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Autowave Self-Organization in Heterogeneous Natural–Anthropogenic Ecosystems

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Abstract—This article considers the spatio–temporal model of natural–anthropegenic ecosystems as a conjugated active media that takes the heterogeneity of anthropogenic and natural factors into account. The approach aims to identify the threshold values of the control parameters. The theoretical basis of the system analysis of the sustainability of the ecosystems is synergistic data on autowave self-organization in active media. The mathematical model is based on the modified FitzHugh–Nagumo system of equations.

Keywords: urban ecosystems, self-organization, active media, autowaves, internal transition layer.

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INTRODUCTION

The development of the recent models that are concerned with the basic principles of evolution in complex biospheric systems is based on the use of physical and mathematical approaches that have proven to be efficient for the description of the processes of self-organization in relatively simple dissipative systems. According to the theory of autowave self-organization in active media, natural–anthropogenic ecosystems, and urban ecosystems (UESs) as their most concentrated variant, may be considered as interacting natural and anthropogenic subsystems characterized by significant heterogeneity of the sources of anthropogenic effects and urban geobiocenoses.

The problem of UES morphology has been widely covered in the literature. However, the approach to these systems is very one-sided. As an example, many researchers focus on studying the population density with an only center of the density-distribution function [1, 2]. Nevertheless, this approach neglects numerous nonlinear effects typical for UESs and is described by a relatively low number of parameters. On the one hand, this simplifies the analytical problem. On the other hand, the processes in complex systems are reduced to lienear ones. One of the promosing complex methods for the investigation of UESs is the empiric method, i.e., a combination of cellular automata theory and Markov chains [3]. This method has been used for studying the probability of consecutive random events associated with urban infrastructure. Application of the theory of fractals for description of the morphology of urban patterns allows one to eliminate the ideas on homogenous space [4]. The above methods make it possible to describe the morphology of a city from the point of view of the spatial and socio-economic structure, but they do not take the nonlinear effect of interaction between numerous anthropogenic and natural factors within the framwork of the described system into account and therefore do not consider cities as a general ecosystem.

Natural–anthropogenic ecosystems, in contrast to natural ecosystems, are characterized by rapid growth, active energy and mass mass tansfer, signficant average population density, and a high metabolic intensity. These factors affect the dynamic balance between the flows of energy, matter, and information, as well as decrease the buffer capacity of natural subsystems and increase the nonlinearity and instability of the system processes.

UESs are characterized by the general patterns of complex crossover anthropogenic effects generated by territorially confined industrial and residential objects, the buffer capacity of natural subsystems, and significant fragmentation of the anthropogenic effects of geobiocenoses. These features of development of UESs are able to form irreversible system processes on the borders of UESs. The necessary prerequisite for the occurrence of the effect of self-organization in the limits of natural–anthropogenic ecosystems is the correspondence of social, biotic, and abiotic factors to the degree of the anthropogenic effect. When the natural processes prevail an ecosystem is capable of natural autowave self-organization. If the anthropogenic processes play a leading role, irreversible system processes occur. We considered the conditions for the self-organization and properties of UESs as hierarchies of conjugated active media in [5–7].

However, large cities and industrial centers are not able to provide the optimum conditions for natural self-organization by themselves, viz., regeneration of the main components of the natural environment that lead to the balance of matter and energy in the system and, thus, compensatory correspondence of the sustainability of the natural subsystem to the level of anthropogenic influence. This is associated to a significant degree with the high density and level of a population, which, in its turn, determines the high intensity of the flows of energy, matter, and information in a general system.

Thermodynamic nonequilibrity, nonlinearity, and bifurcational development is the synergistic basis of UES self-organization in the models of active media [5–7]. The distributed resource of the system is utilized by interrelated nonlinear local transformers: for a physical–chemical system, it is the phase of the process distribution in space. The local transformers of energy and matter convert energy in conjugation with autocatalytic transformation of matter in the presence of a “disturbance” in neighboring cells or when they receive a control signal through a reverse positive or negative connection. Therefore, UESs (outside the critical conditions of functioning) due to their complex and continuously complicating network of direct and reverse connections undergo an increase in the mutual influence of elements of the subsystems and in the number of potential trajectories of development.

