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THEORETICAL AND MATHEMATICAL PHYSICS

A Study of the Equation for an Energy Gap on the Basis of the Fluctuation Theory of High-Temperature Superconductivity

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Abstract—The equation for an energy gap is investigated using the fluctuation theory of high-temperature superconductivity. It is shown that the proposed mechanism can explain an increase in critical temperature, T_c. An explicit expression for the critical temperature is obtained using the technique of rectangular pits.

Keywords: high-temperature superconductivity, spin fluctuations, phonons.

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INTRODUCTION

The theoretical description of low-temperature superconductivity was a puzzle for a rather long time, until in 1956–1957 John Bardeen, Leon N. Cooper, and John R. Schrieffer proposed a theory called the BCS theory, which described the experimental data and phenomena that were available at that time well and, most importantly, predicted further development in this field. In 1958, Bogolyubov showed that the BCS wave function, which was initially obtained from the variational argument, can be built by canonic transformation of the electron Hamiltonian [1, 2]. The BCS theory postulates the existence of a layer near the Fermi surface where electrons with opposite spins form bound pairs and electrons beyond this layer are assumed to be noninteracting.

The BCS theory model predicted the experimental discovery of substances with critical temperatures of up to 40 K with sufficiently high accuracy, although most of the substances that were well-studied at that time had a critical temperatures close to zero. Until 1986, the Nb_3Ge alloy had the highest critical temperature (23 K), which was rather an exception to the rule. However, in 1986, Bednortz and Muller discovered the $La_{2-x}Ba_xCuO_4$ superconductor based on copper, lanthanum, and barium oxides with a critical temperature of 35 K; the $La_{2-x}Sr_xCuO_4$ alloy with $T_c = 40$ K was then obtained [3], which gave rise to the high-temperature superconductor concept. Not long after that, alloys with critical temperatures of up to 130 K were synthesized [4], which could not be correctly described using the existing theory [5].

As a rule, high-temperature superconductivity (HTS) occurs in the family of superconducting ceramics, including copper–oxygen and different impurity layers [6]. Along with ceramics, other families of such substances exist. There are data on the discovery of HTS in intermetallides [7], e.g., in the $MgBr_2$ compound with $T_c = 40$ K, and in iron-based materials [8], e.g., the GdOFeAs compound with $T_c = 55$ K. The family of mercury-based compounds is considered to be promising. However, the family of ceramics exhibits the highest values of critical temperature, T_c. It is commonly supposed that the existence of HTS in ceramics is caused by the presence of easily separated copper oxide layers. This hypothesis is confirmed by the analysis of iron-based HTS materials, since the substances in this family also have a layered structure. It is well-known that, as in low-temperature superconductors, conduction electrons in the oxide layers form copper pairs. In the low-temperature case, pairing occurs via the electron–phonon interaction, while in the high-temperature case the cause of electron pairing remains unclear.

The presence of isotopic effect in superconductors gave rise to one of the research directions in this field. As is known, the isotopic effect consists in the relationship $T_c M^a = \text{const}$ between critical temperature, T_c, and substance mass, M, where a is the index of the isotopic effect. This effect is also observed in HTS materials. Analysis of the a index for different alloys showed that this index decreases with increasing critical temperature, T_c. Based on these results, one can conclude that the phonon mechanism of electron pairing, which prevails in low-temperature superconductors, is combined with some other mechanism at high critical temperatures. As such a mechanism, here

we consider spin fluctuations together with the phonon mechanism [9].

In the experiments with $La_{2-x}Sr_xCuO_4$ alloys with the impurity concentration $x \geq 0.02$, the long-range antiferromagnetic order disappears and the subsequent creation of the superconducting state is accompanied by spin fluctuations [10–12]. It would be reasonable to assume the presence of these fluctuations later, in the superconducting state. The experiments with copper-based high-temperature alloys showed that, in contrast to the low-temperature cases, the spin fluctuations have fairly high energy and the spin excitation rate (10^6 cm/s) exceeds the speed of sound in the substance (10^5 cm/s) by an order of magnitude [11, 12], which may indicate the active participation of this mechanism in the interaction between electrons.

