ISSN 0027-1349, Moscow University Physics Bulletin, 2018, Vol. 73, No. 4, pp. 351–363. © Allerton Press, Inc., 2018.
Original Russian Text © A.V. Belinsky, 2018, published in Vestnik Moskovskogo Universiteta, Seriya 3: Fizika, Astronomiya, 2018, No. 4, pp. 12–25.

Is David Bohm's Quantum Mechanics Interpretation Irrefutable?

A. V. Belinsky*

Department of Physics, Moscow State University, Moscow, 119991 Russia

**e-mail: belinsky@inbox.ru*

Received October 12, 2017; in final form, January 30, 2018

Abstract—The theoretical bases of the so-called "pilot-wave" concept interpretation of quantum mechanics, as well as the performed and possible experiments to test its adequacy are considered.

Keywords: orthodox interpretation, hidden parameters, determinism, quantum superposition, quantum statistics.

DOI: 10.3103/S0027134918040033

INTRODUCTION

It is traditionally considered that the D. Bohm "pilot wave" concept is fundamentally irrefutable, since it is based on the Schrödinger equation and its results completely correspond to results of experiments. In addition, it was enthusiastically adopted by such a giant of "quantum thought" as John Bell [1]:

> But why then had Born not told me of this "pilot wave"? If only to point out what was wrong with it? Why did von Neumann not consider it? More extraordinarily, why did people go on producing "impossibility" proofs, after 1952, and as recently as 1978? When even Pauli, Rosenfeld, and Heisenberg could produce no more devastating criticism of Bohm's version than to brand it "metaphysical" and "ideological"? Why is the pilot-wave picture ignored in text books? Should it not be taught, not as the only way, but as an antidote to the prevailing complacency? To show that vagueness, subjectivity, and indeterminism are not forced on us by experimental facts, but by deliberate theoretical choice?

Thus, the interpretation clearly deserves attention, although certain doubts about its irrefutability still remain. However, in the beginning, we consider the basic prerequisites and propositions of this concept.

1. THE SCHRÖDINGER EQUATION AND BORN'S PROBABILISTIC INTERPRETATION

In 1926, Erwin Schrödinger, while elaborating the Louis de Broglie idea about the occurrence of wave properties of quantum particles [2], formulated the wave equation [3] for the description of the quantum-mechanical system:

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\Delta + V\right)\psi, \qquad (1)$$

where ψ is the wave function, $\hbar$ is the Planck constant, and $V(x, t)$ is a potential field that acts on a particle with mass m.

> ...Recognizing that [this] equation has the structure of a diffusion equation with an imaginary diffusion coefficient, Schrödinger relaxed his original requirement concerning the reality of ψ and admitted complex-valued functions for what he called the "mechanical field scalar ψ."...Schrödinger concluded his paper with a discussion on the physical significance of ψ. He interpreted $\psi\psi^*$ as a weight function in configuration space that accounts for the electrodynamical fluctuations of the electric space density of the charges. He declared: "The ψ function has to do no more and no less than to offer us a survey and mastery over these fluctuations by a single differential equation. It has repeatedly been pointed out that the ψ function itself cannot and may not in general be interpreted directly in terms of three-dimensional space [4].

The famous probabilistic interpretation of Max Born, that brought him the Nobel Prize, was published in the summer of 1926.

> For Born probability, as far as it was related to the wave function, was not merely a mathematical fiction but something endowed with physical reality, for it evolved in time and propagated in space in accordance with Schrödinger's equation. It differed, however, from ordinary physical agents in one fundamental aspect: it did not transmit energy or momentum. Since in classical physics, whether Newtonian mechanics or Maxwellian electrodynamics, only an entity that transfers energy or momentum (or both) is regarded as physically "real," the ontological status of ψ had to be considered as something intermediate. ... Laws of nature, as Born and Heisenberg contended from now on, determined not the occurrence of an event, but the probability of the occurrence.
>
> ... Having interpreted ψ as a probability wave in the sense just explained but realizing that ψ can be expanded in terms of a complete orthonormal set of eigenfunctions ... Born had to ask himself what

> meaning to ascribe to the c_n [coefficients of this expansion]? ... [it] suggested to Born that the integral $\int|\psi(q)|^2dq$ has to be regarded as the number of particles and $|c_n|^2$ as the statistical frequency of the occurrence of the state characterized by the index n. To justify this assumption Born calculated ... the expectation value of the energy ... and obtained ... [for it] the [correct] energy eigenvalue [4].