Self-organization is manifested in the development of autowave dissipative structures [8–11] and forms the degrees of freedom [12]. The control parameters are natural and anthropogenic factors (excitable elements) that determine the length and shape of autowaves. The rates of natural processes are much slower than those of anthropogenic processes [13]. Thus, we assume that anthropogenic processes in this model function as activators, while natural processes are inhibitors of general system processes. In UESs in a similar manner to that in heterogenous active media, subthreshold interactions of the neighboring elements are possible. The considered properties allow one to qualitatively assess the threshold and subthreshold conditions for the distribution of the autowave process depending on the intensity of interaction sources, the locations of excitable, weakly excitable, and non-excitable zones, the availability of latent sources of autowaves, the level and density of the population, as well as other factors.

THE SPATIO–TEMPORAL MODEL OF AUTOWAVE SELF-ORGANIZATION OF NATURAL–ANTHROPOGENIC ECOSYSTEMS

This model was created to reveal the conditions of stability in UES functioning. In [5–7], UESs were considered as conjugated active media based on the modified FitzHugh-Nagumo system of equations [14]:

$$\frac{\partial u}{\partial t} - \varepsilon D_u \Delta u = -\frac{1}{\varepsilon}(u(u-\alpha)(u-1) + uv),$$
$$\frac{\partial v}{\partial t} - \varepsilon D_v \Delta v = (-\gamma v + \beta u), \qquad (1)$$

where u is a function of activator intensity (anthropogenic processes) and v is a function of inhibitor intensity (natural processes).

The uv element that was added by the authors to the right part of the first equation of the system (1) increases the potential for the analysis of interactions between the activator and the inhibitor.

α is a parameter of system activation (in inverse proportion to the population density).

γ is a kinetic parameter of the potential attenuation in the inhibitor, $\gamma > 0$.

β is a kinetic parameter of the interaction between the activator and the inhibitor, $\beta > 0$.

D_u, D_v are the diffusion coefficients of the activator and the inhibitor, $0.1 < \varepsilon D_u < 1$; $0.01 < \varepsilon D_v < 0.1$.

ε is a parameter that describes the rates of changes in the functions of the intensity of the activator and the inhibitor.

The activators of system processes may be technogenic electromagnetic radiation, corrosion of subsurface structures and utilities, changes in temperature, acidity (pH), oxidation–reduction potential (Eh) of ground water, acoustic and vibration loads on the upper lithospheric layer, increase in the concentration of heavy metals in the biogenic matter, the level and density of the population, etc. The inhibitors are the specific electrical resistance, an increase–reduction in the density and humidity of soils, the velocity of the current and mixture of ground waters, natural waste areas, etc.

System (1) is solved numerically using the calculation for determining spatial domain D with the boundary of ∂D. The Neumann boundary conditions are set on the domain boundary: $\left.\frac{\partial u}{\partial \mathbf{n}}\right|_{\partial D} = \left.\frac{\partial v}{\partial \mathbf{n}}\right|_{\partial D} = 0$. The distribution at the initial time is considered as known. For numerical solution of the nonlinear equation, Newton linearization is used with subsequent application of the method of successive approximations. For the one-dimensional problem a uniform grid is introduced on the interval $[-L; L]$, where L is the width of the computational domain; calculation is carried out using the half-sum scheme. Numerical solution of the two-dimensional problem is performed in a rectangle $-L \le x \le L$; $-L \le y \le L$, which introduced a uniform grid. Implementation of the numerical calculation is performed using the scheme of alternating directions [15, 16].

