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PHYSICS OF EARTH, ATMOSPHERE, AND HYDROSPERE

A Hydrodynamic Method for Calculating the Wind Load on the Leeward Slope of a Roof and for Reducing the Pressure Jump

O. N. Melnikova and K. V. Pokazeev

Department of Physics, Moscow State University, Moscow, 119991 Russia

e-mail: olamel@yandex.ru, sea@phys.msu.ru

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Abstract—A hydrodynamic model for calculating the pressure jump on the leeward slope of a roof during a horizontal air flow that is directed to its slope is proposed for the first time. The model takes the formation of vortices in a viscous layer of an air flow that decelerates in the flow direction into account. When a vortex departs from the viscous layer, flow acceleration occurs at the underlying surface, leading to a considerable pressure drop and to tearing the roof off. It is shown that the negative pressure jump at a site that is adjacent to the roof ridge can be decelerated by several percent with a decreased friction factor and an increased length of the portion of the roof where air flows without separation. A method for considerably reducing the pressure drop is proposed for the first time, viz., the installation of expanding air ducts on the roof, within which a compensatory pressure jump occurs. The results are verified by laboratory studies.

Keywords: boundary layer, wind load, destruction of a roof by wind.

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INTRODUCTION

The general concept of designing wind-resistant roofs implies that the projected maximum wind load (the resistance to the roof being torn off) should be higher than or equal to the wind load that acts on the roof. However, it is also necessary to introduce a safety margin for a roof, which takes unavoidable variations in material properties and the aging of material into account. The commonly accepted engineering approach requires the projected wind load to exceed the calculated maximum load by at least two times; only in this case can the roof be considered to be sufficiently wind resistant. Considerable exceedence of the calculated maximum load leads to an unwanted increase in cost.

The wind load on the slopes of a roof is caused by exceedence of the wind pressure on the windward slope and a respective decrease in pressure at the roof surface on the leeward slope when wind blows into slope. When wind blows into the fronton of a roof, pressure drops over the entire roof surface. Tearing-off and breakage of roofs are observed in the zones of the maximum pressure drop: behind the roof ridge on the leeward slope (when wind blows into the slope) and on the frontal edge (when wind blows into the fronton of the roof). The most dangerous area is a small fragment of the leeward slope that is adjacent to the ridge when wind blows into the slope. The calculated value of the average wind-load component p, depending on the height above the ground, is defined by the empirical formula

$$p = Pck, \quad (1)$$

where P is the value of the wind load, which is defined from the map in the appendix to “Changes in SNiP 2.01.07-85, as of 1993”; it takes the influence of the geographic position of an object into account (the maximum possible values for Russia occurs in Anadyr and Vladivostok: P_{max} = 140 kgf/m^2, 1 kgs/m^2 = 9.8 Pa); k is a factor that takes the table-defined change in wind pressure depending on height into account; c is an aerodynamic factor that takes the roof zone and the sign of the pressure change into account (positive in the main part of the windward slope and negative on the leeward slope, where a considerable pressure drop is observed when the wind blows into the slope). The maximum value of the wind load on a leeward slope is, according to (1), $P_{max} = -170$ kgf/m^2. A roof can be torn off at a wind speed $u > 32.5$ m/s (11–12 on Beaufort scale, or F1 on the Fujita–Pearson scale); this speed agrees with the value of the maximum wind load that was mentioned above. However, expression (1) does not take the wind speed and design features of the roof into consideration. Another method [1, 2] estimates the wind speed on a construction using an empirical formula that contains the parameter of the wind speed, u:

$$P = \beta\rho u^2, \quad (2)$$

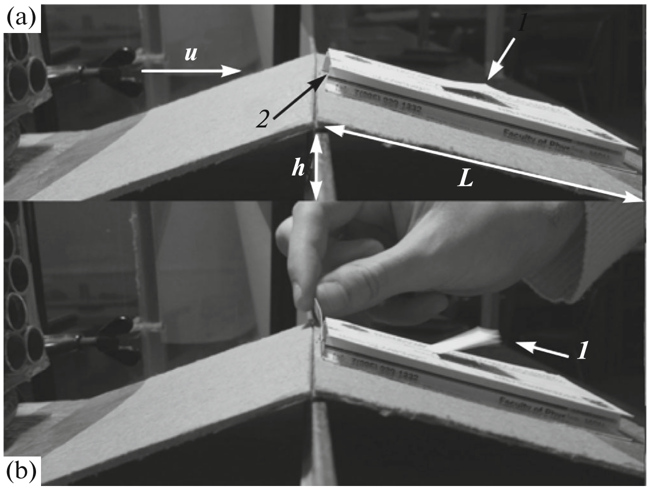

Fig. 1. A roof with a horizontal air flow that blows into the slope: (*1*) the cover at the hole in the upper surface of the air duct; (*2*) the mouth of the air duct near the roof ridge. Panels (a) and (b) illustrate the cases where the mouth of the air duct is opened and closed, respectively.

where ρ is the air density and β is an empirical factor, whose value is 0.5 in Russia and 0.75 in some other countries [2]. The maximum value of the wind load $p_{max} = -170$ kgf/m^2 as calculated from (1) for a leeward slope corresponds to the wind speed of $u = 45$ m/s as calculated by (2). The misfit or inaccuracy of calculations using (1) and (2) can be reduced if the calculations are replaced by empirical formulas with direct measurements of the pressure drop on the leeward slope using a roof model in a wind tunnel; however, this is expensive and is done very rarely. In addition, the problem emerges of how to recalculate the obtained data for real buildings. The problem solution requires that hydrodynamic computation methods should be created that calculate the wind load on the leeward slope at a set wind speed, with the material and roof-design parameters being taken into account. This is possible if the model involves the physical paths by which air passes over the roof. The creation of such a model and the development of a method to reduce the wind load on the leeward slope of the roof are the goals of the present article.

1. CALCULATION OF THE PRESSURE FIELD ON THE LEEWARD SLOPE OF A ROOF

Let a roof with height, h, slope length, L, and a small slope angle, $\alpha < 15\%$ relative to the horizon, be streamlined by a horizontal air flow, u, directed at the roof slope (Fig. 1). Here, the left slope is windward, while the right one is leeward. Let the x axis be directed along the leeward slope of the roof and the y axis be directed along the normal to this slope, while the origin of the coordinates is located at the roof top (ridge). In the viscous layer of a decelerating liquid flow, the wind speed can be written as follows: $u(y) = u_s + \chi y$, where $\chi = \text{const}$ and $u_s = u(0)$ [3, 4]. If we suppose that the streamlines that are near the roof surface satisfy the condition $u_s = 0$, then the pressure that is imposed on the roof from above and below will be equal to the atmospheric pressure. However, practice shows that a considerable decrease in pressure occurs on the top surface of the leeward slope of the roof; this can lead to tearing the roof off during wind gusts. When the roof is streamlined on the windward slope, the air flow narrows and the wind speed increases along the slope; in contrast, on the leeward slope the air flow expands and the wind speed decreases along the slope. It was shown experimentally in [3, 4] that in a stationary air flow that decelerates along its flow direction periodic stoppage of the entire viscous layer with thickness δ occurs. This viscous layer decelerates owing to the effect of frictional force on the underlying surface and that of an inverse pressure gradient on the upper boundary of the layer. During deceleration, a chain of equidistant cylindrical vortices periodically forms in the viscous layer; these vortices rotate as a solid body with the horizontal axis being directed perpendicular to the flow axis [4, Fig. 1]. Vortices roll up at the upper boundary of the viscous layer and remain in the zone where they formed, because the rate of the chain vortices is close, in terms of its speed modulus, to that of the background air flow, but is directed upstream of the flow. When the speed near the underlying surface attains its minimum and the thickness of the viscous layer becomes maximal, vortices depart from the layer and ascend, leading to an abrupt increase in the flow speed in the entire viscous layer [3, Fig. 4]. As a result of this, the value of the flow speed, which is averaged over time, at the near-surface streamline is $u_s \neq 0$. In the locations that vortices departed from the layer, new vortices form; these newly formed ones nearly do not shift from the phase of the viscous layer deceleration. The formation and departure of vortices in the viscous layer of a decelerating air flow above a sand surface is shown in Fig. 2. The acceleration phase of the viscous layer as the vortices leave is approximately two-tenths of the period, T, of departure of the vortices: $\Delta t_{ac} = 0.2T$. This period can be calculated from the following formula [2, 3]:

$$T = \left(\frac{2\delta}{5u_s|u_x|C_f}\right)^2 \arctan\left(\sqrt{\frac{2u_sC_f}{5|u_x|\delta}}\right), \quad u_x = \frac{\partial u_{max}}{\partial x}, \tag{3}$$

where u_{max} is the speed of the flow at the outer limit of the boundary layer; C_f is the factor of the sliding friction on the underlying surface (dimensionless value). According to experiments [3, 4], if the speed of the flow beyond the boundary layer is $u_{max} > 3$ m/s, $u_s =$

$u_{max}/10$ and the thickness of the viscous layer δ exceeds the dimension of the underlying surface roughness by an order of magnitude.

To calculate u_s in (3), the speed drop along the leeward slope of a roof should be estimated. Let a homogeneous horizontal air flow blow with speed u into the leeward slope of the roof. At the roof ridge, the flow speed increases to u_{max} at the outer limit of the boundary layer. Above the roof top, the flow speed decreases in the vertical direction from u_{max} to the speed, u, of the initial air flow that blows into the windward slope; u is attained at some height h_1, whose value depends on the angle α and is found experimentally. For $\alpha = 15°$, $h_1 \approx 3h$. At small values of α, we can assume that an unseparated air flow occurs on the leeward slope of the roof. We can then write the equality of speed flows in the cross section of the expanding layer of the air flow with thickness h_1 above the roof top and $(h + h_1)$ at the terminus of the leeward slope of the roof (the thickness of the viscous layer is neglected):

$$u(h + h_1) = \frac{u + u_{max}}{2} h, \quad h_1 = 3h,$$
$$u_{max} = \frac{5u}{3}, \quad \frac{\partial u_{max}}{\partial x} = -\frac{2u}{3h}\sin\alpha. \tag{4}$$

Relationship (4) was tested in laboratory experiments using the instruments that were described in [4]. For a roof with $L = 12$ cm and $\alpha = 15°$, and at $u = 700$ cm/s, the obtained $u_s = -38$ s^{-1}, which was close to the value that was calculated from (4): -38.9 s^{-1}.

To obtain the estimated ratios of the pressure jump, let us assume that the change in the wind speed at the interface in the phase of flow acceleration is a linear function of time and the speed increases from 0 to u_s per time $\Delta t_{ac} = 0.2T$; the equation for $u_{max} > 3$ m/s can then be written as follows:

$$\dot{u}(0) = \frac{u_{max}}{2T}. \tag{5}$$

We assume that in the phase of viscous layer acceleration after the departure of the vortex the speed of the flow in the layer does not change in the vertical direction. The pressure change relative to the atmospheric one on the streamline near the underlying surface will be defined by the Cauchy–Lagrange equation [5]:

$$p = -\rho\left[\dot{u}X + \frac{1}{2}(u_s)^2 + gy\cos\alpha\right], \tag{6}$$

where X is the distance for which the pressure jump is calculated, ρ is the air density, and g is the acceleration due to gravity. Equation system (3)–(6) enables one to calculate the pressure jump on the leeward slope of a

