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THE PHYSICS OF THE ATOMIC NUCLEUS AND ELEMENTARY PARTICLES

Analogues of the Exotic Hoyle State in the ^{12}C Nucleus

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Abstract—Experimental data on inelastic scattering of alpha particles by the ^{11}B, ^{12}C, and ^{13}C nuclei are analyzed using the modified diffraction model and the radii of these nuclei in some “abnormal” excited states are found. It is shown that the 0_2^+ (7.65 MeV) Hoyle state in the ^{12}C nucleus is the base state for a new 0_2^+–2_2^+–4_2^+ rotational band (in addition to the ground-state band), in which the third member is the discovered 4_2^+ (13.75 MeV) state. The radii of the ^{12}C nucleus in the above-mentioned three states are 25–30% larger than its ground state radius. It is found that the radii of the $1/2^-$ (8.86 MeV) state in the ^{13}C nucleus and the $3/2^-$ (8.56 MeV) state in the ^{11}B nucleus are close to the radius of the Hoyle state in ^{12}C and that a similar rotational band is based on the 8.56 MeV state. The above ^{13}C and ^{11}B states can be regarded as analogues of the Hoyle state. The prediction of the alpha-condensation model that a similar analogue in ^{11}B is the 12.56 MeV state with a radius that is comparable with the nuclear radius of uranium was not confirmed.

Keywords: inelastic scattering, light nuclei, diffraction model, nuclear radius, excited states.

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INTRODUCTION

Modern cluster models have greatly advanced as compared to the primitive alpha-cluster models of the middle of the 20th century and they increasingly rely on first principles. Many new effects have been predicted, which in turn has stimulated the development of experimental-research methods and new types of experiments. One of the most significant theoretical predictions was the hypothesis of excited nuclear cluster states with abnormally large radii. The alpha-particle condensation model is especially popular [1], according to which there could be nuclear states that resemble a Bose condensate in macroscopic objects. Nuclear states of increased size were predicted by other models as well, e.g., antisymmetrized molecular dynamics (AMD) [2] and fermionic molecular dynamics (FMD) [3]. These all are rather rigidly connected to their predicted values of the radii and the experimental determination of these values can be a critical verification of the theories.

Experimental verification of these nuclear models required the development of methods for measuring the radii of short-lived excited nuclear states. For this purpose, we proposed [4] modifying the diffraction model of inelastic scattering. Its applicability has been tested [5] through comparison with two other methods that are based on rainbow scattering [6, 7] and transfer reactions [5, 8].

Theoretical and experimental studies of exotic cluster states concentrated on the famous state of the ^{12}C nucleus with the spin–parity $J^\pi = 0^+$ and excitation energy of 7.65 MeV (Hoyle state). This state plays a crucial part in nucleosynthesis, dictating the elemental composition of the Universe.

In [4, 7] we showed that the Hoyle state had a larger size, although not as large as that predicted by the alpha-condensation model. However, many structural features of this state have been a mystery thus far. One of the unanswered questions is whether the Hoyle state has analogues in the ^{12}C nucleus itself and in its neighbors, ^{11}B and ^{13}C. The particular interest in the analogues arose from the fact that some of them were predicted to have especially large sizes, even in comparison to the Hoyle state.

In this work we analyzed the results of measuring differential cross sections for the $\alpha + {}^{12}$C, ^{13}C, ^{11}B scattering at the alpha-particle energy of 65 MeV [9–11] and 110 MeV [12] (in the case of ^{12}C). The measurements were performed at the cyclotron of the University of Jyväskylä, Finland. Sets of ΔE–E telescopes were used as detectors. The total energy resolution was 200 keV with the beam monochromatization system. The radii of various nuclear states were determined from the experimental data using the modified diffraction model (MDM) [4] and, in some cases, the nuclear rainbow model (NRM) [7].