2. THE BOHM MODEL

In 1952, David Bohm published two papers [5, 6], in which he proposed a nontrivial approach to further development of quantum mechanics. From a purely formal point of view, his proposition was reduced to transfer from a single equation for the *complex* wave function to two equations for two *real* quantities, which are the amplitude $R(\mathbf{x}, t)$ and phase $S(\mathbf{x}, t)$ of the wave function. We denote

$$\psi = R\exp(iS/\hbar). \quad (2)$$

In the case of a single quantum particle, using this expression and the Schrödinger equation, denoting $\rho = R^2$, one can obtain

$$\frac{\partial\rho}{\partial t} + \nabla\left(\rho\frac{\nabla S}{m}\right) = 0, \quad (3)$$

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0, \quad (4)$$

where

$$Q = -\frac{\hbar^2}{2m}\frac{\Delta R}{R}$$

is the so-called quantum potential.

In classical mechanics, S is interpreted as action, with its time derivative $\partial S/\partial t$ as energy, and $\nabla S/m$ as velocity. The obtained relationships can be considered as the continuity equation and the energy-balance equation, however, in this last expression, a fundamentally new term, that is, the *quantum potential Q*, occurs.

In the case of N particles, one can introduce a wave function

$$\psi = R(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N, t)\exp[iS(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N, t)/\hbar)] \quad (5)$$

and determine a $3N$-dimensional trajectory in configuration space, which describes the behavior of each particle in the system. The velocity of the ith particle is

$$v_i = \nabla_i S(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N, t)/m. \quad (6)$$

Similarly to the case with a single particle, the quantum potential is defined using R

$$U(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N) = -\frac{\hbar^2}{2mR}\sum_{k=1}^{N}\Delta_k R(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N), \quad (7)$$

with $\rho(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N) = R^2$ equal to the density of representing points $(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N)$ in our $3N$-dimensional ensemble.

We note again that the occurrence of the quantum potential distinguishes the quantum description from a classical one, where there is no analog of this term. The quantum potential generally provides the so-called entanglement between particles, i.e., the fact that separate trajectories, which have physical meaning in the Bohm interpretation, are not independent from each other and not described by separate independent wave functions. It is very important that the quantum potential in configuration space is considered to *instantaneously* vary when there is a change of the wave function; this mechanism is responsible for the nonlocal correlations that are so characteristic of quantum mechanics. These instantaneous changes can rather be perceived by a human as superluminal information exchange [7].

The quantum potential nonlocality thesis is usually not highlighted anywhere. Indirect considerations are only made, which are based on the fact that using it, the coordinates of one of the particles of the quantum system are found to be dependent on the coordinates of all the other particles of the system; thus, not only does the wave function control the motion of a particle, but the particle affects the wave function of the system as well [8]. However, it logically does not follow from this that nonlocal correlations are instantaneously transferred, but not, say, at the speed of light in a vacuum. Nonetheless, it seems that the violation of the Bell inequalities registered between very distant observers [9] proves both.

On the other hand, the nonlocality of quantum correlations (irrespective of the Bohm model) is a universally recognized fact (for example, [10–13]), which, nevertheless, is paradoxical. In 2017, an attempt was made to explain this paradox by an effect of the relativity theory, which is nearly identical to the known twin paradox [14, 15].

3. THE DUALISM FORMULATION IN THE BOHM MODEL

We focus on two possible pictures of the "wave–particle" dualism. There may be two approaches [16]:

"*Wave OR particle*": Heisenberg, Pauli, Dirac, Jordan and many others considered that depending on the experimental situation one has to choose one description of the behavior of a quantum system. Electrons are associated with probability amplitudes. The corpuscular nature of the electron occurs when we measure its coordinate. In Bohr's words, an object

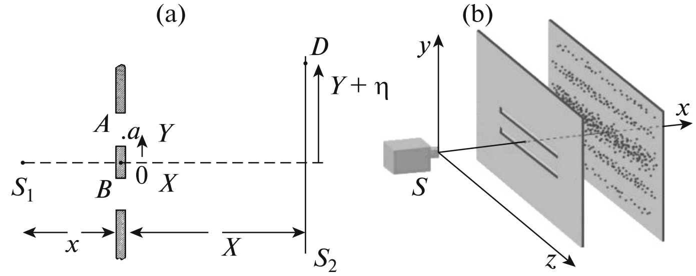

Fig. 1. The configuration and results of an interference experiment with two slits [18, 19].

cannot be both at the same time (this is known as the orthodox interpretation of quantum mechanics and the complementarity principle)[1].