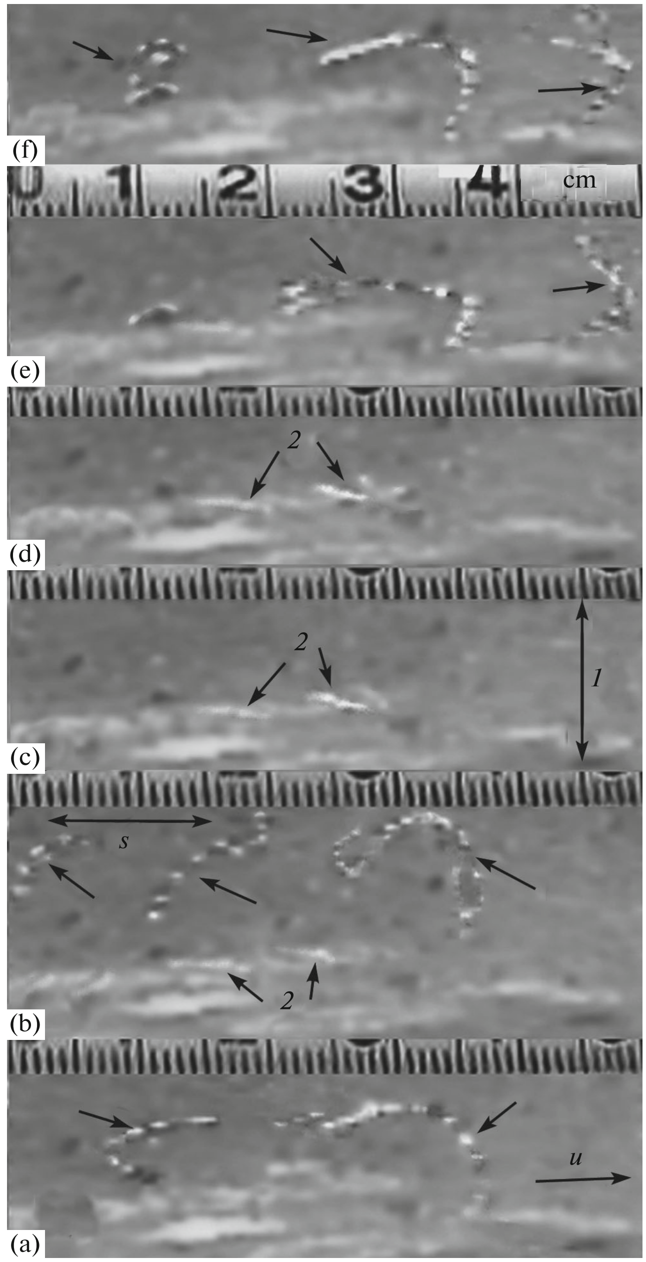

Fig. 2. Sequential shots (a)–(f) of a video record that shows how vortices form above a sand surface in a viscous layer that decelerated in the flow direction, with the camera directed at 45° to the horizontal: (*1*) sand surface; (*2*) cylindrical vortices near the sand surface. Arrows indicate vortices that rose above the sand surface; s is the distance between the vortices.

roof, where a roof is most often torn off during strong wind gusts. The calculations were performed at a wind speed of 33 m/s (at which a roof is typically torn off), $\alpha = 15°$, $\delta = 0.001$ m, and $C_f = 0.05$. The pressure was calculated per square meter (in (6), $X = 1$ m). The values of the second and third summands in (6) appear to be two–three orders of magnitude lower than the

Table 1. The pressure jump at different values of slope length

Slope length, m	4	5	6
The pressure jump relative to atmospheric pressure, kgf/m^2 (1 kgf/m^2 = 9.8 Pa)	−199	−174	−156

Table 2. The pressure jump at different values of the friction factor

Friction factor, C_f	0.05	0.06	0.07
The pressure jump relative to atmospheric pressure, kgf/m2 (1 kgf/m2 = 9.8 Pa)	−174	−188	−200

value of the first summand and can be neglected. For this case, a simple calculation formula can be used:

$$P \approx -\rho\frac{u_{\max}^2}{2}\frac{X}{2F}, \quad F = \sqrt{q\frac{\delta}{C_f}}\arctan\left(\frac{1}{5}\sqrt{q\frac{C_f}{\delta}}\right), \quad q = \frac{3L}{2}. \tag{7}$$

The values of the pressure drop on the leeward slope, as calculated using (7) for slope lengths of 4, 5, and 6 m, are given in Table 1.

