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PHYSICS OF NUCLEI AND ELEMENTARY PARTICLES

Neutrinos and Probes of High Scale Physics[1]

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Abstract—In light of the theoretical and experimental developments in the neutrino sector and their importance, we study its connection with the new physics above the electroweak scale. In particular, by considering the neutrino oscillations with the possible effective mass, we investigate, according to the experimental data, the scale as well as the signature of the underlying new physics beyond the SM.

Keywords: standard model, neutrino and high scale physics.

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1. INTRODUCTION

In spite of the exciting successes of the Standard Model (SM) as a theory of fundamental particles and interactions at energies up to about ~100 GeV [1, 2], the SM satisfactory is still far and physics beyond the SM is widely expected to reside with new characteristic mass scales, perhaps up to, likely, the GUT scale [3, 4]. In fact, the hierarchy among the fermion masses is not explained, and to most of the experimental data, some input parameters are required. All these unc-known parameters reflect our lack of understanding of flavor physics [5–7]. Moreover, with the progressive interest and results in neutrino physics, the SM description of the neutrino sector remains criticized and to be revised [8].

The history of the neutrino is very interesting, pulsating and illuminating. In the SM, with the absence of any direct evidence for neutrinos mass, the latter were presented as verily massless fermions for which no corresponding gauge-invariant renormalizable mass term can be constructed, and, thus no mixing occurs in the lepton sector [8–10]. However, the recent evidence of neutrino oscillations found in the SuperKamiokande [11], SNO [12], KamLAND [13], and other solar and atmospheric neutrino experiments bring the first sign of lepton mixing implying the non-zero neutrino masses [14–16], which are many orders of magnitude smaller than masses of charged leptons and quarks. In this way, neutrino oscillations can be connected to new physics beyond the SM which seems to have manifested itself in the form of an effective scale behind the possible neutrino masses.

The purpose of this paper is to investigate the interplay between the neutrino sector and new physics. More precisely, the connection of neutrino oscillations with the high mass scale M_s. For that, after introducing the present status of the neutrinos within the SM and envisaging their possible effective masses $m_{\nu_{j=1,2,3}}$, we derive from their flavor oscillations $\nu_{\alpha=e,\mu,\tau} \to \nu_{\beta=e,\mu,\tau}$ the implication of the high mass scale M_s and use the experimental neutrino masses and source data as a positive indication to explore the range and the possible signature of the underlying high scale parameter M_s.

2. NEUTRINOS WITHIN THE STANDARD MODEL

In the SM [1, 2], neutrinos belong to the left-handed states that carry the weak charge but their masses were compatible with zero when the SM was established, they were postulated to be Weyl fermions: i.e., a left-handed particle and a right-handed antiparticle. They are arranged as doublets for chiral left-handed fields,

$$l_{jL} = (\nu_{e_j} \;\; e_j)^T, \quad e_{j=1,2,3} = e, \mu, \tau. \tag{1}$$

The l_{jL} are the three family $j = 1,2,3$ left-handed $SU(2)$ doublet lepton fields. There are three known flavors of neutrinos. We shall define the neutrino of a given family $j = e, \mu, \tau$ in terms of leptonic W-boson decay. This decay produces a charged lepton, which may be an e_j, plus a neutrino of the same flavor ν_{e_j}. These are defined as the neutrino flavors that accompany the three charged leptons. In particular, the neu-