Depending on the ratio of the α, β, and γ parameters, the polynomial on the right side of the first equation (1) can have either three real roots (an excitable medium) or one real root that is equal to zero. In the

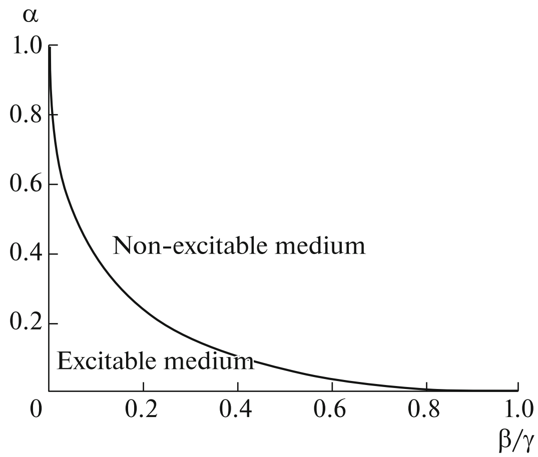

Fig. 1. A diagram that shows the state of the active medium depending on the ratio between the α, β, and γ parameters.

latter case, the only possible stable state of the system is zero. In this system, any initial fluctuation decays with time: natural factors are capable of inhibiting negative anthropogenic processes (a non-excitable medium). In the case of this ratio between the parameters α, β, and γ, at which the model describes an excitable medium, the solution of system (1) has the form of autowaves. The activator wave is supported by the presence of the sources of information matter energy distributed it. The threshold value of the parameter α_c as the functions of the ratio β/γ is defined implicitly by the equation $(\alpha_c - 1)^2 + (\beta/\gamma)^2 - 2(1 + \alpha_c)\beta/\gamma = 0$. If $\alpha(\beta/\gamma) > \alpha_c(\beta/\gamma)$, the system (1) describes a non-excitable medium. If $\alpha_c(\beta/\gamma) < \alpha(\beta/\gamma)$, the medium is excitable (Fig. 1). The higher the level and density of the population are, the more direct and reverse feedbacks are formed and the higher the value of the kinetic parameter of the interaction between the activator and inhibitor is (β).

In the case of the heterogeneity of the spatial distribution of excitable and non-excitable zones, sharp changes occur on the borders of these zones, which is characteristic of UESs, in the functions of the intensity of the activator and inhibitor. In this case, the internal transition layers are formed on the boundaries of the zones. The heterogeneity of the system is determined by its structural and functional complexity related to the network of direct and reverse feedbacks, as well as to the presence of barriers (urban biocenoses). A barrier is the area of the active medium in which the activator wave front is leveled when the barrier width is greater than the width of the transition layer (autowave locking). When the barrier width is smaller, tunneling through the barrier occurs (Figs. 2 and 3) [5].

In this case, the ratio of the correlation coefficients α, β, and γ in the barrier zone must correspond to the area of the non-excitable medium in the diagram (Fig. 1). The system parameters are presented in Table 1. L denotes the width of the computational domain.

The considered effects of the autowave behavior process are associated with the synergy of effects produced by various factors on the ecosystem: fragmentation and the area of biocenoses, their quantitative and qualitative composition, the density of the distribution of industrial and residential objects, various anthropogenic factors (as the result of the functioning of industrial and residential objects), and the level and density of the population, etc. In most cases, the anthropogenic factors that are characteristic of UESs are associated with the formation of technological fields that cover almost the entire upper part of the lithosphere (within the ecosystem) and provide the total nonadditive effect on the formation of the structure and functions of the ecosystem.

Thus, the intensity of anthropogenic impacts on UESs is directly related to the density of construction and, therefore, to the formation of heterogeneous electromagnetic fields (EMFs) of industrial frequencies, which are generated by numerous electrical appliances in residential and office buildings [5]. The sizes of the characteristic zones of EMF intensity heterogeneities of industrial frequencies correspond to