"*Wave AND particle*": de Broglie, followed by Bohm, considered that the wave and particle concepts merge at the level of the atomic scale, where a "pilot wave" guides the electron trajectory. There are two objects at once but not either one of them.

The difference between these two approaches can be easily seen from the interpretation of the double-slit experiment, in which a beam of electrons with low intensity (such that electrons are injected one by one) is directed upon an opaque surface with two slits in it (see Fig. 1). Discrete traces of hits of electrons are registered on the detecting screen from the other side of the surface. Even if we consider that traces on the screen correspond to particles; they group into the interference fringes, which are characteristic of waves. Thus, both wave (interference) and corpuscular (dots on the screen) behavior is reproduced.

According to the de Broglie–Bohm "wave AND particle" approach, the wave function (whose modulus gives the electron probability density of being at a certain coordinate, regardless of the measurement process) passes through BOTH slits. At the same time, a well-defined trajectory is associated with the electron. However, this trajectory passes through only ONE of the slits. The final position of the particle on the detecting screen and the slit through which the particle passes are determined by the initial state of the particle. Such an initial state is not controlled by the experimentalist; thus, there is an effect of randomness of the detected pattern. The wave function guides the particle in such a way that it rarefies traces of particles in the region where the interference is destructive and condenses them in the region where the interference is constructive, giving rise to the interference fringes on the detecting screen. In this regard, Bell wrote [1]: "This idea seems to me so natural and simple, to resolve the wave-particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored."

The Young double-slit experiment long served as a decisive experiment for the interpretation of the "wave–particle" dualism. This simple experiment possesses two properties of a quantum phenomenon: a wave nature at the macroscopic level, related to the interference phenomenon of the wave function, and a corpuscular nature at the microscopic level, related to traces of collisions on the screen. The double-slit interference experiments were performed with such massive objects as electrons, neutrons, cold neutrons, atoms, and, recently, also with coherent ensembles of ultracold atoms and mesoscopic single quantum objects.

A correct numerical simulation of the double-slit experiment on the basis of the Bohm interpretation was successfully carried out in [18] for the first time, and then again in [19]. The quantum potential was calculated there for a typical double-slit configuration, which includes a source of electrons S_1, two slits A and B, and screen S_2. The centers of the slits have coordinates $(0, Y)$ and $(0, -Y)$ in the frame with origin at O, shown in Fig. 1a.

The slits are sufficiently long (infinite) along the z axis (normal to the figure plane); thus, there is no diffraction effect along this axis. Therefore, only a wave function along the y axis was considered under the simulation; the variable x was classically interpreted as $x = vt$. Electrons emitted by an electron gun were represented by the same initial wave function

$$\psi^0(y) = (2\pi\sigma_0^2)^{-1/4}\exp(-y^2/4\sigma_0^2) \qquad (8)$$

with standard deviation $\sigma_0 = 3$ μm. The continuum Feynman path integrals method allowed one to calculate the time dependent wave function. The wave function *before the slits* is:

$$\psi(y,t) = [2\pi s_0^2(t)]^{-1/4}\exp[-(y - vt)^2/4\sigma_0 s_0(t)], \qquad (9)$$

where

$$s_0(t) = \sigma_0(1 + i\hbar t/2m\sigma_0^2). \qquad (10)$$

[1] The in-depth theoretical analysis of the complementarity relationship leads to the inequality $V^2 + D^2 \le 1$, which puts an upper bound on the maximum values of simultaneously determined parameters of the interference visibility V and path distinguishability D [17]. Obviously, the $V = 1$, $D = 0$ and $V = 0$, $D = 1$ cases are the limiting ones.