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trino sector, consisting of the propagation and interaction terms, is described by the leptonic part of SM Lagrangian,

$$\zeta_\nu = \sum_{j=1,2,3} i\bar{l}_{jL} D_\mu \gamma^\mu l_{jL}, \tag{2}$$

where γ^μ are the usual gamma matrices. The neutral components of left-handed lepton fields, i.e., neutrinos ν_{jL}, are coupled to the $SU(2)_L \times U(1)_Y$ weak gauge fields $W_\mu^{\pm,0}$ by the corresponding covariant derivative as,

$$\begin{aligned}\zeta_\nu = \sum_{\alpha=e,\mu,\tau} \Bigg[i\bar{\nu}_{\alpha L}\partial_\mu \gamma^\mu \nu_{\alpha L} + \Bigg(\frac{g_2}{\sqrt{2}} \bar{\nu}_{\alpha L} \gamma^\mu e_{\alpha L} W_\mu^+ \\ + \frac{g_2}{4\cos\theta_W} \bar{\nu}_{\alpha L}\gamma^\mu \nu_{\alpha L} W_\mu^0 + \text{h.c.} \Bigg) \Bigg],\end{aligned} \tag{3}$$

written in terms of the three flavor $\alpha = e, \mu, \tau$ states. The g_2 is the $SU(2)_L$ weak coupling constant, θ_W is the Weinberg angle and $e_{\alpha L}$ are the left-handed components of the charged lepton fields. The neutrino of flavor α couples then only to the neutral and charged leptons of the same flavor through the neutral and charged currents respectively. The absence of the right-handed neutrinos in the minimal SM framework is responsible for the missing of the neutrino mass term in this Lagrangian.

The simplest way to add neutrino mass to the SM is to invoke a neutrino term made out of the SM lepton doublets ℓ and Higgs field h consistent with the SM symmetries as,

$$\zeta_{\nu_{\text{mass}}} = \frac{y_\nu}{M_s^{2n+1}} (\ell h)^2 (h^\dagger h)^n, \tag{4}$$

where y_ν / M_s^{2n+1} stands now for the effective Yukawa coupling constant and n a positive integer specifying the term mass dimension which will be investigated later on [17]. This term (4) is dimensionally reduced by inverse powers of a mass scale M_s at which lepton number is expected to be violated. After electroweak symmetry breaking: $SU(2)_L \times U(1)_Y \to U(1)_{\text{EM}}$ by the Higgs vev $\langle h \rangle$, this term leads to the suppressed neutrino masses,

$$m_{\nu_j} \simeq y_{\nu_j} \frac{\langle h \rangle^{2n+2}}{M_s^{2n+1}}. \tag{5}$$

The appearance of such neutrino mass is more general, and would be expected to occur in any high-scale theory $M \geq M_s$. Moreover, if the underlying scale is huge, the corresponding neutrino masses might be too small $m_{\nu_\alpha} \ll$ eV to explain their likely range. Thus low mass scales $M_s \ll M_{\text{Planck}}$ must exist to give the desired masses to neutrinos. Though we have got an approach to the neutrino masses in an effective way, since we still don't have a fully consistent neutrino theory at hand, the question of wether the latter had mass or no was still open untill the observation of their flavor oscillations [11–13].

3. FLAVOR OSCILLATION MECHANICS

Neutrinos are normally identified by their flavors $\alpha = e, \mu, \tau$ rather than their masses m_{ν_j}. That neutrinos have masses means that there exists a spectrum of neutrino mass eigenstates ν_j, whose masses m_{ν_i} we would like to determine. That leptons mix means that the neutrinos of definite flavor ν_α are not the mass eigenstates ν_j. Instead, the neutrino flavor state $|\nu_\alpha\rangle$ which is the neutrino state that is created in leptonic $W^{\pm,0}$ processes together with the charged lepton of the same flavor $e_{\alpha L}$, is a quantum superposition of the mass eigenstates $|\nu_i\rangle$,

$$|\nu_\alpha\rangle = \sum_{i=1,2,3} U_{\alpha i} |\nu_j\rangle, \tag{6}$$

where the coefficients $U_{\alpha i}$ are elements of the neutrino mixing 3×3 unitary matrix U that transforms the neutrino flavor states to their mass states. With this mixing, the neutrino Lagrangian (3) becomes,