1. THE HOYLE STATE IN ^{12}C

Many theoretical works on the structure of the Hoyle state have been published in the past few years. A summary (probably incomplete) of calculated rms radii is presented in Table 1, together with the value that was obtained from the MDM analysis of the experimental data at eight energies [4] (column 10). The spread of the predictions is rather large, but almost all of the theoretical models expect a considerable increase in the size of the Hoyle state as compared to the ground state (2.34 fm). The best agreement with experiments is obtained with one of the AMD versions [18]. The largest radii are predicted by the alpha-condensation model (columns 1 and 3).

The strong discrepancy between the predictions of the alpha-condensation model and experiment does not automatically disprove the model. The point is that, unlike other theories, the alpha-condensation model predicts another important parameter of the Hoyle state, the probability $W_s(\alpha)$ for all three alpha particles to be in the zero-moment state. Theory predicts $W_s(\alpha) = 0.7–0.8$ [20], and experiments yield a rather similar value, $W_s(\alpha) = 0.6$ [21]. Since the radius and $W_s(\alpha)$ are related to each other [20], the result can be interpreted, with the details omitted, as a manifestation of the rudimentary alpha condensate (ghost condensate). The more important issue is to investigate other states that are genetically related to the Hoyle state.

2. ROTATIONAL BANDS IN ^{12}C

The ground-state rotational band 0^+(0.00 MeV)–2^+(4.44 MeV)–4^+(14.08 MeV) has long been known in ^{12}C. The existence of the rotational band based on the Hoyle state logically follows from the first model of this state [22], which treats it as a chain of three alpha particles. Recent experiments [23, 24] identified a 2_2^+ level in ^{12}C at the excitation energy of 9.6–9.8 MeV, which can be the second member of the rotational band under discussion. However, another model [15] treats the Hoyle state as vibrational, while in the condensation model it is almost spherical. In the latter case, the 2_2^+ level is produced by the transition of one alpha particle from the *s* orbit to the next *d* orbit, which gives the nucleus an anomalously large radius of ~6 fm [14, 20].

Recently we determined the radius of the 2_2^+ state, which is ~3.1 fm [25], i.e., it is almost as large as that of the Hoyle state. The rms radius that was obtained under the assumption of a rigid rotator is very close to that value (2.7 fm). Both facts favor the assumption that the 0_2^+ and 2_2^+ states are really members of the second rotational band in ^{12}C. However, the ultimate conclusion about the existence of this band can be made only after identification of the corresponding 4_2^+ state. Some indications of its existence were obtained in [26], which reported observation of a 4^+ state with a large width $\Gamma = 1.7$ MeV at the excitation energy $E^* = 13.3$ MeV.

To identify the 4_2^+ state, we used not only the data [9] at 65 MeV but also the earlier data on the inelastic scattering of alpha particles at 110 MeV [27]. Typical spectra at both energies are shown in Figs. 1a and b, respectively. Decomposing the spectra into components, we took all known ^{12}C states in the excitation energy range of 11 to 15.5 MeV into account with the widths given in [12]. Inclusion of a new state with $E^* = (13.75 \pm 0.12)$ MeV and $\Gamma = (1.4 \pm 0.15)$ MeV considerably decreased χ^2. The best description was achieved under an additional assumption that the differential cross section of the new level at 13.35 MeV with $J^\pi = 2^-$ (according to [27]) coincided with the cross section for the formation of the neighboring 2^- (11.83 MeV) state. However, recent data indicate that the 13.35 MeV state actually has $J^\pi = 4^-$ (see, for example, [28] and references therein). Therefore, the above-mentioned decomposition of the spectra should be regarded only as one of the possible variants.

Figures 2a, 2b show the angular distributions that were obtained at the energy of 110 MeV and corresponding to the excitation of the 4^+ (14.08 MeV) state and the new 4^+ (13.75 MeV) state. Standard calculations using the distorted wave Born approximation (DWBA) are also presented (solid curves). These cal-

Table 1. The root-mean-square radius of the ^{12}C nucleus in the Hoyle state

	1	2	3	4	5	6	7	8	9	10
Ref.	[13]	[2]	[14]	[15]	[3]	[16]	[17]	[18]	[19]	Exp. [4]
R_{rms}, fm	3.83	3.27	4.31	3.47	3.38	3.22	3.53	2.90	2.4	2.89 ± 0.04

