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PHYSICS OF EARTH, ATMOSPHERE, AND HYDROSPHERE

Estimation of Water-Surface Deformation by Vortices in a Viscous Horizontally Decelerating Air Flow

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Abstract—The maximum height of a disturbance on the water surface, which occurs when a spatial (non-point) vortex departs from a viscous layer of a horizontal air flow that decelerates in the direction of motion, is estimated. The dependence of the initial disturbance parameters on the air flow speed over the water surface is derived. It is shown that the height of the disturbance increases and its width decreases with an increase in the air flow speed. The suggested model is verified experimentally.

Keywords: generation of wind waves, deformation of the water surface by vortices, vortices in a shift flow, a viscous layer of an air flow near the water surface.

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INTRODUCTION

T. Stanton was the first person who studied the process of wind wave excitation experimentally [1] and showed that stable waves 5–6 cm long occur at the wind speed $u \geq 300$ m/s. Phillips [2] and Kononkova [3] assumed that initial waves on the water surface are manifestations of vertical pulses of a turbulent air flow. The conditions for excitation of low-amplitude capillary waves by a weak point vortex located in air over the interface between two ideal fluids were derived in [4]. Conditions for excitation of stable waves on the fluid surface by a horizontal air flow decelerating in the motion direction were found in [5] for the first time: stable waves occur when the period $T_{\rm ed}$ of the departure of a chain of vortices that are formed in a viscous air flow coincides with the free oscillation period T defined by the dispersion equation for a group of waves:

$$s \approx \lambda, \quad T_{\rm ed} = T = \frac{s}{U}, \tag{1}$$

where s is the space between vortices, λ is the wavelength, and U is the wave group velocity. According to [6], vortices with the angular velocity $\omega = \chi/2$ are formed in the airflow decelerating zone (the vertical speed profile $u(y) = u_s + \chi y$, where $\chi = 4u_s/\delta$, u_s is the airflow speed at the lower boundary of the viscous layer). The period of departure of the vortices and the space between them are defined by the semi-empirical relations [6]:

$$s = T_s u_s - \frac{\delta}{2C_f}\ln\left(1 + u_s\frac{2C_f}{5|u_x|\delta}\right), \tag{2}$$

$$T_{\rm ed} = \left(\frac{2\delta}{5u_s|u_x|C_f}\right)^{1/2}\arctan\left(\sqrt{\frac{2u_sC_f}{5|u_x|\delta}}\right), \tag{3}$$

where C_f is the air sliding coefficient on the water surface (dimensionless, $C_f = 0.01$ for 20°C). According to experimental data [6], if the flow speed $u > 1$ m/s beyond the boundary layer, then $u_s = u/10$, and the viscous layer thickness $\delta = 0.05$ cm. The viscous layer and interface surface parameters χ, δ, and C_f are found experimentally.

The dependences of the wavelengths of stable waves on the airflow speed over a clean water surface and a surface covered with a film of light oil were derived in [5] and verified in a laboratory experiment. However, the authors did not estimate the height of initial disturbances on an initially smooth water surface, which occur when spatial vortices depart from the viscous air layer under the action of the generalized Joukowski force. The action of weak point vortices on the water surface was studied in [4] for the case where the disturbance height is low compared to the vortex–interface space. Vortices formed in the viscous layer of descending air are finite in size, with the diameter $a \approx \frac{2}{3}\delta$; the water surface disturbance height can exceed the thickness of the viscous layer [6]. The impact of these vortices on the water surface should be studied experimentally, since even approximated estimates of the interface deformation by high-power spa-

The parameters of the water-surface disturbance versus the wind speed

U, cm s^{-1}	λ_0, cm	L, cm	h_{max}, cm	h_{exp}, cm
300	3.8	1.3	0.20	0.21
350	2.5	1.1	0.23	0.22
400	2.0	0.9	0.26	0.27
500	1.4	0.7	0.29	0.30
600	1.2	0.5	0.32	0.31

tial (non-point) vortices that depart from a viscous air layer have not been carried out thus far.