$$\begin{aligned}\zeta_\nu &= \sum_{i=1,2,3} i\bar{\nu}_{jL}\partial_\mu\gamma^\mu \nu_j \\ &+ \sum_{\substack{\alpha=e,\mu,\tau \\ j=1,2,3}} \Bigg(\frac{g_2}{\sqrt{2}} U^*_{\alpha j} \bar{\nu}_{jL}\gamma^\mu e_{\alpha L} W_\mu^+ \\ &+ \frac{g_2}{4\cos\theta_W} \bar{\nu}_{jL}\gamma^\mu \nu_{jL} W_\mu^0 + \text{h.c.} \Bigg),\end{aligned} \tag{7}$$

where now ν_j is a neutrino mass eigenstate. We then see that the amplitudes of the production of the neutrino ν_j in all possible $W^{\pm,0}$ weak processes are $\frac{g_2}{\sqrt{2}} U^*_{\alpha j}$ and $\frac{g_2}{4\cos\theta_W}$ respectively. A weak eigenstate (6) produced at time $t = 0$ in a pure $|\nu_\alpha\rangle$ state,

$$|\nu_\alpha(0)\rangle = \sum_{i=1,2,3} U_{\alpha i} |\nu_j\rangle, \tag{8}$$

will evolve after a time t to the state,

$$|\nu_\alpha(t, L)\rangle = \sum_{j=1,2,3} U_{\alpha j} e^{-i\varphi_j} |\nu_j\rangle, \tag{9}$$

where L is the traveled distance and $\varphi_j = \mathbf{p}_j \cdot \mathbf{r} = E_j t - p_j L$ is the phase factor with E_j and p_j are the energy and momentum associated with the mass eigenstate j which can be derived from the dynamics of the elementary process in which the neutrino is produced. In the extreme relativistic limit $L \simeq t$, appropriate for the tiny neutrino masses, the phase factor could be approximated to $\varphi_j \simeq m_{\nu_j}^2 L/2E$. With this and the expression of the mass eigenstates $|\nu_j\rangle$ back in terms of weak eigenstates $|\nu_\alpha\rangle$, the neutrino state evolution (9) reads,

$$|\nu_\alpha(L)\rangle = \sum_{j=1,2,3} U_{\alpha j}\left(\sum_{\alpha=e,\mu,\tau} U_{\alpha j}^*|\nu_\alpha\rangle\right)e^{-im_{\nu_j}^2 L/2E}. \quad (10)$$

At this level, if the masses of $|\nu_j\rangle$ are the same $m_{\nu_1} = m_{\nu_2} = m_{\nu_3} = m_\nu$, the mass eigenstates remain in phase $\varphi_1 = \varphi_2 = \varphi_3$, and the state remains the linear combination corresponding to $|\nu_\alpha\rangle$,

$$|\nu_\alpha(L)\rangle = 3e^{-im_\nu^2 L/2E}|\nu_\alpha\rangle, \quad (11)$$

which, in a weak interaction will produce a charged lepton of the same flavor e_α. However, if the neutrino masses are different, the neutrino state no longer remains a pure $|\nu_\alpha\rangle$, but a time-variable linear combination of the three flavor states $|\nu_{e,\mu,\tau}\rangle$,

$$\begin{aligned}|\nu_\alpha(L)\rangle = \sum_{j=1,2,3}\Big(&U_{\alpha j}U_{ej}^* e^{-im_{\nu_j}^2 L/2E}|\nu_\alpha\rangle \\ &+ U_{\alpha j}U_{\mu j}^* e^{-im_{\nu_j}^2 L/2E}|\nu_\mu\rangle + U_{\alpha j}U_{\tau j}^* e^{-im_{\nu_j}^2 L/2E}|\nu_\tau\rangle\Big),\end{aligned} \quad (12)$$