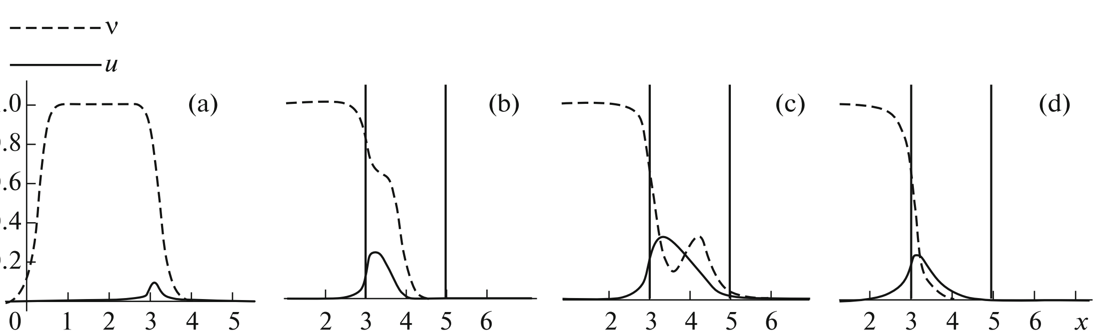

Fig. 2. The effect of blocking the impulse of the activator: (a), (b), (c), the stages of the process, (d), stationary distribution. Barrier width $2L/25$. The position of the barrier shown by vertical lines.

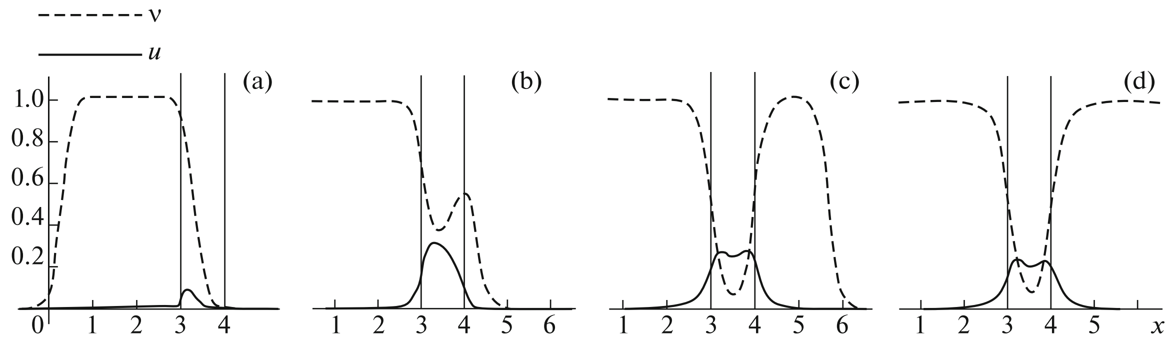

Fig. 3. The effect of tunneling of the impulse of the activator: (a), (b), (c), stages of the process, (d), stationary distribution. The barrier width $L/25$. The position of the barrier is shown by vertical lines.

the sizes of UES biocenoses (from 30 to 100 m depending on the number of floors in buildings) and the average value of energy absorbed by the soil and groundwater environments and radiated by the terrestrial sources is up to 20%. Reduction of the distances between tall residential buildings, as well as an increase in their height and the intensity of the development of subsurface utilities significantly enhances not only the heterogeneity of the electromagnetic background, but also the heterogeneity of the natural subsystem, increasing the fragmentation of urban geobiocenoses, the reduction of local areas, and the nonlinear qualitative changes in geobiocenoses.

Compared to the atmosphere and terrestrial waters, the soil, as a part of the environment, is more vulnerable to anthropogenic effects, which is due to its heterogeneity and numerous processes that are responsible for the transfer of different substances. A link (or links) of the trophic chain may be lost not only due to the effect of EMGs, but also as a result of chemical impacts and soil infections, as well as mechanical damage to the soil body. Disturbances of the physical and chemical parameters of soils, the primary active medium, can lead to the qualitative and quantitative disbalance of trophic networks with a higher probability. As an example, periodic movement of the saline solution of ground waters in an ion exchanger (which soil is to a certain degree), the alternation of sorption–desorption processes of substances dissolved in soil by the water solution caused by the synchronous periodic changes in temperature leads to the division and redistribution of dissolved substances and ions in the soil horizons [17]. Microorganisms are the most important factor that influence the process of soil formation and develop intensively only under particular conditions, humidity, and pH of the environment. The optimum temperature regime often varies within the range of +20–35°C. The pH values of the environment differ for various groups of microorganisms. The process of nitrification occurs as a result of the joint activity of bacteria from two groups (Nitrosomonas, Nitrocystus, Nitrosospira, and Nitrobacter) in well-aerated soils with neutral and alkaline reactions (pH values from 6.2 to 9) in the presence of a significant amount of humus and sufficient humidity. On the other hand, changes in the ratio of CO_2 in soils and ground waters result in changes of the pH of the environment, as well as the productivity and species composition of biocenoses. Another important factor is the redox potential [Eh]. Outside the boundaries of the domain $Eh_{min} \leq Eh \leq Eh_{max}$, the medium becomes thermodynamically unstable: $0.5 \leq Eh_{max} \leq 10$, $0.5 \leq Eh_{min} \leq 0.7$ [18]. Changes in the natural Eh disturb the ratio of dissolved and non-dissolved forms of some minerals, migratory substances, and anaerobes. Biodegradation of the organic substances (with free gas exchange with the atmosphere) in ground waters and soils enables the formation of redox zones, viz., methane release as a result of CO_2 reduction and sulfate reduction, as well as Fe^{3+} and Mn^{4+} reduction.