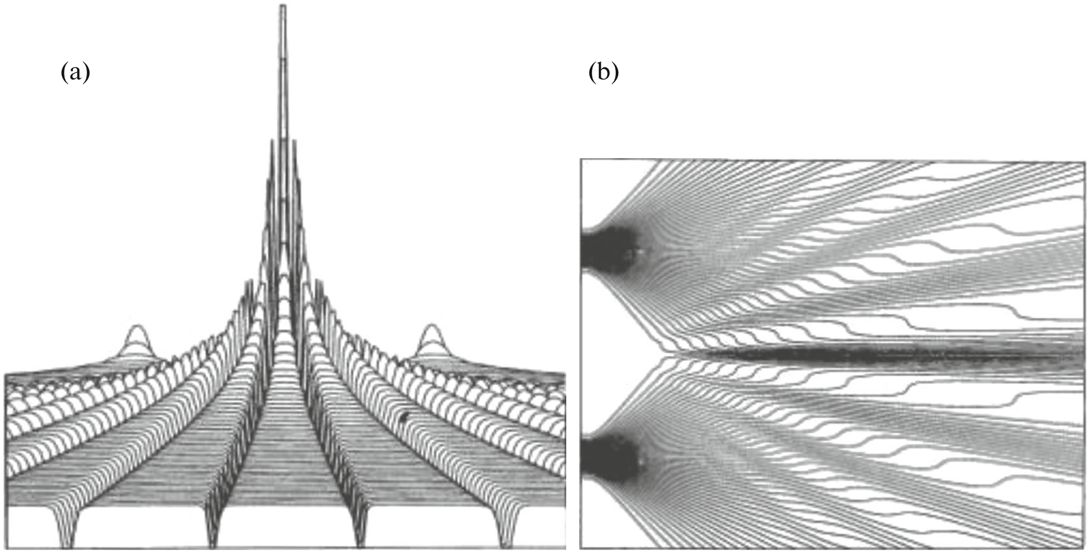

Fig. 2. (a) The quantum potential for two Gaussian slits with respect to S_2 [18]). (b) The ensemble of trajectories that traverse through two Gaussian slits [18].

After the slits, the wave function is the sum of wave functions *A* and *B*:

$$\psi(y,t) = \psi_A(y,t) + \psi_B(y,t), \tag{11}$$

where

$$\psi_A(y,t) = \int_A [m/2i\hbar t_1]^2 \exp[(im(y-y_a)^2/2\hbar(t-t_1)]\psi(y_a,t_1)dy_a \tag{12}$$

and

$$\psi_B(y,t) = \int_B [m/2i\hbar t_1]^2 \exp[(im(y-y_b)^2/2\hbar(t-t_1)]\psi(y_b,t_1)dy_b. \tag{13}$$

Probability amplitudes $\psi_A(y, t)$ and $\psi_B(y, t)$ were found by integration over all coordinates of the slits. Calculation of the wave function was carried out using the continuum path integrals method, via which the quantum potential from the previously given expression (4) was obtained, see Figs. 2a and 2b.

Trajectories were calculated by integration over time of $\nabla S = mv$, which links the S function with the velocity of particles in the usual way. In the beginning, the trajectories diverge from each slit in such a manner that they are compatible with a single Gaussian slit. Sequential breaks of trajectories coincide with "troughs" in the quantum potential. These occur because when a particle falls into the region of the trough it seems to be experiencing a significant effect in the y direction, which rapidly accelerates the particle from the trough to the trough in the region where "force" becomes weak again. As a consequence of this, most of the trajectories are along the plateau and give a bright interference pattern, when troughs coincide with dark fringes.

It is interesting that according to Fig. 2b, the Bohm trajectories cannot intersect, since the velocity field is single valued with respect to the x axis, otherwise, what determinacy could one ever discuss? However, then, it appears that particles that passed through the upper slit can never be found below the plane $y = 0$, and vice versa. However, it is clear that due to diffraction of the slit quite feasible trajectories exist. The problem is serious, since it calls into question the entire logic of David Bohm. Works [8, 20] were devoted to discussion of it.

4. "SURREALISTIC" BOHM TRAJECTORIES

An interesting situation occurs if close to each slit we place a "yes/no" one-bit detector, which indicates whether the particle traversed through a given slit or not [20]. These detectors give us information of a "which-way" type and allow one to distinguish two classes of trajectories: those that traverse through one slit and those that traverse through the other one. At the same time, of course, the interference pattern on the screen is lost (see Fig. 3).

When these "yes/no" one-bit detectors are used, the contribution of the upper slit is now *correlated* with the "which-way" information registered by the upper detector, and similarly, with respect to the lower slit and lower detector.