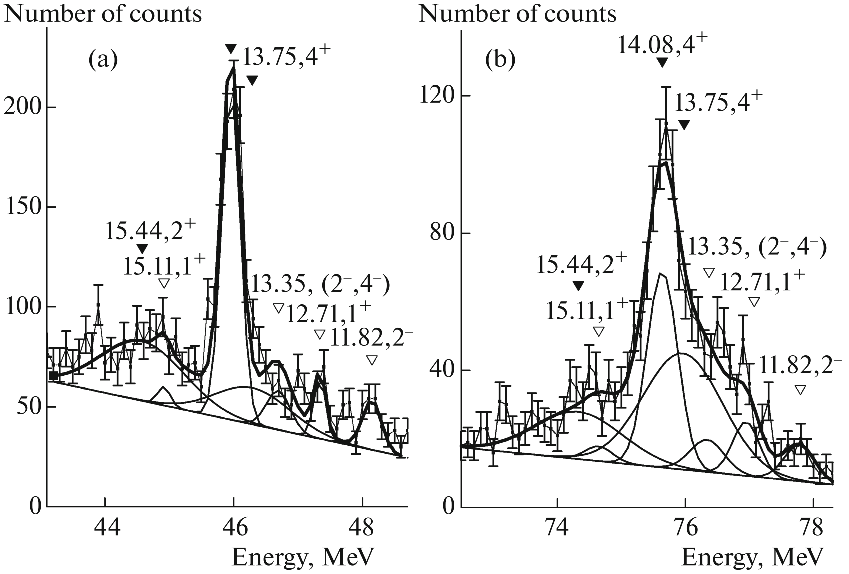

Fig. 1. Examples of the alpha spectra at $E_{\text{lab}} = 65$ MeV, $\theta_{\text{lab}} = 30.8°$ (a) and $E_{\text{lab}} = 110$ MeV, $\theta_{\text{lab}} = 43.6°$ (b). The dashed line is the background. The decomposition of the spectrum into Gaussian functions for the known ^{12}C levels and the state at the excitation energy of 13.75 MeV is shown.

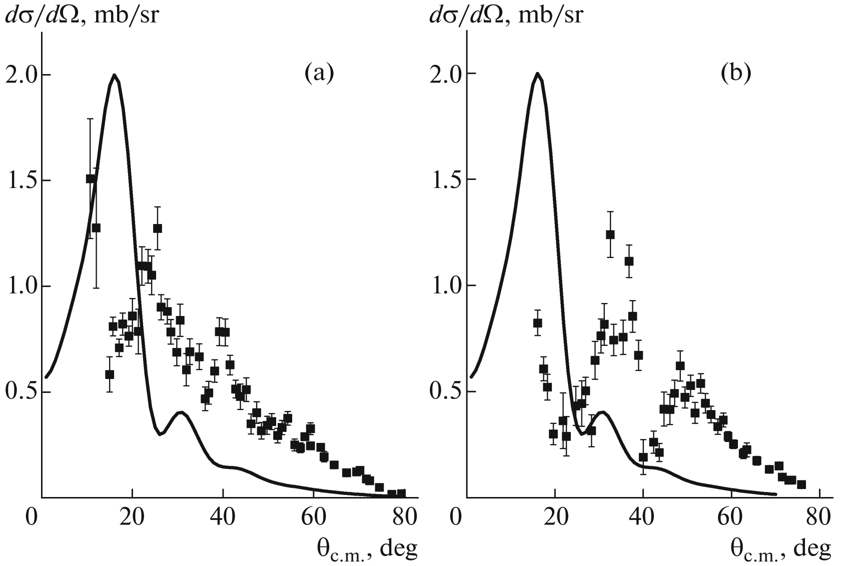

Fig. 2. Differential cross sections for the inelastic $\alpha + {}^{12}$C scattering with the excitation of the 4^+ (14.08 MeV) state (a) and the 4^+ (13.75 MeV) state (b) at $E_{\text{lab}} = 110$ MeV. The solid curves correspond to the DWBA calculations with $L = 4$.

culations were performed with the energy-dependent optical potential parameters in the entrance and exit channels [21] and the collective-model inelastic form factors in the form of the derivative of the entrance optical potential. No good agreement was obtained with this form factor.