leading automatically to neutrino flavor oscillations whose the probabilty is,

$$\begin{aligned}&P(\nu_\alpha \to \nu_\beta) \\ &= |\langle\nu_\beta|\nu_\alpha(L)\rangle|^2 = \left|\sum_{j=1,2,3} U_{\alpha j}U_{\beta j}^* e^{-im_{\nu_j}^2 L/2E}\right|^2,\end{aligned} \quad (13)$$

which, consequently, requires different neutrino masses for oscillation. At this stage, straight forward calculations and the neglection of CP violation and the consideration of the experimental results [11–13], lead to the expression,[2]

$$\begin{aligned}P(\nu_\alpha \to \nu_\beta) &\simeq -4U_{\alpha 1}U_{\beta 1}U_{\alpha 2}U_{\beta 2} \\ &\times \sin^2\frac{\Delta m_{\nu_{21}}^2 L}{4E} + 4U_{\alpha 3}U_{\beta 3}\sin^2\frac{\Delta m_{\nu_{32}}^2 L}{4E},\end{aligned} \quad (14)$$

for neutrino $\alpha \to \beta$ oscillation probability and whose the two associated wave lengths read,

$$\lambda_{ji} = \frac{4\pi E_\nu}{\Delta m_{\nu_{ji}}^2}, \quad ji = 21, 32, \quad \to \Delta m_{\nu_{ji}}^2 = \frac{4\pi E_\nu}{\lambda_{ji}}, \quad (15)$$

resulting in a neutrino masses hierarchy $\Delta m_{\nu_{ji}}^2 = m_{\nu_j}^2 - m_{\nu_i}^2$, and therefore in the new mass parameter M_s, related to neutrino masses (5), through,

$$\lambda_{ji} = \frac{4\pi}{\Delta y_{\nu_{ji}}^2}\frac{E_\nu M_s^{4n+2}}{\langle h\rangle^{4n+4}}, \quad ji = 21, 32. \quad (16)$$

In this sense, the oscillatory character of the neutrino wave relies on the scale of the underlying mass M_s. Indeed, if the latter is huge $M_s \sim M_{\text{Planck}}$, regardless of their energy E_ν, the corresponding wave lengths might be too long to explain the neutrino oscillation phenomenon. Thus, low mass scales $M_s \ll M_{\text{Planck}}$ must exist to give the observed neutrino oscillation wave lengths, mesured by solar and atmospheric experiments [11–13].

4. PROBES OF HIGH SCALE PHYSICS

We have shown that no oscillation phenomena can happen if neutrinos are massless which contradicts the SM in non-conservation of the lepton flavour and non-zero neutrino masses. In particular, we have now convincing evidence that the three active neutrinos of the SM have different masses and they mix with each other. The results of the solar and atmospheric experiments have now narrowed the neutrino masses [11–13]. More precisely, the range of the differences of the squared neutrino masses as,

$$\Delta m_{\nu_{21}}^2 \equiv \Delta m_{\nu_{21}\text{solar}}^2 \sim 8\times 10^{-5}\,\text{eV}^2, \quad (17)$$

$$|\Delta m_{\nu_{32}}^2| \simeq |\Delta m_{\nu_{31}}^2| \equiv |\Delta m_{32}^2|_{\text{atmos}} \sim 3\times 10^{-3}\,\text{eV}^2, \quad (18)$$

the relevant ones for solar and atmospheric neutrinos, respectively. Although oscillation experiments are insensitive to the absolute scale of neutrino masses, since the knowledge of $\Delta m_{\nu_{21}}^2 > 0$ and $|\Delta m_{\nu_{32}}^2|$ leads to two possible hierarchy schemes characterized by the sign of $\Delta m_{\nu_{32}}^2$, it appears sensitive to new physics beyond the SM. In fact, according to the associated wave length expressions (15), (16), the above differences of the squared neutrino masses allow to express the involved high scale as,

$$M_s^{\text{solar}} = \sqrt[4n+2]{\Delta y_{\nu_{21}}^2 \frac{\langle h\rangle^{4n+4}}{\Delta m_{\nu_{21}}^2}}, \quad (19)$$

$$M_s^{\text{atmos}} = \sqrt[4n+2]{\Delta y_{\nu_{32}}^2 \frac{\langle h\rangle^{4n+4}}{\Delta m_{\nu_{32}}^2}}, \quad (20)$$