The main aim of this study, in the authors' opinion, was to build an expression for T_c in the most general case or, if necessary, with the minimum number of simplifying assumptions. The expression for T_c is obtained sequentially by building a model Hamiltonian based on the Frolich model [13] and then diagonalizing this Hamiltonian using the $u - \upsilon$ Bogolyubov transformation, and finally, solving the integral equation for the energy gap, Δ

$$\Delta(\omega) = \int_0^{\hbar\omega_c} \frac{\Delta(q)}{q} Q(q,\omega) th\left(\frac{q}{2kT}\right) dq. \qquad (1)$$

1. BUILDING A KERNEL OF THE INTEGRAL EQUATION FOR THE GAP, Δ

As mentioned above, electron pairing can be explained by the interaction between lattice oscillations and spin fluctuations of conduction electrons [14–20]. In other words, electrons interact with one another through exchange by quasiparticles, which represent quanta of coupled oscillations of ions in the crystal lattice with spin fluctuations of electrons.

We choose the Hamiltonian that describes the phonon and spin parts and their interaction. Note that we investigate the model case of the isotropic crystal lattice. The operator $\hat{A}_k^\alpha$ has the form $\hat{A}_k^\alpha = \partial\Omega/\partial x_k$ and is the magnetization vector and $\alpha_s = J_0\langle r_c\rangle^2$, where $J_0 = \int dx J(x)$ is the exchange coupling potential and $\langle r_c\rangle^2$is the exchange correlation radius. To describe the spin fluctuations in our system, we introduce small variations in spin variables $\delta\hat{\Omega}$ and $\Delta\hat{A}$. The z axis is chosen to be the quantization axis.

$$\hat{H} = \hat{H}_s + \hat{H}_{ph} + \hat{H}_{s-ph} =$$
$$= \int d\vec{x}\left[\frac{\hat{m}_z^2}{2\chi} + \frac{1}{2}\alpha_s(\Delta\hat{A}_{\nu z}^2) - J_0\delta\hat{\Omega}^2 + \frac{\hat{p}_\nu^2}{2\rho} + \frac{1}{2}\lambda\hat{u}_{i\nu}^2 + \right. \qquad (2)$$
$$\left. + \frac{\hat{p}_\nu}{\rho}\hat{m}_z\delta_{\nu'\nu}(\Delta\hat{A}_{\nu z}) + \frac{\chi}{2\rho^2}p_{\nu'}p_{\nu''}\delta_{\nu'\nu_1}\delta_{\nu''\nu_2}(\Delta\hat{A}_{\nu z}^2)\right],$$

where $\hat{\Omega}^2$ is the electron magnetization operator, χ is the effective paramagnetic susceptibility, $\hat{m}^2/2\chi$ is the operator of the kinetic energy of a spin system, s is the electron spin, $\hat{p}_\nu$ and $\hat{u}_i$ are the phonon operators, and ρ is the density of a substance [14, 15].

The next step is the transition to the operators in the form of secondary quantization, which obey the Bose statistics. The Hamiltonian in the new operators has the form

$$\hat{H} = E_s + E_\nu + E_3 +$$
$$+ \sum_{k,\nu=1,2}\omega_{ck\nu}\hat{b}_{k\nu}^+\hat{b}_{k\nu} + \sum_k\left[\omega_{sk}\hat{a}_{kz}^+\hat{a}_{kz} + \omega_{ck3}\hat{b}_{k3}^+\hat{b}_{k3} - \right. \qquad (3)$$
$$\left. - q_k\left(\hat{b}_{k3}\hat{a}_{-kz} - \hat{b}_{k3}\hat{a}_{kz}^+ - \hat{b}_{-k3}^+\hat{a}_{-kz} + \hat{b}_{-k3}^+\hat{a}_{kz}^+\right)\right].$$

The first terms E_s, E_ν, and E_3 do not contain operators of creation and annihilation and therefore can be interpreted as the energies of zero oscillations in the spin and phonon systems, ω_{sk} is the longitudinal spin wave frequency, and ω_{ck3} is the phonon mode, which is linearly related to the spin mode. It is important that the transverse waves interact nonlinearly with phonons and are not considered below.

