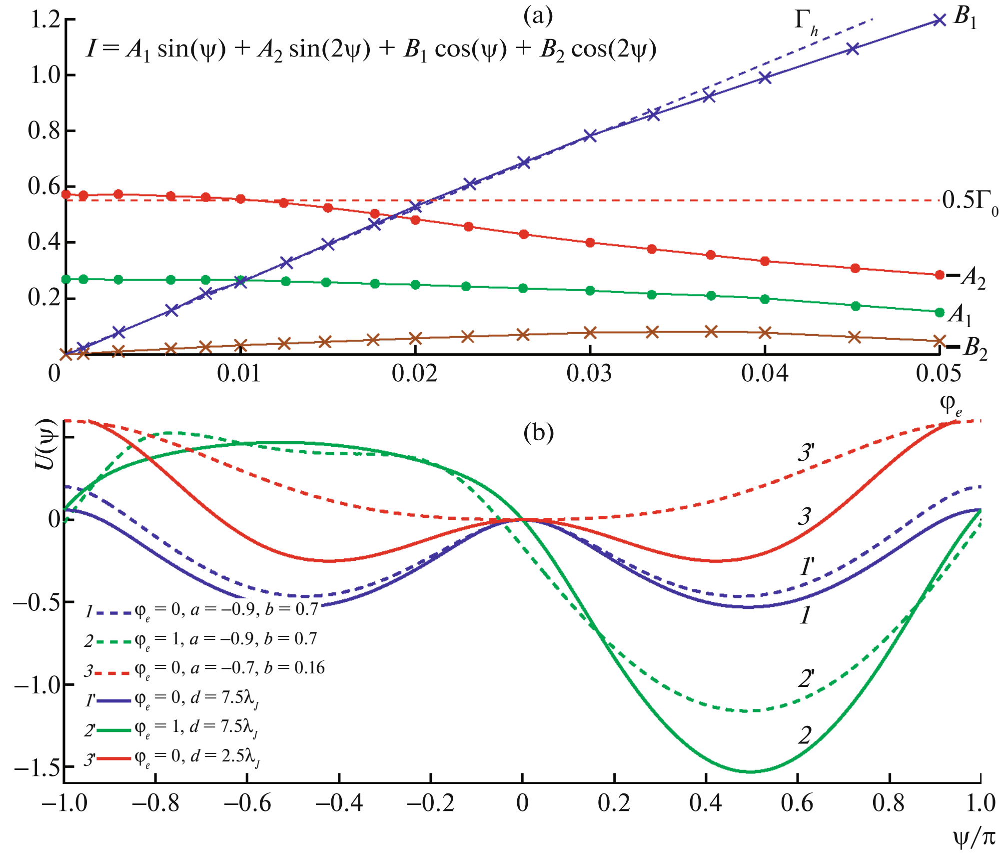

Fig. 2. (a) The dependence of the amplitudes of the first two harmonics (both odd and even) on the external dimensionless flux for $N = 80$, $N_\pi = 40$, $J_{c,\pi} = -0.7 j_{c,0}$, and $l = 0.001$.The dashed lines illustrate similar results for CPR components of the distributed contact that was obtained on the basis of the expression $\Gamma_0 = -\frac{(N-1)l}{12}\left(\frac{j_{c,0} - j_{c,\pi}}{j_{c,0} + j_{c,\pi}}\right)^2$ etc. transformed from [18]; (b) the Josephson potential as a function of the phase and its evolution under the action of the applied flux. The dashed lines present similar results for a two-contact interferometer that contains 0 and π junctions. The parameters of the model are specified in the figure.

method. The problem with this approach is that from the solution of equations for the dynamics of the phase it is possible to find only stable states that satisfy the condition $\frac{\partial i}{\partial \psi} > 0$. This problem was solved by the approximation of the obtained stable points via a dependence of the form

$$I(\psi) = \sum_{j=1}^{M} (A_j \sin(j\psi) + B_j \cos(j\psi)). \qquad (3)$$

The algorithm for the approximation without an external magnetic field contained three steps.

(1) The matrix equation $\hat{A}\hat{\psi} = \hat{i}$ was solved for M = 1, where in the general form $\hat{\psi} = \begin{pmatrix} \sin\psi_1 & \ldots & \sin(M\psi_1) \\ \ldots & \ldots & \ldots \\ \sin\psi_M & \ldots & \sin(M\psi_M) \end{pmatrix}$, $\hat{i} = \begin{pmatrix} i_1 \\ \ldots \\ i_M \end{pmatrix}$, $\hat{A} = \begin{pmatrix} A_1 \\ \ldots \\ A_M \end{pmatrix}$ and i_j and ψ_j are obtained from the solution of a system of differential equations (2).