However, through which slit did the particle traverse? We assume that the upper detector gave "yes" and the lower one produced "no." Then, the probability to find a trace of the particle on the screen does not completely vanish on the lower half of the screen, although none of the possible trajectories can intersect in view of single-valuedness of the velocity field. Consequently, the case is possible where the particle *traversed* through the upper detector and, therefore, through the *upper* slit, and then fell into the lower half of the screen; thus, the corresponding "Bohm trajectory traverses through the lower slit." In other words, when "yes/no" one-bit detectors are used, the Bohm trajectories of the particle can be characterized by contradictory behavior: they can start at one slit, whereas readings of the detectors indicate that the particle traversed through the other slit. In short, the Bohm trajectories are now not realistic; they are "surrealistic."

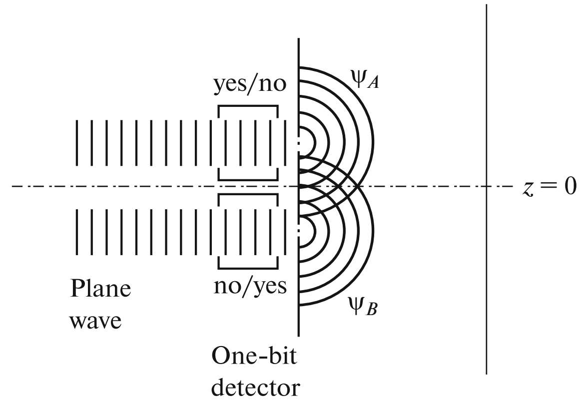

Fig. 3. A double-slit interferometer with "yes/no" one-bit detectors: a collimated plane wave falls onto the detecting screen through the slits. The interference on the screen disappears due to which-way information for particles. However, could the trajectories intersect?

The experimental work [8], in which the theoretical analysis of the situation was given as well, was performed in accordance with this theoretical model. The experiment was carried out with two photons entangled by trajectories using the "weak" measurement method (for example, [21]). It was shown that the trajectories of the first particle (its position and velocity) are in fact coupled with the behavior of the second particle, and at distance, i.e., *nonlocally*. This is certainly not new, but gives confidence in the nonlocality of quantum correlations and the Bohm potential Q in (4).

It is very simple to determine which slit the particle traversed: in the case of photons, mutually orthogonal polarizers can be put before the slits; thus, a state, for example $|H\rangle$, will correspond to traversing the upper slit and $|V\rangle$ will reflect traversing through the lower one. However, if we observe electrons, then these can be two opposite spins. In any case, the states are mutually orthogonal; thus, $\langle H|V\rangle = 0$.

Trajectories of a single photon (particle 1) are measured and post-selected by detecting the second photon, which is entangled with the first one (particle 2). To this end, one of the entangled photons is directed to the scheme of measuring parameters of the trajectory and the second photon is used as a source of the control action, opening or blocking the arrival of the first photon into the measurement scheme.

The reading of a detector of which-way information (which-way measurement, WWM) can be found at the discretion of an experimentalist, at the time when particle 1 is in the near region (Fig. 4a) or middle region (Fig. 4b). Meanwhile, the reading for particle 2 is not registered until particle 1 arrives at the required region. The corresponding slits for the initial Bohm trajectories were indicated in Fig. 4 and the height shows the WWM result (position x_2 of the second, "control" particle, i.e., which slit it traversed). When a WWM reading is registered if particle 1 is at the near region, the Bohm trajectories perfectly correlate with the value of this result. However, if a WWM reading is carried out when particle 1 is in the middle region, the Bohm trajectories correlate with the WWM result only at the edges of the diagram. Near to the device symmetry axis, both WWM results are equally probable irrespective of the slit at which the Bohm trajectory starts. The authors explained that the state of particles as they move away from the screen with the slits changes to a superposition of $|H\rangle$ and $|V\rangle$, which is certainly true; this leads to an ambiguity of determining the slit through which the particle traversed. However, if the polarization state had changed when propagating photons, then their interference due to a gradual "erasing" of the "which-way" information would have observed, similarly to the "quantum eraser" effect, for example [22]. This, certainly, cannot happen since there are no physical reasons or such a change. The superposition occurs only as a result of overlapping the scattering indicatrices of the slits; the result of the polarization measurement unambiguously corresponds to one slit or the other.

Thus; the difficulty in the interpretation of the "surrealistic" trajectories is impossible to avoid.