[2] We have used $|z_1 + z_2 + z_3|^2 = |z_1| + |z_2| + |z_3| + 2\,\text{Re}(z_1 z_2^* + z_1 z_3^* + z_2 z_3^*)$, the unitarity deduced relation $\sum_{j=1,2,3} U_{\alpha j}U_{\beta j}^* = 0$ and the fact that $|\Delta m_{\nu_{21}}^2|_{\text{solar}} < |\Delta m_{\nu_{31}}^2|_{\text{atmos}} \simeq |\Delta m_{\nu_{32}}^2|_{\text{atmos}}$.

Neutrino time delays from different distant sources

Neutrino source	Distance (d)	Energy (E_ν)	Time delay ($\delta t_{\nu-c}$)
Atmosphere (upper atmosphere)	$d_{\rm atmos} \sim 10^7$m	~ GeV	$\delta t_{\nu-c} \leq \sim$fs
Sun	$d_{\rm solar} \sim 10^{11}$m	~ MeV	$\delta t_{\nu-c} \leq \sim\mu$s
Supernovae (SN1987A)	$d_{\rm snovae} \sim 10^{21}$m	~ MeV	$\delta t_{\nu-c} \geq \sim$h

where the new physics seems to have manifested itself in the form of the mass scales $M_s^{\rm solar}$ and $M_s^{\rm atmos}$ characterizing the neutrino masses behind their oscillatory behavior (15). At this stage, according to the oscillation experimental bounds (16), (17), with the known SM data $\langle h\rangle \sim 10^2$ GeV for the Higgs vev and the coupling constants taken $\Delta y^2_{\nu_{ji}} \sim y^2_{\nu_j} : 10^{-2} < y_\nu < 1$ accounting for the neutrino family hierarchy, we approach the scales of the underlying mass parameters,

$$M_s^{\rm solar} \simeq M_s^{\rm atmos} = M_s \sim 10^{\frac{4n+15}{2n+1}}\ \text{GeV}, \tag{21}$$

which, roughly, appear to have the same order of magnitude. Then, according to the extreme values of the integer number n, we finally explore the possible range of the new physics scale,

$$\begin{aligned} M_s^{\rm low} &= M_s(n)\big|_{n\to+\infty} \simeq 10^2\ \text{GeV}, \\ M_s^{\rm high} &= M_s(n)\big|_{n=0} \simeq 10^{15}\ \text{GeV}, \end{aligned} \tag{22}$$

starting from the already accessible electroweak energy scale $M_s^{\rm low} \sim M_{\rm EW} \sim 10^2$ GeV in accelerators where the electroweak and strong forces have very different strengths, up to the scale $M_s^{\rm high} \sim M_{\rm GUT} \sim 10^{15}$ GeV at which their strengths become the same.

5. SM AND ASTROPHYSICAL CONSTRAINTS

The existence of the higher mass scale M_s behind neutrino oscillations (16) is likely expected to manifest itself in the neutrino sector at low energy. This could be considered in the neutrino propagation. Indeed, the dispersion relation for the propagating neutrino with the mass (5) reads,[3]

$$E_\nu^2 - p_\nu^2 \simeq y_\nu^2 \frac{\langle h\rangle^{4n+4}}{M_s^{4n+2}}. \tag{23}$$

[3] We use natural units $c = \hbar = 1$.