For further analysis of the production cross section for the 14.08 MeV level we used the data from the measurements [29] of the cross section for production of the 4^+ (10.36 MeV) state in the inelastic $\alpha + {}^{16}$O scattering at almost the same center-of-mass energy of 83.2 MeV (E_{lab} = 104 MeV) as in [12] for the $\alpha + {}^{12}$C scattering, 82.5 MeV (E_{lab} = 110 MeV). This level is also the third member of the rotational band in the ^{16}O nucleus. Both angular distributions almost coincide in shape within the overlapping region of angles (Fig. 3). The DWBA calculations that were performed in [29]

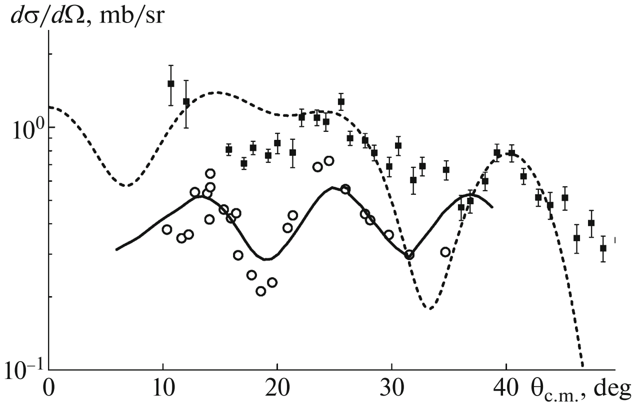

Fig. 3. Differential cross sections for the inelastic $\alpha + {}^{16}\text{O}$ scattering with the excitation of the 4^+ (10.36 MeV) state at $E_{\text{lab}} = 104$ MeV (white dots) [29]; the solid curve is the calculation using the coupled-channels method [29]. The black squares are the experimental cross section for the inelastic $\alpha + {}^{12}\text{C}$ scattering with the excitation of the 4^+ (14.08 MeV) state at $E_{\text{lab}} = 110$ MeV. The dashed curve is the calculation using the diffraction model with the momentum transfer $L = 4$ and radius $R_{dif} = 4.2$ fm.

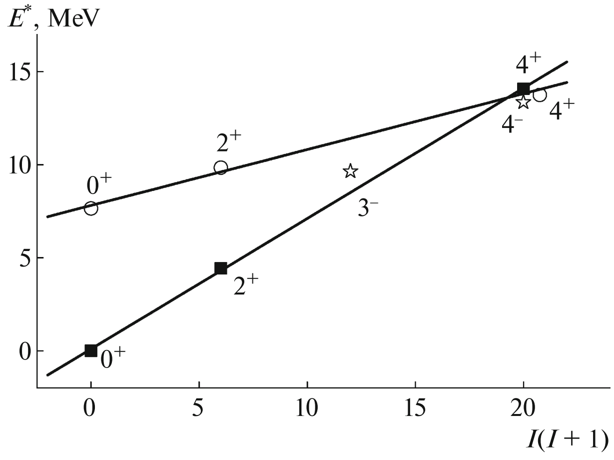

Fig. 4. States that are associated with the rotational bands based on the ground state (black squares) and the Hoyle state (white dots) in ${}^{12}\text{C}$. For clarity, the point for the 4^+ (13.75 MeV) is moved to the right. The solid lines correspond to the linear approximation of the points. Stars denote the 3^- (9.64 MeV) state and the 13.35 MeV state (assumed I^π is 4^-).

also failed to describe the production cross section for the 4^+ state. However, the calculations that use the coupled-channels method that were performed in [29] and shown in Fig. 3 describe the experiment well.

This justifies the interpretation of the minima and maxima as diffraction ones. In fact, the MDM rather adequately reproduces their positions (Fig. 3). However, this agreement is achieved with the diffraction radius $R_{dif} \approx 4.2$ fm, which is approximately 1 fm smaller than the diffraction radius of the ground state that was found from the differential cross section for the elastic scattering.