5. THE BOHM THEORY AND THE PROBLEM OF MEASUREMENT

In [23], von Neumann drew special attention to a fundamental difference between the "eigen" evolution of the quantum system (described by a time-reversible Schrödinger equation) and "reducing" evolution. The latter generally has an irreversible character and occurs when measuring the state of a particle. A measurement procedure leads to collapse of the quantum state, when, as is conventionally thought, superposition of possible states with orthogonal measurements is instantly replaced by one and only one of the states of the superposition, at which the system resides before the measurement. Von Neumann called this measurement "projective," since the initial vector of state in Hilbert space is instantly transformed ("reduced" or "collapsed") into one of its basis components in this

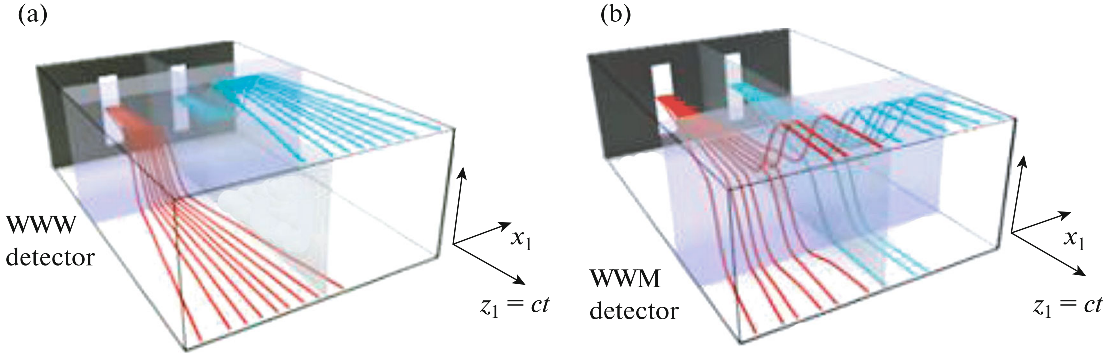

Fig. 4. The Bohm trajectories in the double-slit experiment. The vertical difference of the trajectories shows which slit a particle arrived from. Ambiguity of trajectories occurs in diagram (b), which indicates that either the trajectories still intersect or the correspondence of the |H⟩ and |V⟩ states to their slits is not strict (from [8]).

space. This means that the initial (before measurement) wave function is replaced by one of the *eigen states* of a specific projection operator $\hat{P}$. Unlike the deterministic evolution law of the wave function defined by the Schrödinger equation, the collapse is not deterministic, since the final wave function is randomly chosen among the eigen states of the operator $\hat{P}$. This description appear to be a "trick" [24], and, above all, arbitrarily "isolates" the quantum system under measurement from the rest of the world.

Bohm proposed a completely different approach to the problem of measurement. In its theory, the process of measurement is interpreted as any other process of interaction of particles; thus, the difficulties of the orthodox interpretation described above simply disappear, in particular, there is no need to introduce projection operators. Here, the entire quantum system is described by a trajectory plus a wave function, but not a wave function alone. Both the wave function and trajectory are associated with the system *as a whole*, i.e., with a quantum system under measurement plus the measuring device. During the process of measurement, a term that describes the interaction appeared for this composed system in the Hamiltonian in the general Schrödinger equation; thus, the Schrödinger equation remains valid both during the measurement and after termination of the interaction. This term introduces coupling between the state of the system and the measuring device, thereby enabling one to carry out the measurement. At the same time, changes in the state of the system under measurement and measuring device occur during the measurement, i.e., their degrees of freedom, which are mutually independent before the measurement, are found to be correlated (entangled) with each other after the measurement. As Bohm pointed out, it is important that when the measurement is performed the coupling between the system and device is sufficiently strong, while the interaction itself should continue for a certain minimal interval of time, but not long enough to cause distortions (in [25], Bohm gave too short or too long exposures during photography as an analogy).

A double-slit experiment with a continuous spectrum of possible trajectories may serve as a good example of measurement. According to Bohm's ideas, unlike the orthodox interpretation of the quantum theory, a particle chooses a certain trajectory due to the effect of a nonlocal hidden parameter (a phase of the wave function): one of the possible trajectories in accordance with the Schrödinger equation. During the experiment, the general wave function, which corresponds to a superposition of potentially possible states, is split into wave packets by a number of these trajectories and the distance between the centers of the packets increases; thus, they cease to overlap in the space. Eventually, the Bohm trajectory corresponds to one of the packets, whereas all the other ones are found to be "empty" (Fig. 5; see also [6]).