(2) The square deviation for each exact solution was calculated for the obtained $\hat{A}$ coefficients.

(3) If the total square deviation turned out to be larger than the given accuracy value, we returned to issue (1) with an increase in M by 1. If the deviation turned out to be less than the given accuracy value, it was considered that the function $I(\psi)$ is the CPR of the distributed contact with the given accuracy.

In the presence of an external magnetic field the algorithm remained the same; however, in addition to the sinusoidal harmonics, the even harmonics with respect to the Josephson phase were also considered.

2. SIMULATION RESULTS

The performed numerical simulation made it possible to take the effect of the quasicontinuous variation

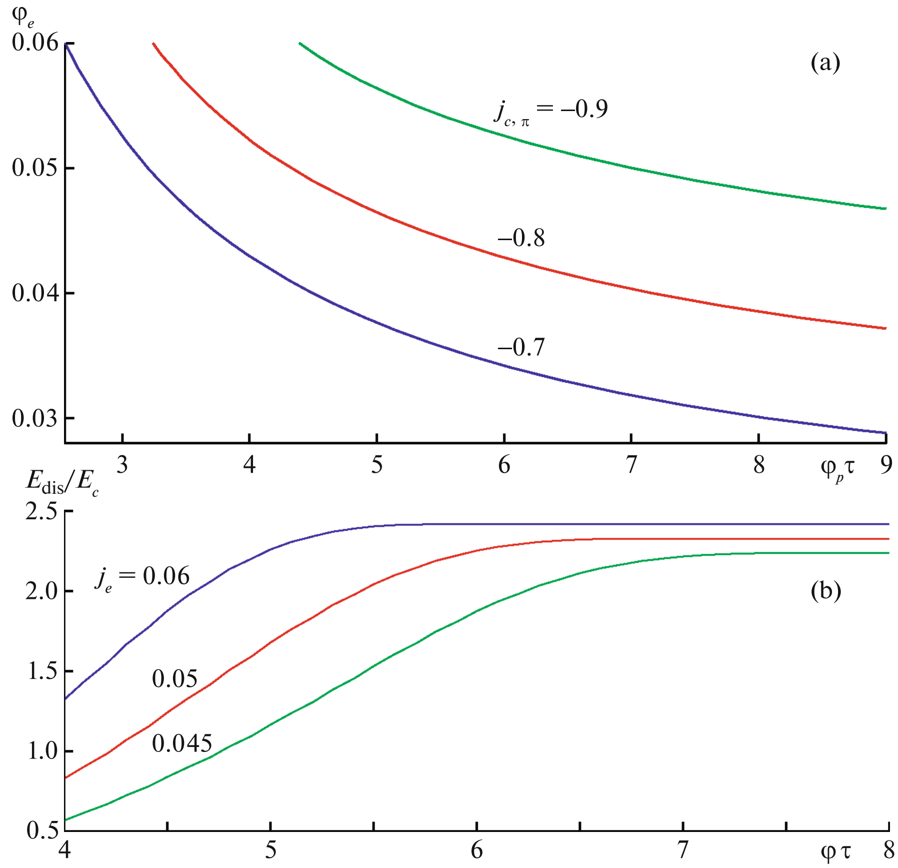

Fig. 3. (a) The dependence of the amplitude of the magnetic flow pulse on its minimum duration that is necessary for the write operation at different critical—current densities in the π region; (b) the dependences of the normalized dissipation energy for the write operation for different amplitudes of the magnetic flux pulse (dimensionless decay is 1 everywhere).

of the Josephson phase along a spatially inhomogeneous structure on the CPR form into account, which is particularly noticeable in the presence of an external magnetic field. Figure 2a shows the calculated dependences of the amplitudes of the first two harmonic components (both odd and even) from the normalized external flux into the cell $\varphi_e = 2\pi\frac{\Phi_e}{\Phi_0}$ (the parameters were taken based on the example of the model: $N = 80$, $N_\pi = 40$, $j_{c,\pi} = -0.7 j_{c,0}$ and $l = 0.001$, and the sizes of the 0 and π regions of the system, d_0 and d_π, are approximately 1.25). For comparison we should mention that the solution of the continuous sine-Gordon equation with respect to the Josephson phase $\frac{\Phi_0}{2\pi l_J}\varphi - j_c(x)\sin(\varphi) = -j$ (l_J is the specific inductance and j is the current density of the power supply) by expansion over the small parameter, viz., the deviation of the phase from the average ψ value, to the second order of magnitude gives the following form of the current-phase dependence [19]:

$$I(\psi) \propto \left(\sin\psi + \frac{\Gamma_0}{2}\sin 2\psi + h\Gamma_h\cos\psi\right), \tag{4}$$

$$\Gamma_0 = -\frac{d_0^2 d_\pi^2}{3}\frac{(j_{c,0} - j_{c,\pi})^2}{(j_{c,0}d_0 + j_{c,\pi}d_\pi)^2},$$
$$\Gamma_h = -\frac{d_0 d_\pi}{2}\frac{j_{c,0} - j_{c,\pi}}{j_{c,0}d_0 + j_{c,\pi}d_\pi}. \tag{5}$$

The parameters of the simulated system that were chosen for the example (the length of the distributed contact is approximately $2.5\lambda_j$ and the condition $|A_2| > A_1/2$, $A_2 < 0$ is met for the amplitudes of the odd harmonic components) correspond to the bistable Josephson potential, for which in the absence of an external magnetic field the minima are separated by a barrier, U_b, which almost excludes the transitions between stable states under the action of fluctuations.

A system with two stable states can be considered as an elementary memory cell that holds one bit of information. To perform the write operation in such a cell, it is necessary to apply a certain external magnetic flux: Fig. 2b shows the evolution of the Josephson potential under the action of the applied flux, as a

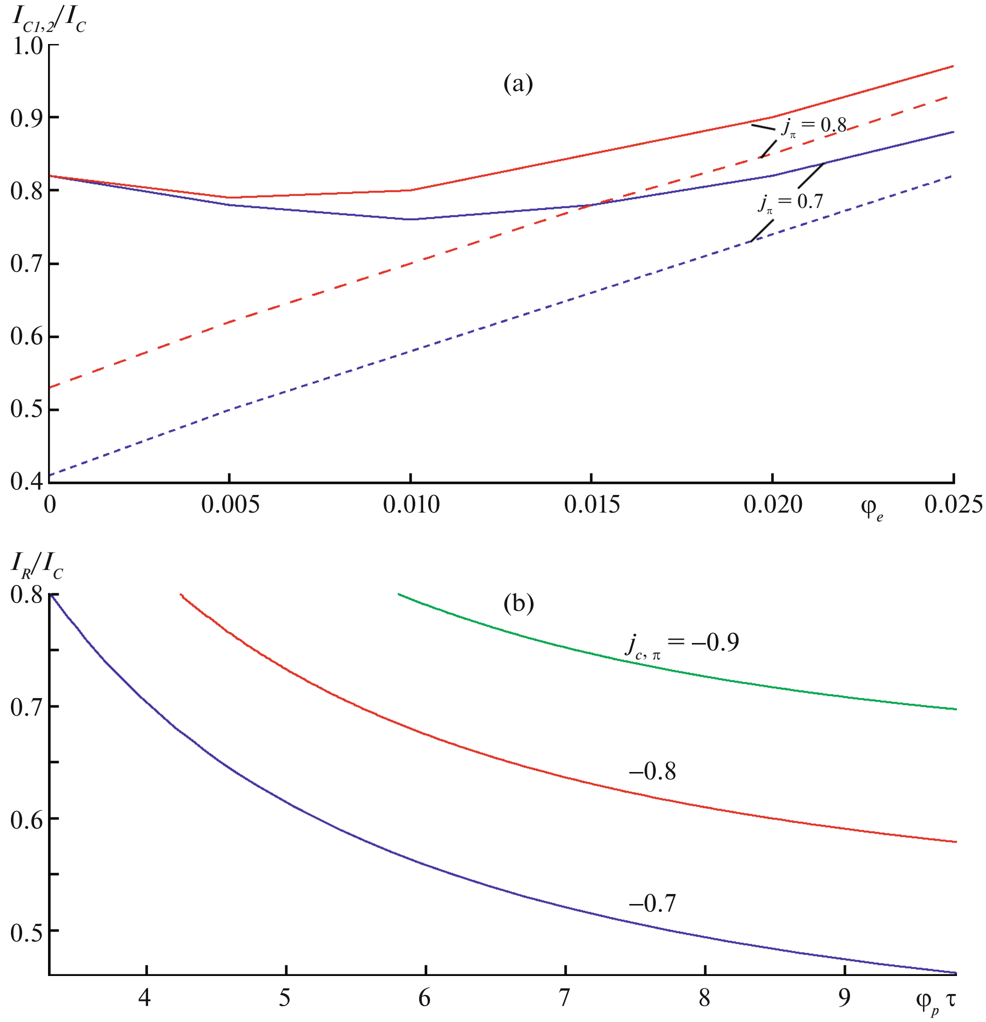

Fig. 4. (a) The dependences of two values of the critical—current of the φ junction on the external magnetic flux at different critical—current densities in the π region; (b) the dependences of the amplitude of the magnetic flux pulse on its minimum duration that is necessary for the write operation at different critical—current densities in the π region.