The main assumption that underlies the MDM is that the rms radius of an excited nucleus is

$$\langle R^* \rangle = \langle R^0 \rangle + \Delta, \tag{1}$$

where

$$\Delta = [R^*_{dif} - R^0_{dif}], \tag{2}$$

where $\langle R^0 \rangle$ is the rms radius of the ground state (which is usually known), and R^*_{dif} and R^0_{dif} are the diffraction radii of the excited and ground states that are found from the positions of the minima and maxima in the angular distributions of the inelastic and elastic scattering, respectively.

Obviously, the negative sign of the difference $\Delta = (R^*_{dif} - R^0_{dif})$ cannot mean this strong decrease in the true radius of the excited state, relative to the ground state at least, because the 4^+ level in ${}^{12}\text{C}$ is a member of the ground state rotational band. A change in the diffraction radius can result from a transfer of a large angular momentum in the inelastic scattering reaction when the scattered particle energy considerably changes (this effect could not be observed in previous applications of the MDM and will be discussed in a separate publication). However, since the difference Δ turned out to be negative, we cannot determine the rms radius for the 14.08 MeV state by directly applying the MDM, i.e., by adding Δ to the ground-state radius.

In Fig. 2b differential cross sections are shown for the excitation of a new state with $E^* = 13.75$ MeV. The general character of the angular distribution is close to the one that was obtained at the excitation of the 14.08 MeV state (Figs. 2a, 3). This indicates that the momentum $L = 4$ is transferred; thus, the spin parity of this state is $I^\pi = 4^+$. Note that if the corresponding group in the spectrum were associated with the 4^- state, this similarity would hardly be probable, because a state with an anomalous parity cannot be excited by a one-step process.

The diffraction radius of the 13.75 MeV state can be estimated with respect to the diffraction radius of the 14.08 MeV state. This is ≈5.0 fm, which is 0.8 fm larger than for the 14.08 MeV state. This value agrees well with the differences of the radii for the excited 0^+_2 and 2^+_2 states (0.6 fm and 0.8 fm, respectively [25]) and the ${}^{12}\text{C}$ ground state.

Thus, there are grounds to believe that the 13.75 MeV state is the third member of the rotational band built on the Hoyle state (Fig. 4). However, in view of the reservations in the above description of the

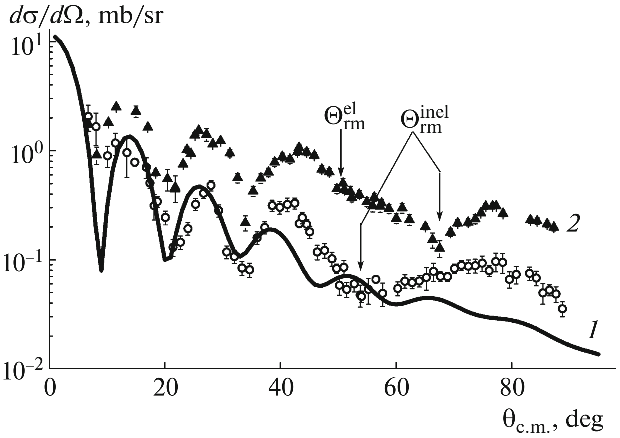

Fig. 5. Differential cross sections for the inelastic α + ^{13}C scattering at E_{lab} = 65 MeV leading to the 8.86 MeV state in ^{13}C (white dots, *1*). The solid curve corresponds to the DWBA calculations with $L = 0$. The differential cross section for the inelastic scattering with the excitation of the Hoyle state in ^{12}C is shown for comparison (black triangles, *2*). The arrows indicate the positions of rainbow minima.

procedure for the separation of the 13.75 MeV state, these results require additional verification. This applies to the excitation energy of the 4_2^+ state (13.75 or 13.3 MeV) and to the spin parity of the 13.35 MeV state (2^- or 4^-).