Using the standard u–υ Bogolyubov transformation, Hamiltonian (3) is diagonalized:

$$\hat{H} = \sum_{k,\nu=1,2}\omega_{\nu ck}\hat{b}_{k\nu}^+\hat{b}_{k\nu} + \sum_k\varepsilon_{ks}\hat{c}_{kz}^+\hat{c}_{kz} + \sum_k\varepsilon_{k3}\hat{d}_{k3}^+\hat{d}_{k3}. \qquad (4)$$

Since during diagonalization we passed to the new operators of creation and annihilation, now it is appropriate to speak about quasimagnons (operators $\hat{c}_{kz}^+$ and $\hat{c}_{kz}$) and quasiphonons (operators $\hat{d}_{k3}^+$ and $\hat{d}_{k3}$), while ε_{sk} and ε_{3k} are the frequencies of coupled spin-phonon oscillations, where $z = \frac{\xi}{\sqrt{1-\xi^2}}$ is the dimensionless parameter of spin-phonon coupling, and ξ is the parameter of the spin-phonon coupling

$$\varepsilon_{ks}^2 = \frac{1}{2}\left[(\omega_{sk}^2 + \omega_{ck3}^2) + \sqrt{(\omega_{sk}^2 - \omega_{ck3}^2)^2 + 4z^2\omega_{sk}^2\omega_{ck3}^2}\right],$$
$$\varepsilon_{k3}^2 = \frac{1}{2}\left[(\omega_{sk}^2 + \omega_{ck3}^2) - \sqrt{(\omega_{sk}^2 - \omega_{ck3}^2)^2 + 4z^2\omega_{sk}^2\omega_{ck3}^2}\right].$$

The next step is building the kernel $Q(q,\omega)$ of the integral equation for the energy gap, Δ. Separating the

part that is responsible for the interaction with spin fluctuations, we obtain

$$\hat{H}_{e-ph}=\frac{g_{ph}}{\sqrt{V}}\sum_{q_1,q_2}\frac{1}{\sqrt{2\langle\omega_D\rangle}}\Big[\sqrt{\omega_{cq_1\nu}}(n_\nu e_\nu)(\hat{b}_{q_1\nu}+\hat{b}_{-q_1\nu}) + \sqrt{\omega_{cq_13}}(n_3e_3)(\hat{b}_{q_13}+\hat{b}^+_{-q_13})\Big]\hat{c}^+_{q_2}\hat{c}_{q_2-q_1}, \quad (5)$$

where g_{ph} is the electron–phonon coupling constant, ω_{cq_ν} is the frequency of phonons with polarization ν, e_ν is the phonon polarization vector, $\langle\omega_D\rangle$ is the Debye energy or the maximum energy of phonons involved in the interaction, and n_ν are the unit vectors corresponding to wave vector q_ν.

Presenting the kernel of the integral equation for the gap as superposition of three Green's functions and taking all the coefficients in Hamiltonian (5) into account, we arrive at

$$Q(q,\omega)=N(0)\frac{g^2_{ph}}{9\langle\omega_D\rangle}\left\{2\frac{\omega_{cq_1\nu}}{\omega^2+\omega^2_{cq_1\nu}} + \omega_{cq_13}\left(|u_{33}(q_1)+\upsilon_{33}(q_1)|^2\frac{\varepsilon_{cq_13}}{\omega^2+\varepsilon^2_{cq_13}} + |u_{3z}(q_1)+\upsilon_{3z}(q_1)|^2\frac{\varepsilon_{sq_1}}{\omega^2+\varepsilon^2_{sq_1}}\right)\right\}, \quad (6)$$