With the smallness of the neutrino mass compared to its energy $y_\nu\langle h\rangle^{2n+2}/M_s^{2n+1} \ll E_\nu$ and the use of (21), the corresponding effective neutrino velocity is,

$$v_\nu(E_\nu) \simeq 1 - \frac{y_\nu^2\langle h\rangle^{4n+4}}{2E_\nu^2 M_s^{4n+2}} \simeq 1 - \frac{y_\nu^2 \times 10^{-22}(\text{GeV})^2}{2E_\nu^2}, \tag{24}$$

where now $\delta v_{\nu-c}(E_\nu) \simeq y_\nu^2 \times 10^{-22}(\text{GeV})^2/2E_\nu^2$ is the energy-dependent retardation effect, from the speed of light, undergone by the propagating neutrino. This could be then experienced, with respect to a light ray propagating with speed $c = 1$ and emitted by the same source at a distance d from the detector, by the neutrino time delay δt_ν^c,

$$\delta t_{\nu-c}(E_\nu) \simeq \delta v_{\nu-c}(E_\nu) d \simeq d\,\frac{y_\nu^2 \times 10^{-22}(\text{GeV})^2}{2E_\nu^2}. \tag{25}$$

For not enough distant sources, the corresponding time delay values could be less than femto seconds, and an experimental time resolution at least of such order is then required to detect such effects. In order to probe such conjectured NP effects, which must be distinguished from the effects of conventional media, there is a premium on distant pulsed sources that emit neutrinos at the highest available energy [11–13]. The neutrinos with their low interaction cross sections may then provide the best prospects for the highest-energy quanta from the largest distances. We display in the following table the different distant neutrino sources and the corresponding time delays.

Thus, it is clear that, Core collapse supernovae are formidable sources of neutrinos as almost the total energy of the explosion is carried away by a burst of neutrinos [18–22]. The large numbers of neutrinos produced by the nuclear processes in stellar cores are of energies MeV. The feeble interaction of neutrinos with matter ensures that they exit the core and star with near 100% transmission. This makes neutrinos a unique probe of stellar astrophysics. The detection of neutrinos from SN1987A has proven to be among the most fruitful experiments in the heavenly laboratory to constrain the possible NP effect. In principle crossing data from sources of neutrino and gamma rays can allow to check time coincidence or delay, due to the huge distance of the source of the neutrinos, which is in the Large Magellanic Cloud at about $d_{\text{snovae}} \sim 10^{21}$ m from Earth, offering an opportunity to observe neutrinos over a baseline that is roughly 10^{10} times longer than that traveled by solar neutrinos $d_{\text{solar}} \sim 10^{11}$ m. With such far high energy neutrino emiting source, the handful of events recorded provide then a powerful tool to bound scenarios of the high scale $\sim M_s$. In particular, the resulting time delay might be largely amplified $\delta t_{v-c} \geq \sim$hour due to the corresponding long intergalactic path $d_{\text{snovae}} \sim 10^{21}$ m traveled by the energetic neutrinos.

6. CONCLUSIONS

Neutrinos play an important role in probing new physics and constraining the possible high scales beyond the SM. These particles that complement the efforts of laboratory experiments, will remain an active research field in the next years and provide information about the hidden shape of nature.

In this paper, we were interested in the connection between neutrino oscillation and new physics scale. We have based on the standard description of three active neutrino species and their effective masses beyond the SM. We have then described in detail how the evolution of neutrino states is generated by their masses which have been confirmed experimentally, and how this behavior leaves an imprint in the existence of an underlying high mass M_s parameter above the electroweak scale. In particular, altough the experimental results do not fix the absolute neutrino mass scale, we saw how the use of flavour neutrino oscillation mechanics with the analysis of the experimental oscillation data can provide a positive indication on the new mass scale and an upper bound close to the GUT one $M_s^{\text{high}} \sim M_{\text{GUT}} \sim 10^{15}$ GeV, corresponding to neutrino masses $m_{\upsilon_j} \sim 10^{-3}$ eV, in the most probable mass range as indicated by oscillation data and other laboratory results.