The amplitude of fluctuations in the level of ground waters in urban territories is significantly higher than in natural ecosystems and, as a rule, is not subjected to seasonal changes. As the intensity of anthropogenic loads on an aquatic basin increases sharply, the response of the hydrosphere becomes more pronounced (an explosive or gradual decrease in the quality and level of ground waters): hydrodynamical, temperature, hydrochemical, and hydrobiological anomalies occur. Contamination of ground waters causes changes in the properties of natural geochemical barriers (pH, gleyed, and sorption barriers), hydro-

Table 1. The parameters of the system for description of the barrier in an urban ecosystem

ε	D_u	D_v	α	γ	L	β/γ outside barrier	β/γ in barrier
0.04	5	1	0.1	0.5	25	0.02	1.4

Table 2. The parameters of the one-dimensional problem for description of an urban ecosystem with a "spotted" population density

Control parameters of equation system (1)					Parameters of distribution function $\rho(x)$					
ε	γ/β	D_u	D_v	L	α_0	α_d	d	x_1	x_2	$\Delta x/d$
0.04	6.7	5	1	5	0.2	0.9	$0.25L$	$0.25L$	$0.75L$	0

dynamic traps (drainless water bodies, swamps), and bottom sediments. Under these conditions, even at the maximum water capacity, hydrobiological processes are disturbed (the water is less available for absorption by plants and the activity of soil bacteria decreases) [18].

A characteristic feature of large urban agglomerations is disturbance of the thermal regime at the depth of 100–300 m. The temperature of rocks and ground waters is often 10–40°C higher than the background level and even 60–100°C higher in the areas of underground transport and other utilities [19]. An increase in the thermal flow by 2–3 times relative to the background level disturbs the physical and chemical properties of the environment and causes growth of the complex influences of polluting substances and biodestruction rates (an increase in the temperature of the upper layer of lithosphere by 10°C increases the rate of biodestruction by 1.5–2 times) [18]. If the source of a thermal influence is located near the level of ground waters, convective transfer of heat (increased dissipation) occurs, which enlarges the areas of thermal anomalies by 30–50%, depending on the flow rate of ground waters.

The above examples are indicative of the content of the model and the essence of non-additivity in multicomponent system process of UESs formed by the loops of negative and positive reverse connections.

Thus, the primary factor of the sustainable or disturbed development of the anthroposphere is the settlement of the human population. In the framework of the considered model, two major types of formation of the population density in urban settlements were considered: that according to Gauss and "spotted" superposition of two Gaussian distributions with different centers, which are typical for most cities. In the latter case, the maxima of the function of density distribution are located in the points with the coordinates $x = x_i$, $i = 1, 2$ (Table 2, Fig. 4).

For the one-dimensional model (1), the density distribution is determined by the function:

$$\rho = \rho_0(\exp(-\eta(x - x_1)^2/2) + \exp(-\eta(x - x_2)^2/2)),$$

where $\eta = 2/d^2 \ln \rho_0/\rho_d = 2/d^2 \ln\alpha_0/\alpha_d$, $\rho_0 = \alpha_0^{-1}$; the density at the maximum, $\rho_d = \alpha_d^{-1}$; the density at the distance d from the maximum: $\rho(x_i \pm d) = \rho_d$. The parameters of the system are given in Table 2. L designates the domain of the calculation: $\Delta x = x_1 - x_2$ is the distance between the maxima of the function of the population distribution.