where $N(0)$ is the normalization factor. In addition, we averaged the unit vectors and polarization vectors, which finally yielded a factor of 1/9. We introduce the notation $\lambda_0=N(0)g^2_{ph}(9\langle\omega_D\rangle)^{-1}$, take the explicit form of functions of the u-υ transformation into account, and pass to the long-wavelength limit, in which our kernel is written as

$$Q(q,\omega)=\lambda_0\left\{2+\frac{4q_k^2\omega_{sk}\omega_{ck3}}{(\varepsilon^2_{k3}-\varepsilon^2_{ks})(\varepsilon^2_{k3}-\omega^2_{ck3})}+\frac{\varepsilon^2_{ks}-\omega^2_{sk}}{\varepsilon^2_{ks}-\varepsilon^2_{k3}}\right\}. \quad (7)$$

2. THE SOLUTION OF THE INTEGRAL EQUATION FOR THE GAP Δ

It can be seen that the solution, as the analysis of Eq. (1), with the use of kernel (7) seems to be an unsolvable problem. We choose the resonance region where the electron pairing is the most intense for further investigation, so that the separation of this region will not narrow the physical meaning of the model. The resonance region $\omega_{sk}\cong\omega_{ck3}=\omega_r$ will allow us to simplify the kernel with the use of the relations

$$\varepsilon^2_{k3}-\varepsilon^2_{ks}=-2\omega^2_r z_r,\quad \varepsilon^2_{k3}-\omega^2_{ck3}=-\omega^2_r z_r,\quad \varepsilon^2_{ks}-\omega^2_{sk}=\omega^2_r z_r. \quad (8)$$

Using Eq. (8), we write the kernel as

$$Q(q,\omega)=\lambda_0\left\{\frac{5}{2}+\frac{2}{z_r^2}\frac{q^2}{\omega^2}\right\}. \quad (9)$$

Below, we discuss the resonance variables q_r and ω_r; therefore, for simplicity, we omit the index r. We write the final equation for the energy gap as

$$\Delta(\omega)=\lambda_0\int_0^{\hbar\omega_c}\frac{\Delta(q)}{q}\left\{\frac{5}{2}+\frac{2}{z_r^2}\frac{q^2}{\omega^2}\right\}th\left(\frac{q}{2kT}\right)dq. \quad (10)$$

To solve Eq. (10), we found an algorithm that allows the expression for the gap, Δ, to be obtained without further simplifying the current kernel, $Q(q,\omega)$. This algorithm is applicable to equations such as

$$y(x)-\lambda\int_a^b[Ag(x)+Bg(t)]h(t)y(t)dt=f(x). \quad (11)$$

Regrouping the terms in Eq. (10), we can easily establish the relationship with model equation (11):

$$\Delta(\omega)=\lambda_0\int_0^{\hbar\omega_c}\Delta(q)\left\{\frac{2}{z_r^2}\frac{1}{\omega^2}+\frac{5}{2}\frac{1}{q^2}\right\}qth\left(\frac{q}{2kT}\right)dq. \quad (12)$$

Selecting the coefficients, we can solve Eq. (12) with the use of the algorithm for model Eq. (11). The first step is building the characteristic values of Eq. (12) using the formula

$$\lambda_{1,2}=\frac{(A+B)s_1\pm\sqrt{(A-B)^2s_1^2+4ABs_0s_2}}{2AB(s_1^2-s_0s_2)},$$

where

$$s_0=\int_a^b h(x)dx,\quad s_1=\int_a^b g(x)h(x)dx,\quad s_2=\int_a^b g^2(x)h(x)dx. \quad (13)$$