There are several other open questions about excited states of ^{12}C. In [30] it was reported that the known 0_3^+ state with $E^* = 10.3$ MeV and width Γ = 3 MeV was actually a combination of two 0_3^+ states with $E^* = 9.4$ MeV and $E^* = 10.8$ MeV. We believe it is premature to discuss the effect of this result on the current concept of the structure of the ^{12}C nucleus.

Recently, observation of the 5^- (22.4 MeV) state has been reported [28], adding intricacy to the issues that are related to the structure of excited rotational states. It has been hypothesized [28] that the new state together with the 3^- (9.64 MeV) and 4^- (13.35 MeV) levels that are shown in Fig. 4 make up a negative-parity branch in a unified rotational band based on the ground state. However, the question arises of how to reconcile the fact that the members of this unified band have equal moments of inertia with the fact that the radius of the 3^- state was found to be 2.88 ± 0.11 fm [4], i.e., appreciably larger than the radii of positive-parity states. Note that allowance for the above-mentioned possible effect of the centrifugal barrier would result in an even larger radius. An increased radius in the 3^- state as compared to the ground state has been predicted by a number of cluster models as well (see Table 4 in [4]).

Thus, many uncertainties remain that should be investigated further in the structure of the highly excited states of ^{12}C, which is a key nucleus in dealing with the issue of nucleon clustering in light nuclei.

3. AN ANALOGUE OF THE HOYLE STATE IN ^{13}C

For many years the ^{13}C and ^{11}B nuclei have been considered as good examples of the manifestations of shell effects in light nuclei. Various versions of the shell model reproduced the entire spectrum of levels up to the excitation energies of 8–10 MeV. New cluster models predict states with a much more complicated structure in these nuclei, in particular those that are obtained from the Hoyle state by removing a proton (^{11}B) and adding a neutron (^{13}C). The question is to what extent these states retain the initial cluster configuration, that is, if they can be considered as analogues of the Hoyle state.

In [31] it was assumed that in the $1/2^-$ (8.86 MeV) and $1/2^+$ (11.00 MeV) states of ^{13}C the valence neutron occupies the $1p_{1/2}$ and the $2s_{1/2}$ orbit, respectively, and the core is the Hoyle state.

The measured differential cross sections at an initial energy of 65 MeV [10] that lead to the excitation of the $1/2^-$ (8.86 MeV) state are presented in Fig. 5 together with the DWBA calculations that were performed with the energy-dependent parameters of the optical potentials [21] in the entrance and exit channels that were determined from the elastic scattering data and with the inelastic form factors of the collective model in the form of the derivative of the Woods–Saxon potential with free parameters. These cross sections are compared to the cross sections at the same energy that lead to the excitation of the Hoyle state in ^{12}C. In both cases, similar behavior is observed for the diffraction part of the cross sections that correspond to the transfer momentum $L = 0$. The rms radius in the 8.86 MeV state obtained within the MDM turned out to be 2.68 ± 0.10 fm (diffraction radius R_{dif} = 5.66 ± 0.10 fm), which is slightly smaller than for the Hoyle state (2.89 ± 0.04 fm). All this allows the states to be regarded as analogues.

Rainbow minima (Airy minima) were identified in both angular distributions; their positions are indicated by arrows. For the 8.86 MeV state the rainbow minimum is at an angle that is slightly larger than in

Table 2. The root-mean-square radius of the ^{11}B nucleus in the $3/2_3^-$ (8.56 MeV) state

	MDM, this work	MDM [38]	AMD [35]	OCM [36]	MDM,Hoyle state of ^{12}C [4]
R_{rms}, fm	2.87 ± 0.13	2.99 ± 0.18	3.1	3.0	2.89 ± 0.04