The analysis of the diagrams in the presence of several peaks of population density shows that the system is characterized by the formation of autowaves as an activator and inhibitor. The distribution becomes stationary with time.

The analysis of the conditions of the existence of the stationary transition layer obtained by the equation set for $\alpha(x) = \rho^{-1}(x)$ [20] shows that the $\Delta x/d$ ratio plays a key role in the formation of a stable stationary solution with an inner transition layer. In an excitable medium, the condition for the existence of transition layers between the peaks with the values of the parameters α_0 and α_d indicated in Table 2 is $\Delta x/d \geq 1.8$. The value $\Delta x/d$ corresponds to the value $\alpha = 0.336$. This value of the α parameter acts as the critical one: according to the suggested model, an urban environment with a localization that is stable in time and space is formed at $\alpha > 0.336$. At $\Delta x/d = 2$ (Fig. 4) the solution has clear inner transition layers at $\rho = 1/0.336$ between the points x_1 and x_2.

All the above considerations are also true for two-dimensional models developed on the basis of the system of equations (1) [21]. In Fig. 5, the results of the numerical experiment for the two-dimensional case with a "spotted" population density are given:

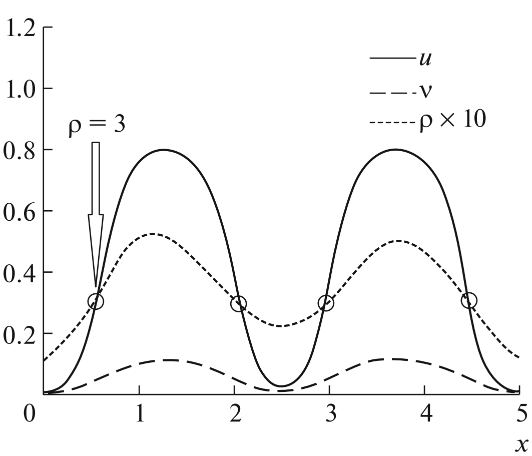

Fig. 4. A diagram that shows the dependence of the functions of the activator, inhibitor, and population-density distribution on the x coordinate at $\Delta x/d = 2$ (the one-dimensional model): stationary distribution.