Formally, the solution of Eq. (12) is reduced to calculating integrals (13) and the subsequent reasonable comparison of their different combinations. Unfortunately, only the integral s_0 of the three integrals s_0, s_1, and s_2 can be analytically calculated in the explicit form; however, the presence of the Euler logarithm in this solution, as well as the presence of the exponent with the temperature dependence of T_c under the dilogarithm argument, makes further separation of the critical temperature in the final solution nearly impossible. The authors used upper and lower estimations of these integrals. Although the chosen approach is rough, it allows the final result for the energy gap, Δ, to be obtained. The explicit expressions for integrals s_0, s_1, and s_2 have the form

$$s_0=\lambda_0\frac{\hbar^2\omega_c^2}{2},\quad s_1=\lambda_0\ln\left[\frac{e\hbar\omega_c}{2kT_c}\right],\quad s_2=\lambda_0\left(\frac{3}{2}-\frac{2k^2T_c^2}{\hbar^2\omega_c^2}\right). \quad (14)$$

Using the expressions for integrals s_0, s_1, and s_2, the expression for the characteristic values of Eq. (12) can be written as

$$\lambda_{1,2} = \frac{(4+5z_r^2)\ln\left[\frac{e\hbar\omega_c}{2kT_c}\right] \pm \sqrt{(4-5z_r^2)^2\ln^2\left[\frac{e\hbar\omega_c}{2kT_c}\right] + 4z_r^2(3\hbar^2\omega_c^2 - 4k^2T_c^2)\cdot 5}}{5\cdot[4\ln^2\left[\frac{e\hbar\omega_c}{2kT_c}\right] - (3\hbar^2\omega_c^2 - 4k^2T_c^2)]}. \quad (15)$$

The general solution of Eq. (12) is

$$\Delta(\omega) = C\Delta_{1,2}, \quad \Delta_{1,2} = g(\omega) + \frac{1-\lambda_{1,2}As_1}{\lambda_{1,2}As_0}, \quad (16)$$

where C is the constant independent of variable ω and $\Delta_{1,2}$ has the form

$$\Delta_{1,2} = \frac{1}{\omega^2} + \frac{(-4+5z_r^2)\ln\left[\frac{e\hbar\omega_c}{2kT_c}\right] \mp \sqrt{4z_r^2(-4k^2T_c^2 + 3\hbar^2\omega_c^2)\cdot 5 + (4-5z_r^2)^2\ln^2\left[\frac{e\hbar\omega_c}{2kT_c}\right]}}{4\hbar^2\omega_c^2}. \quad (17)$$

Changing the variables

$$x = (-4+5z_r^2)\ln\left[\frac{e\hbar\omega_c}{2kT_c}\right], \quad (18)$$

$$y = 4(-4k^2T_c^2 + 3\hbar^2\omega_c^2)\cdot 5, \quad (19)$$

we arrive at the final expression for the energy gap, Δ

$$\Delta = \frac{1}{\omega_r^2} + \frac{x}{4\hbar^2\omega_c^2} + \frac{\sqrt{x^2 + z_r^2 y}}{4\hbar^2\omega_c^2}. \quad (20)$$

Although it seems impossible to obtain the critical temperature, T_c, in the explicit form with the use of expression (20) at the chosen level of assumptions, we can make certain qualitative estimations.

In particular, it can be seen that at low frequencies ω, the energy gap, Δ, increases. Recall that the frequencies, ω, are assumed to be resonance frequencies ω_r. Thus, we established that with an increase in resonance frequencies, ω_r, the energy gap, Δ broadens and the resonance frequencies, ω_r, decrease due to an increase in the coupling parameter, ξ. In other words, with an increase in the coupling parameter, ξ, the branches of coupled spin-phonon oscillation frequencies increase, which leads to a decrease in the resonance frequencies, ω_r, which, in turn, cause broadening of the energy gap, Δ. As is known, broadening of the energy gap, Δ, leads to an increase in critical temperature, T_c. We can state that the proposed model makes it possible to theoretically explain the growth of the critical temperature, T_c, in some substances.