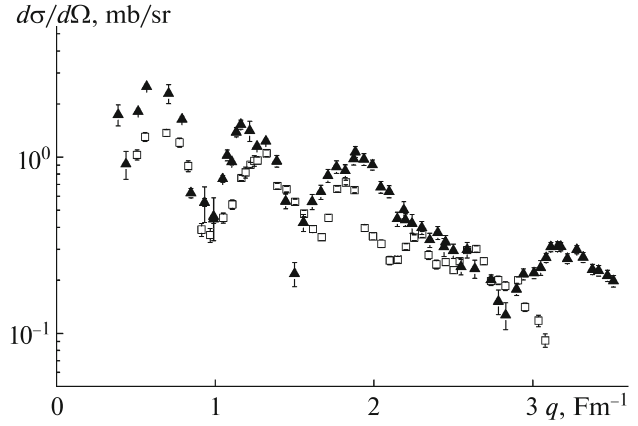

Fig. 6. Differential cross sections for the inelastic $\alpha + {}^{11}B$ scattering at E_{lab} = 65 MeV with the excitation of the 8.56 MeV state (white squares) and for the inelastic $\alpha + {}^{12}C$ scattering at E_{lab} = 65 MeV with the excitation of the Hoyle state (black triangles).

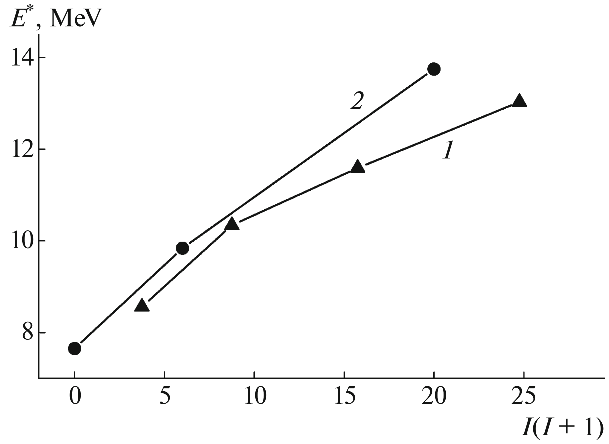

Fig. 7. Rotational band $K = 3/2^-$ in the ^{11}B nucleus built on the 8.56 MeV state (black triangles, curve *1*). For comparison, the rotational band [9] based on the Hoyle state in ^{12}C is also shown (black dots, curve *2*).

the case of elastic scattering and slightly smaller than for the Hoyle state, which qualitatively agrees with results of the MDM analysis.

The 8.86 MeV state has another noteworthy feature. It is almost never excited in the alpha transfer reactions $^9Be(^6Li, d)^{13}C$ and $^9Be(^7Li, t)^{13}C$ [32, 33]. This means that its structure does not comply with the $^9Be + \alpha$ configuration. This difference is probably due to the position of the valence neutron. It is believed (see, for example, [34]) that in the ^9Be nucleus the neutron is part of the ^5He cluster. In the 8.86 MeV state as an analogue of the Hoyle state the neutron orbit can "cover" the entire core.

4. AN ANALOGUE OF THE HOYLE STATE IN ^{11}B

It was initially assumed [2] that the analogue of the Hoyle state in ^{11}B was the $3/2^-$ state with E^* = 8.56 MeV, which was not described in any of the shell-model versions. Various theoretical approaches (AMD [2, 35], orthogonal condition method (OCM) [3]) treated it as a cluster state with the $2\alpha + t$ structure and predicted its radius to be larger than that of the ground state. Later, the idea was proposed [36] that the true analogue of the Hoyle state was the state with E^* = 12.56 MeV. We investigated excited states of ^{11}B in [37, 38].

Figure 6 shows differential cross sections (as a function of the momentum transfer) for the inelastic $\alpha + {}^{11}B$ scattering at the energy E_{lab} = 65 MeV with the excitation of the 8.56 MeV state in comparison with the similar cross sections for the inelastic $\alpha + {}^{12}C$ scattering with the excitation of the Hoyle state. These almost coincide.

We analyzed the available experimental data within the MDM and compared them with the theoretical predictions (see Table 2).

The MDM analysis that was performed in this work yielded an rms radius of the 8.56 MeV state that coincides within the error with the result of the previous MDM analysis [38] of the previously published data [37, 39, 40] and is quite close to the predictions of theoretical models and to the rms radius of the Hoyle state.