1. TECHNIQUE AND EQUIPMENT

To solve the stated problem, we studied wind-wave generation in a straight channel 3.5 m long and 20 cm wide with transparent walls on a water layer with a 30–cm depth experimentally. An air flow was produced by a fan and input into the channel through a grid of straight smooth pipes 1 cm diameter. The wind speed was measured by the anemometers described in [6]. The wave parameters were estimated by a video record made through the side wall of the channel. Waves of approximately several centimeters in length were formed on the surface of a water layer with a depth of h = 30 cm; the deep-water condition $\lambda \ll h$ was fulfilled.

2. EXPERIMENTAL STUDY OF DEFORMATION OF THE WATER SURFACE

Figure 1a shows the water surface deformation by a wind flow at the beginning of the acceleration. The first steep narrow crests appear on the water surface in the segment $19 < x < 23$ cm (x) is the longitudinal coordinate, with the origin on the water surface at the channel entrance); the intercrest space λ = 1.8 cm. The crests have such a form only at the instant of vortex departure, which is detected by shifts of light foam plastic particles caught by the vortex from the water surface (Fig. 1b). Downstream ($x > 24$ cm, Fig. 1a) the disturbance amplitude quickly drops; low-amplitude harmonic waves with a length close to the intercrest

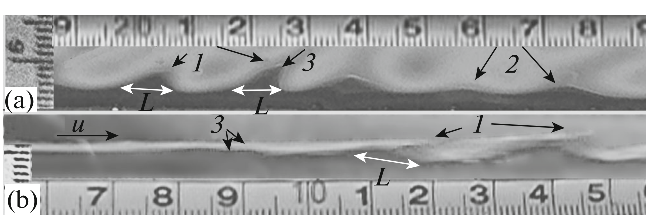

Fig. 1. The water-surface deformation by a horizontal air flow in the zone of generation: (a) wind speed u = 415 and (b) 330 cm/s.

space at the upper segment remain on the surface. The mean wind speed in the upper segment, where short steep crests are observed, u = 415 cm/s (u is the airflow speed in the homogeneous part of the vertical wind profile); it decreases with the longitude coordinate: $u_x \approx \frac{\partial u}{\partial x} \approx -4.8\,\mathrm{s}^{-1}$. In this region vortices are generated, with the space between them s = 1.8 cm according to Eqs. (2) and (3). Lower, in the segment $s = \lambda$. The calculation by Eqs. (2) and (3) gives s = 6.3 cm, which exceeds the length of this segment. This explains the reason that no vortices depart and no steep crests are observed at x = 24 cm. Finding the area of the longitudinal vertical cross section of the first crest in the upper segment of L in length and h_{max} in height, one can derive S_{per} = 0.12 cm^2. The cross section area of the second crest is almost two times larger, which can be explained by the serial action of two vortices under fulfillment of the stable wave generation condition. Experiments have shown that the cross section area of the first crest S_{per} and its width L near the base of the initial disturbance increase with the wind speed and the maximal height h_{max} decreases. Figure 1b shows the water surface in the region of wave generation by wind (u = 330/cm/s, u_x = –2 s^{-1}) over the first and second crests at the instant when vortices depart: condition for stable wave generation (2) is fulfilled; the wave amplitude increases. The dependence of the parameters of initial disturbances on the wind speed derived in our experiments is given in the table.