To probe the high scale M_s, we have investigated its possible effect at low energy in the neutrino dynamic. In particular, we have considered the corresponding neutrino propagation velocity and discussed high scale physics manifestation through the time delay $\delta t_{v-c}(E_v)$ with respect to the speed of light which requires long baseline experiments. For that, we have investigated the neutrino time delays from the known distant neutrino sources, i.e., atmospheric, solar and supernovae, where we have shown that the observation of such effect becomes significant for distant high energy neutrino astrophysical sources such as the famous SN1987A event where during the long intergalactical path $d_{\text{snovae}} \sim 10^{21}$ m the accumulated time delay $\delta t_{v-c} \geq \sim$hours becomes significant.

Though neutrinos have occupied much of our reflection about physics at the GUT scale, our knowledge of this sector is still imprecise and incomplete. We still believe that these elusive particles could play a crucial role in many areas of physics, from particle physics, at very short distances to astrophysics and cosmology, and hopefully, in Quantum Gravity effects or the existence of extra dimensions which have enriched dramatically our perspectives in searching for physics beyond the SM [23, 24]. One way to judge this long-held belief is to build powerful particle accelerators and reach the energy scale directly. Future measurements are likely to bring more surprises.

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REFERENCES

1. C. Quigg, *Gauge Theories of the Strong, Weak, and Electromagnetic Interactions* (Benjamin- Cummings, New York, 1983).
2. T.-P. Cheng and L.-F. Li, *Gauge Theories of Elementary Particle Physics* (Oxford Univ. Press, 1984).
3. G. Bertone, D. Hooper, and J. Silk, Phys. Rep. **405**, 279 (2005).
4. S. Dimopoulos, S. Raby, and F. Wilczek, Phys. Rev. D **24**, 1681 (1981).
5. L. Camilleri, E. Lisi, and J. F. Wilkerson, Annu. Rev. Nucl. Part. Sci. **58**, 343 (2008).
6. H. Fritzsch and Z.-Z. Xing, Prog. Part. Nucl. Phys. **45**, 1 (2000).
7. S. E. Ennadifi, A. Belhaj, and E. H. Saidi, J. Phys. G **38**, 115005 (2011).
8. G. Amelino-Camelia et al., Nature **393**, 763 (1998).
9. F. Halzen and D. Hooper, Rep. Prog. Phys. **65**, 1025 (2002).
10. J. F. Beacom, Annu. Rev. Nucl. Part. Sci. **60**, 439 (2010).
11. S. Fukuda et al. (Super-Kamiokande Collab.), Phys. Rev. Lett. **86**, 5651 (2001).
12. Q. R. Ahmad et al. (SNO Collab.), Phys. Rev. Lett. **89**, 011301 (2002).
13. K. Eguchi et al. (KamLAND Collab.), Phys. Rev. Lett. **90**, 021802 (2003).
14. M. Altmann et al. (GNO Collab.), Phys. Lett. B **490**, 16 (2000).

15. W. M. Allison et al. (Soudan 2 Collab.), Phys. Lett. B **449**, 137 (1999).
16. J. N. Abdurashitov et al. (SAGE Collab.), Phys. Rev. C **60**, 055801 (1999).
17. S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979).
18. F. Halzen and D. Hooper, Rep. Prog. Phys. **65**, 1025 (2002).
19. J. F. Beacom, Annu. Rev. Nucl. Part. Sci. **60**, 439 (2010).
20. K. Hirata et al. (Kamiokande-II Collab.), Phys. Rev. Lett. **58**, 1490 (1987).
21. R. M. Bionta et al. (IMB Collab.), Phys. Rev. Lett. **58**, 1494 (1987).
22. E. N. Alekseev, L. N. Alekseeva, V. I. Volchenko, and I. V. Krivosheina, JETP Lett. **45**, 589 (1987).
23. S. E. Ennadifi, Phys. At. Nucl. **76**, 662 (2013).
24. N. Arkani-Hamed et al., Phys. Rev. D **65**, 024032 (2001).