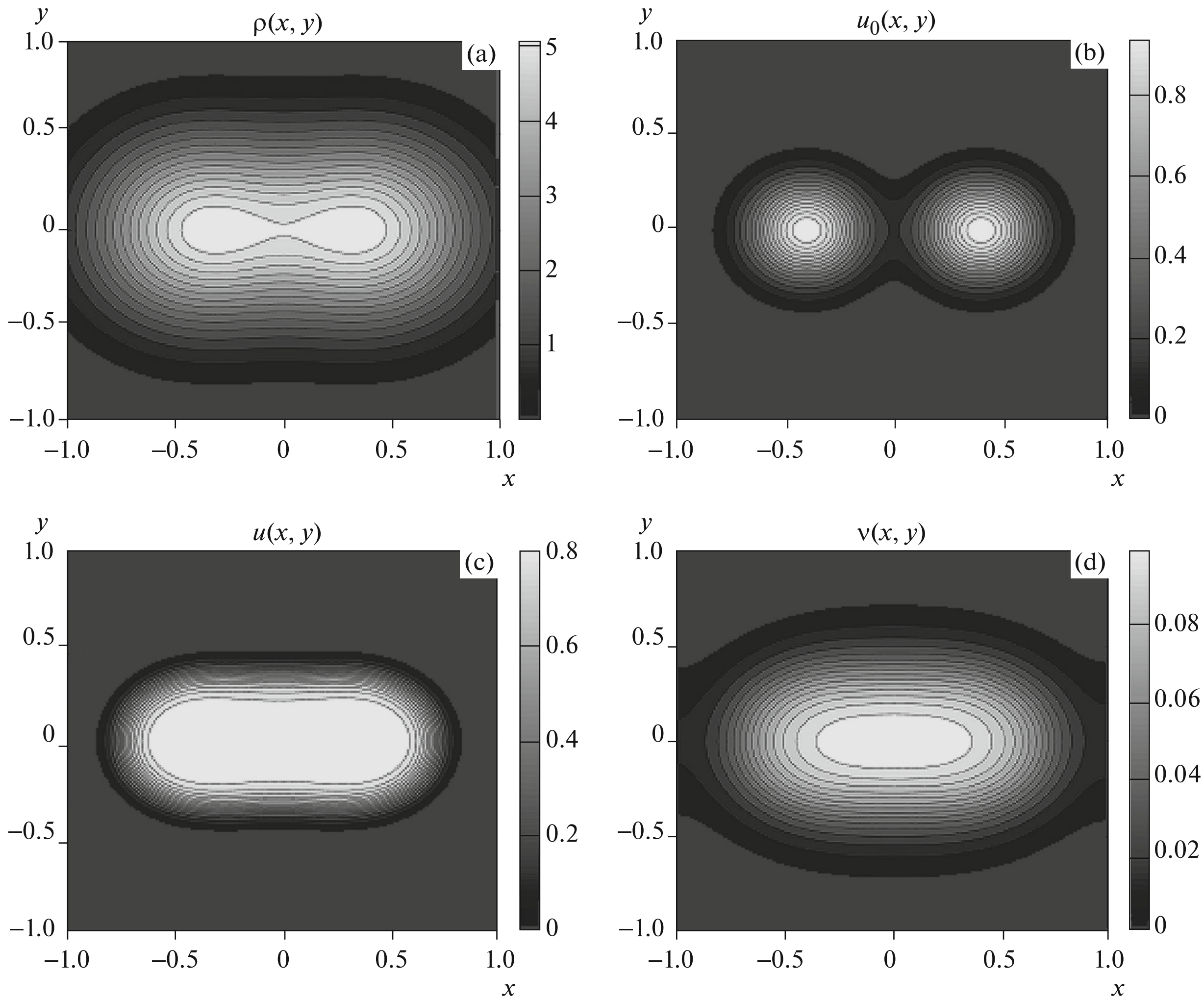

Fig. 5. A diagram that shows the dependence of the functions of the activator, inhibitor, population density distribution on the coordinates x, y (two-dimensional model): (a) the stationary distribution of the population density, (b) the initial distribution of the function of the activator intensity, (c) the stationary distribution of the function of the activator intensity, (d) the stationary distribution of the function of the inhibitor.

$$\rho = \rho_0\left(\exp\left(-\frac{\eta}{2}((x-x_1)^2+(y-y_1)^2)\right)\right) + \left(\exp\left(-\frac{\eta}{2}((x-x_2)^2+(y-y_2)^2)\right)\right)$$

and the initial distribution

$$u_0(x,y) = \exp(-15((x-x_1)^2+(y-y_1)^2)) + \exp(-15((x-x_2)^2+(y-y_2)^2)), \quad \nu_0 = 0.$$

The parameters of the system are given in Table 3.

CONCLUSIONS

The approach presented in this work allows one to develop a direct and adequate evaluation of anthropogenic impacts in an excitable medium of natural–anthropogenic ecosystems as complex structures that conjugate the values of the system and events that differ by orders of magnitude. Control over the parameters of autowave self-organization and natural–anthropogenic ecosystems by local influences can be very promising, because it can influence the macroscopic situation of the entire system. In principle, the

Table 3. The parameters of the two-dimensional problem for description of an urban ecosystem with a “spotted” population density

ε	γ	β	D_u	D_v	L	d	Δx	$\Delta x/d$	ρ_0	ρ_d
0.007	1	0.15	5	1	2	$0.3L$	$0.4L$	1.3	5	1.1

model allows one to analyze and forecast the ecological dynamics of the region when introducing particular data on the natural characteristics and anthropogenic effects.

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