result of which the system will occur in the stable +φ state. Curves *1* and *1*' along with *2* and *2*' in the figure show that on the qualitative level it is possible to describe the behavior of the system, even using the "ultra-concentrated" model of a two-contact interferometer containing 0 and π junctions (let the ratio of the critical currents in such a model be the parameter a, and the dimensionless inductance of the shoulders be b). It is possible to conclude from the shape of curves *3* and *3*' that the requirements of the "effective size of the system" for the performance of the bi-stable state that are given by the value of the parameter b turn out to be strongly overestimated.

This method for the analysis of the dynamic characteristics of the bistable Josephson system may permit the optimization of the described write operation with allowance for the effect of the quasicontinuous change of the Josephson phase along a spatially inhomogeneous structure on the CPR. The systematized results of such optimization are given in Fig 3a. As expected, the minimum duration of the "rectangular" pulse, τ, that is necessary for the operation is inversely proportional to the value of the applied magnetic flux; the τ value also grows with the increase in the normalized critical—current density in the π region, which is associated with the increase in the amplitude of the second odd harmonic component in the CPR and the potential barrier, U_b, in the "initial" state. Finally, the minimal impact duration that is required for the write operation also increases with an increase in the decay exponent of the plasma oscillations in the Josephson junctions of the model α.

The change of the Josephson phase of the system in the course of operation in accordance with the nonstationary Josephson relationship causes a voltage pulse on the junction and energy dissipation that is associated with this voltage of the order $E_c = I_c\Phi_0/2\pi \times 10^{-19}$ J [20]:

$$E_{dis} = E_c\alpha\int_{-\infty}^{\tau}\left(\frac{\partial\psi}{\partial t'}\right)dt'. \qquad (6)$$

It is seen in Fig. 3b that when the duration of the pulse increases, the dissipation energy tends to a constant value, which corresponds to the complete decay of the phase fluctuation in the system, and the pulse amplitude affects the value of the dissipated energy more strongly than its duration. For lower energy release it is necessary to use long weak pulses; however, this will lead to an increase in the writing time. In addition, systems with low modules of the critical—current density in the π region reach a constant (and somewhat smaller) value of the energy dissipation faster.

The performance of the read operation is based on the existence of two critical power-supply current values for the memory cell on the basis of the φ junction (Fig. 4a shows the calculated dependences of these values on the external magnetic flux). The excess of the first critical value, I_{c1}, will transfer the total Josephson element in the resistive state, which forms the detected voltage pulse only if it initially was in the stable state +φ state. Figure 4b illustrates the relationship between the minimum duration of the rectangular current pulse, τ, that is necessary for performing the read operation and the amplitude of the impact I_R for different parameters of the system, whose effect is analogous to the write operation, which was considered in detail.

CONCLUSIONS

Thus, the optimization results show that the characteristic time for the read and, of particularly importance, the write operation is determined by the Josephson processes and, as a consequence, in many orders of magnitude less than the writing time record in typical cryogenic magnetic-memory cells, which makes it possible to use the bistable φ junction as a promising element of superfast random-access memory (RAM). It should be noted separately that the traditional opinion that Josephson π junctions are "slow" elements is not related to reality. For S—IsF—S structures that contain an additional superconducting layer in the region of the weak bond, the possibility of π junctions with characteristic voltage I_CR_N and characteristic frequency values that are close to typical for tunnel transitions has already been demonstrated [21, 22]. Analogously, the addition of the auxiliary superconducting layer with the tunnel barrier *I* in the region of the weak bond of structures that is shown in Fig. 1 will make it possible to solve the problem of speed. The calculated energy dissipation in the considered processes demonstrates the achievable high-energy efficiency of the memory cell: less than a unit of the characteristic Josephson energy (on the order of 10^{-19} J) per one operation.

This method for the analysis of charge transport in a distributed Josephson junction with an alternating critical—current density may make it possible to optimize the characteristics of calm qubits [23, 24], semiflaxon Josephson oscillators [25], and perspective Ratchet systems [26], as well.

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