Since this approach formally pretends to be general, despite the successful description of the physical meaning of the model the correlation between the final result with the a priori known result of the BCS theory remains unclear. We are interested in this correlation because most of the steps to the final result were based on well-known standard approaches. The authors suggest that this theory will be developed in analyzing the expression that is obtained by the nullification requirement or elimination of the term that contains the root in Eq. (20). Then, after simple transformations, one can obtain the expression for the critical temperature, T_c, with a structure resembling the classical result of the BCS theory

$$T_c = \frac{e\hbar\omega_c}{k} e^{-\tilde{A}\tilde{f}},$$

where $\tilde{A} = 4\hbar^2(-4+5z_r^2)^{-1}$ and $\tilde{f} = \frac{\omega_c^2}{\omega_r^2}(\omega_r^2\Delta - 1)$.

3. SOLUTION OF THE INTEGRAL EQUATION FOR THE GAP, Δ, WITH THE USE OF THE TECHNIQUE OF RECTANGULAR PITS

To find the expression for critical temperature, T_c, in the explicit form, we use the technique of rectangular pits, which suggests dividing the integration interval in Eq. (1) into subintervals and subsequent approximation of the kernel in these subintervals by constants. Then, it is proposed to solve a set of algebraic equations and thus find the expression for the temperature, T_c.

In this model, we propose to take three intervals: $(0,\omega_D)$, (ω_D,ω_s), and (ω_s,ω_c). In the first interval $(0,\omega_D)$, the main contribution to the kernel is made by the phonon system; therefore, taking three phonon polarizations and the rough averaging 1/2 into account we obtain the constant $3\lambda_0$. In the second interval (ω_D,ω_s), the spin subsystem and its interaction with the phonon subsystem will prevail; therefore, taking only two actively interacting modes into account and averaging, we obtain the constant λ_0. The last term of our model potential, μ, characterizes Cou-

lomb repulsion of electrons and therefore exists in all the intervals. In the chosen designations,

$$Q_{11} = 3\lambda_0 - \mu,$$
$$Q_{12} = Q_{22} = Q_{21} = \lambda_0 - \mu,$$
$$Q_{13} = Q_{23} = Q_{33} = Q_{32} = Q_{31} = -\mu.$$

To establish the temperature dependence, it is proposed to assume the existence of three energy gaps, $\Delta_{1,2,3}$, one per each chosen interval.

The system of algebraic equations has the form

$$\begin{cases} -\Delta_1 = Q_{11}Z_1\Delta_1 + Q_{12}Z_2\Delta_2 + Q_{13}Z_3\Delta_3, \\ -\Delta_2 = Q_{21}Z_1\Delta_1 + Q_{22}Z_2\Delta_2 + Q_{23}Z_3\Delta_3, \\ -\Delta_3 = Q_{31}Z_1\Delta_1 + Q_{32}Z_2\Delta_2 + Q_{33}Z_3\Delta_3, \end{cases} \quad (21)$$

where $Z_{1,2,3}$ within a weak bond :

$$Z_1 = \ln\left[\frac{1{,}14\hbar\omega_D}{kT_c}\right], \quad Z_2 = \ln\left[\frac{\omega_s}{\omega_D}\right], \quad Z_3 = \ln\left[\frac{\omega_c}{\omega_s}\right].$$

We can write the expressions for $\Delta_{1,2,3}$ using the Cramer's formulas for the system of algebraic equations, so that $\Delta_{1,2,3} = D_{1,2,3}/D_0$:

$$\begin{cases} \Delta_1 = -\frac{\Delta_1 - \Delta_2}{2Z_1\lambda_0}, \\ \Delta_2 = \frac{\Delta_1 - 3\Delta_2 + 2\Delta_3}{2Z_2\lambda_0}, \\ \Delta_3 = \frac{\mu\Delta_2 + \Delta_3(-\mu + \lambda_0)}{\mu Z_3\lambda_0}. \end{cases} \quad (22)$$

Now, sequentially separating Δ_3 and Δ_2, we obtain the expression for Δ_1, which contains the temperature dependence of T_c in term Z_1:

$$\Delta_1 = -\frac{\Delta_1(-1 + \mu Z_3 + Z_2(\mu + (-1 + \mu Z_3)\lambda_0))}{Z_1(\mu + \lambda_0(-3 + 3\mu Z_3 + 2Z_2(\mu + (-1 + \mu Z_3)\lambda_0)))}. \quad (23)$$