Another piece of evidence in favor of the genetic relationship between the 8.56 MeV state and the Hoyle state is the similarity of the rotational bands that are based on them. The AMD calculations [35] predict a rotational band built on the 8.56 MeV state. As assumed in [35, 41], this band is a sequence of states 10.33 ($5/2^-$)–11.60–13.14 ($9/2^-$) MeV (the assumed spin parties are in parentheses).

In our experiment we observed all of the states that belong to this rotational band. This is shown in Fig. 7, together with the rotational band based on the Hoyle state in ^{12}C.

There are several noteworthy features of the excitation-energy dependence on $I(I + 1)$ that are shown in Fig. 7. First, the bands have comparable moments of inertia. Second, anomalously large radii were obtained for all states of the bands. In most cases they are approximately 0.7 to 1.0 fm larger than the rms ground-state radius of ^{11}B.

Thus, both the radii and the moments of inertia of these states turn out to be close to those of the Hoyle state in ^{12}C, which allows the 8.56 MeV state in ^{11}B to be regarded as an analogue of the Hoyle state.

Particular attention should be paid to the ^{11}B states in the excitation-energy range of 12.0 to 12.9 MeV. It was previously believed that the range included only one state at 12.56 MeV with $J^\pi = 1/2^+$ and isospin $T = 3/2$ [27]. In [36] it was proposed that the 12.56 MeV state actually has the isospin $T = 1/2$ and a "giant" radius $R_{\rm rms} \approx 6$ fm that is comparable with the radius of the uranium nucleus (!) and was a true analogue of the Hoyle state.

In an experiment [37] we observed a state with $E^* = 12.6 \pm 0.1$ MeV and a probable spin parity $J^\pi = 3/2^+$. As we investigated inelastic alpha scattering, it is natural to assign $T = 1/2$ to this state. In the ^{7}Li(α, α') reaction [41], which was investigated with high resolution, a state with $E^* = 12.63 \pm 0.04$ MeV was found, and the state with $E^* = 12.56$ MeV was not observed at all. In addition to the obvious value $T = 1/2$, the 12.63 MeV state was assigned $J^\pi = 3/2^+$ or $9/2^+$. It is thus most likely that the same ^{11}B state was observed in [37] and [41].

Our MDM analysis showed that the 12.6 MeV state had a "normal" rms radius $R_{\rm rms} = 2.24 \pm 0.37$ fm. Thus, the predictions [36] of an anomalous radius of the ^{11}B nucleus in the state under consideration have not been confirmed and it is not an analogue of the Hoyle state.

CONCLUSIONS

Differential cross sections for inelastic scattering of 65 MeV alpha particles by the ^{12}C, ^{13}C, and ^{11}B nuclei and 110 MeV alphas by ^{12}C have been analyzed. Using the modified diffraction model, we found the radii of the nuclei in the excited state whose structures have attracted great attention in the past decade. The results indicate that the known 0^+ (7.65 MeV) Hoyle state in ^{12}C, which is larger in size than the ground state, does have analogues in both the ^{12}C nucleus itself and in the neighboring ^{13}C and ^{11}B nuclei. It was shown that the Hoyle state was a base state for a new rotational band (in addition to the known ground state band). A new 4^+ state with the excitation energy of 13.75 MeV (the third member of rotational band) was observed in ^{12}C and it was shown that its radius was similar to the radius of the Hoyle state and the 2_2^+ (9.8 MeV) state, which is the second member of the band.

The $1/2^-$ (8.86 MeV) state in ^{13}C and the $3/2^-$ (8.56 MeV) state in ^{11}B show a strong resemblance to the Hoyle state. The angular distributions with the excitation of these states are very similar and the radii are identical within the measurement error. As well, some data indicate that the 8.56 MeV state in ^{11}B is a base of a rotational band that is similar (in terms of radii and moments of inertia) to the Hoyle state band.

At the same time, a number of open questions remain. Comparison of the results with the predictions of the theoretical models demonstrated good agreement in several cases, but appreciable discrepancies have also been observed. In particular, concerning the alpha-condensate model, which is rather popular now, it can be said that manifestations of this condensate are rudimentary, if any exist at all.

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