The obtained estimate is close to the maximum values that were obtained by empirical methods (for roofs with an average slope length of 5 m): −170 kgf/m^2; this verifies the validity of solution by (7).

2. THE INFLUENCE OF ROOF-DESIGN PARAMETERS ON THE PRESSURE JUMP

In contrast to empirical methods, solution by (7) takes the design features of a roof into account; this is essential because such parameters as slope angle, slope length, and the roughness of the surface considerably affect the results for the same wind speed. The decrease in pressure at $L = 5$ m and different values of C_f (0.05, 0.06, and 0.07, with the remaining parameters remaining unchanged) is seen in Table 2.

The obtained data show that a decrease in the roughness of the surface leads to a smaller (by a few percent) pressure drop at the upper surface of the leeward slope of the roof. Nevertheless, at the wind speed $u > 33$ m/s, such a decrease in the pressure drop cannot prevent the roof from being torn off.

For this reason, the pressure drop between the upper and lower surfaces of the roof should be reduced. Such a reduction can be attained if the pressure on the lower surface of the roof is decreased. This pressure jump will occur if the air flow can be directed along the lower surface and decelerate in the flow direction. A negative pressure jump will then occur on the lower surface of the roof and compensate for the jump on the upper surface. It should be noted that this flow can be made only in the short fragment near the roof top (where a roof is usually torn off): the length of this fragment is no more than one-fifth of the slope length. The easiest way to solve this problem is to design the roof with expanding air ducts that open on both sides. The short side of the air duct should be placed along the roof ridge so that air flows into the duct through it (Fig. 1), thus forming a decelerating flow within the expanding air duct. At the Subdepartment of Marine and Inland Waters Physics at the Department of Physics of Moscow State University, a series of experiments was carried out to test this design. An expanding air duct that caught the air flow at the roof ridge was installed on a roof with $L = 12$ m. On the upper surface of the air duct a rectangular hole was made and closed with a light cover. The cover was placed on the hole of the air duct in such a manner that no roughness was observed on the surface of the air duct. The upper edge of the cover was attached to the surface of the air duct and the remaining edges were free, in such a manner that the cover could turn up. The angle of the cover was defined by the pressure difference between the upper and lower surfaces. Horizontal air flow was directed into slope of the roof. In Fig. 1a, the mouth of the air duct near the roof ridge is opened and the air freely flows through it. It is seen that cover on the air duct surface is turned at a small angle; this is possible only if the pressures at the upper and lower surfaces are nearly equal. This condition is satisfied, because there are decelerating air flows both above and below the cover and the negative pressure jumps nearly equalize each other. In Fig. 1b, the mouth of the air duct is closed and air does not flow into it. In this case, the cover sharply turns upward and is not torn off only because of its attachment to the surface of the air duct. The experiment shows that placement of an expanding air duct on the roof considerably decreases the pressure difference between the upper and lower surfaces of the roof on its leeward slope. These air ducts can be placed on both slopes of the roof, preventing it from being torn off via strong winds from both directions. On the leeward slope, the air flow in the duct will be decelerating (a negative pressure jump at the lower surface of the roof), whereas it will accelerate on the windward slope (no pressure jump).

CONCLUSIONS

A hydrodynamic model for calculating the pressure jump on the leeward slope of a roof for a horizontal air flow that was directed to the slope was proposed for the first time. On the windward slope, in an accelerating air flow at the upper and lower roof surfaces, the flow speed on the near-surface streamline is zero. On the leeward slope, in a decelerating air flow within the viscous air layer, vortices are formed and periodically move upwards, departing from the viscous layer and leading to an abrupt acceleration of the air flow and therefore a decrease in pressure on the upper surface of the roof. It is shown that the negative pressure jump can be reduced by few percent with a decrease in the friction factor and an increase in the length of the segment with an unseparated air flow along the roof. We proposed a method to considerably reduce the pressure jump via the placement of an expanding air duct that catches the air flow near the roof ridge and tested it under laboratory conditions.

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