Substituting the expressions for Z_1, Z_2, and Z_3 in the explicit form, we obtain

$$\ln\left[\frac{1{,}14\hbar\omega_D}{kT_c}\right] = -\frac{-1 + \mu\ln\left[\frac{\omega_c}{\omega_s}\right] + \ln\left[\frac{\omega_s}{\omega_D}\right]\left(\mu + \left(-1 + \mu\ln\left[\frac{\omega_c}{\omega_s}\right]\right)\lambda_0\right)}{\mu + \lambda_0\left(-3 + 3\mu\ln\left[\frac{\omega_c}{\omega_s}\right] + 2\ln\left[\frac{\omega_s}{\omega_D}\right]\left(\mu + \left(-1 + \mu\ln\left[\frac{\omega_c}{\omega_s}\right]\right)\lambda_0\right)\right)}. \quad (24)$$

It is convenient to change the variables:

$$\varphi = -1 + \mu\ln\left[\frac{\omega_c}{\omega_s}\right] + \ln\left[\frac{\omega_s}{\omega_D}\right]\left(\mu + \left(-1 + \mu\ln\left[\frac{\omega_c}{\omega_s}\right]\right)\lambda_0\right),$$

$$\phi = -3 + 3\mu\ln\left[\frac{\omega_c}{\omega_s}\right] + 2\ln\left[\frac{\omega_s}{\omega_D}\right]\left(\mu + \left(-1 + \mu\ln\left[\frac{\omega_c}{\omega_s}\right]\right)\lambda_0\right).$$

In the new variables, the expression for T_c acquires the form

$$T_c = \frac{1{,}14\hbar\omega_D}{k}\exp\left[-\frac{\phi}{\mu + \lambda_0\varphi}\right]. \quad (25)$$

The technique of rectangular pits, despite its simplicity and definite roughness in approximating the kernel, $Q(q, \omega)$, of Eq. (1) for the energy gap, Δ, yields a sufficiently good result. As an example, the final expression (25) has a recognizable structure that is characteristic of the classical result of the BCS theory. Having qualitatively analyzed the functions ϕ and φ, as well as the construction that is formed by them in (25), we can state a definite increase in the critical temperature, T_c, with allowance for spin interactions.

CONCLUSIONS

Based on the fluctuation theory of high-temperature superconductivity, we obtained the coupled spin-phonon oscillation spectrum, which makes it possible to qualitatively explain the growth of the critical temperature, T_c, in magnetic superconducting systems.

Based on the Frolich model of the interaction between phonons and electrons as the main superconductivity mechanism, we built the model Hamiltonian, which takes the phonon and spin systems and the interaction between phonons and spin fluctuations into account.

Using the Bogolyubov unitary transformation, we brought the Hamiltonian to the diagonalized form, in which the interaction between quasiphonons and quasimagnons is described. Based on the diagonalized Hamiltonian, we obtained the kernel $Q(q, \omega)$ of the equation for the energy gap, Δ.

We considered the general solution of the obtained equation in the resonance region, where the peak interaction of electrons occurs. The approach used yielded the expression for the energy gap, Δ, which takes into account the interaction with spin fluctuations. Unfortunately, it appeared impossible to obtain the explicit expression for critical temperature T_c; however, based on the qualitative analysis, the trend to an increase in T_c with allowance for the introduced

pairing mechanism was established. Since the main solution pretends to be general, a particular case was shown, when the well-known classical results can be easily obtained.

To complete the investigations, we proposed to use an updated technique of rectangular pits to obtain the explicit dependence of critical temperature T_c. Despite the rough approximation of the kernel $Q(q, \omega)$, the physical meaning of the model is retained and the result obtain can qualitatively describe an increase in critical temperature T_c for some substances.

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