3. ASSESSMENT OF THE VORTEX DISTURBANCE HEIGHT OF THE WATER SURFACE

The experiments in [7] showed the generation of three vortices in a viscous layer of decelerating fluid flows, i.e., primary (near the upper boundary of the viscous layer), with a radius that is three times smaller then the viscous layer thickness, due to the inverse pressure gradient at the upper boundary of the viscous layer and the friction at the lower boundary, and two secondary vortices, between the primary vortex and the interface, due to viscosity forces. The primary and secondary vortices rotate in opposite directions. Densely packed vortices overlap the viscous layer, which results in a sharp decrease in the speed of the background flow before a vortex and disappearance of the vertical speed shift, which causes the absence of the shift of vortices observed in experiments [6, 8]. However, interaction of vortices with each other and with the interface surface results in a change in this structure. Secondary vortices rotate about the primary one and gradually twist it [7, 9]. As a result, the background flow is restored, which promotes to the departure of the primary vortex upward from the viscous layer. It was shown experimentally in [7] that the primary vortex motion with the satellite vortices in a viscous layer can be described by the model of cylindrical vortex motion in a shift flow of an ideal fluid [10], if the Reynolds number of the vortex $\mathrm{Re} = \gamma/\nu \approx \frac{4u_x\delta}{3\nu} > 10$ ($\gamma = 2\omega\pi a^2$, ν is the kinematic viscosity). The air vortex Reynolds number Re = 13 ($\nu = 0.14$ cm^2/s) for the minimum wind speed under which stable waves are generated. This allows the use of results from [10] for the calculation of the vortex acceleration at the initial time point

$$\ddot{\eta}_{t=0} = \left(\chi - \frac{\gamma}{2\pi a^2}\right)(u_x + 2a\chi), \quad (4)$$

where η is the vertical coordinate of the vortex center. One may assume that background flow acceleration near the interface surface $\dot{u}_s$ at the instant of vortex departure is close in modulus to the acceleration of the vortex.

To roughly estimate the deviation of the water surface $h(t, x)$ (t is the time) under the action of the negative pressure jump during the vortex departure, we assume that a quiescent air layer of δ in thickness starts moving from the water interface under the action of acceleration (4). The fluids are considered ideal. The drift of the water surface is ignored, since the region under study is at the beginning of the acceleration, where the drift has not been formed yet. Let us write the pressure balance condition at the interface surface:

$$p_a = p + T_s K, \quad K = h_{xx}(1 + h_x^2)^{-\frac{3}{2}}, \quad h_x = \frac{\partial h}{\partial x}, \quad (5)$$

where p_a and p are the pressures at the interface in the air and in the water, respectively; T_s is the surface tension coefficient, and K is the surface curvature. The background flow motion is potential in this case and the pressure at the intermediate level can be written as

$$p_a = -\rho_a\left[\varphi_t + gy + \frac{1}{2}(\nabla\varphi)^2\right], \quad p = -\rho g y, \quad (6)$$

where ρ_a and ρ are the air and water densities, respectively; g is the acceleration due to gravity; $\varphi = u_s x$ is the speed potential in air written for the frozen water surface at a segment of $x = L$ in length exposed to the action of the negative pressure jump; y is the vertical coordinate (with the origin on the water surface). To calculate the pressure by Eq. (5), we should find the parameter $\varphi_t = \dot{u}_s L$. Assuming $\ddot{\eta} = \dot{u}_s$, we derive φ_t

$$\varphi_t = \dot{u}_s L = 7u_s^2 \frac{L}{\delta}. \quad (7)$$

Assessment of the terms in Eq. (6) shows that the first term in square brackets, caused by the spatial vortex acceleration, is an order of magnitude higher than the others. Neglecting squares of small magnitudes, we derive the approximate estimate of the maximum water surface height under the action of the negative pressure jump:

$$h_{\max} \approx \frac{\rho_a}{\rho}\frac{7u_s^2}{g}\frac{L}{\delta}. \quad (8)$$

The maximum height of the initial disturbance calculated versus the wind speed is given in the table along with the experimental data. The good agreement between the calculation results and experimental data within the confidence interval (for a probability of 0.67), which does not exceed 10% of the measured value, proves the admissibility of the simplifications that were made for the experimental conditions.

CONCLUSIONS

A water-surface disturbance was assessed under a decrease in the pressure when a spatial vortex departs from a viscous layer of a horizontal air flow moving over the water surface with its speed decreasing in the direction of motion. The calculation results are in a good agreement with the experimental data.

It is shown that the departure of a vortex from the viscous air layer provides the main contribution to the pressure jump near the water surface under the vortex, which deforms the water surface.

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