The splitting of the wave function closely resembles Everett's many worlds interpretation of quantum mechanics [26] with the only difference being that a particle belongs to all packets there, but they are in different "worlds" (for example, [27–29]), whereas all packets but one are empty according to Bohm.

From the modern point of view, the process of measurement and the evolution of vectors of state in it are called decoherence. Both the system M under measurement and the measuring device (or environment) ε participate in this [30].

Before the measurement the system M is characterized by the density matrix that corresponds to this state. The elements that are on the main diagonal of the matrix give the probabilities of obtaining each of the possible (basis) results of the measurement, while the elements that are outside of the main diagonal correspond to correlations (phase relationships) between the basis states. The environment ε is described in a similar way as well.

The degrees of freedom of M and ε are entangled when M interacts with ε (i.e., under measurement).

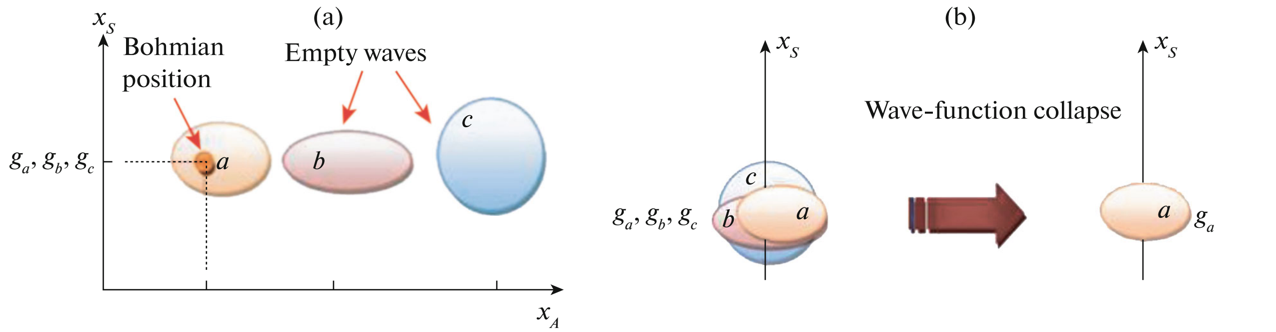

Fig. 5. Diagrams that illustrate the differences of the Bohm and orthodox interpretations (from [16]). (a) the Bohm explanation of measurement in the $[x_S, x_A]$ (system–device) space: from the nonoverlapping wave function, only the g^a part of the wave function where the trajectory is present is needed to compute the evolution of the Bohmian system, (b) the Orthodox explanation of measurement in the $[x_S]$ (system) space: the (system) wave function collapses into the g^a part when the measurement takes place.

The general density matrix that describes them, which could be expanded (factorized) into two factors that separately correspond to M and ε before the measurement, loses this property, since there is a correlation between the degrees of freedom of M and ε, which was not the case before the measurement. In particular, if M and ε originally were in pure states, then, at this stage, each part of the composed system (M and ε) can no longer be described by a separate vector of state, but only by the density matrix.

The final stage is reduced to the decoherence process. This is characterized by destruction of the phase relationships between separate states; the elements of the density matrix outside of the main diagonal (coefficients of correlation) are damped, whereas the elements on the main diagonal (probabilities of the basis states) are not substantially changed. Eventually, the system under measurement changes to a statistical mixture of the possible results of measurements, which can formally correspond to the von Neumann projection operator, in the case where the density matrix is a projector.

6. CRITICISM OF A HYPOTHETICAL SPLITTING OF THE WAVE FUNCTION INTO EMPTY AND NONEMPTY WAVE PACKETS

If we follow the Bohm hypothesis of splitting the wave function into empty and nonempty wave packets, then the particle itself should only be present in one channel in the interference schemes with a two-channel division of particles, otherwise either it will not have a trajectory at all or one has to refer to the Feynman interpretation, according to which each particle simultaneously propagates along all possible trajectories at once [31]. We note that the Bohm interpretation differs greatly from the formalism of the Feynman paths in quantum mechanics, where the transition probability between two points in a phase space is calculated using all possible paths between these two points. In contrast to the Feynman formalism, the Bohm mechanics asserts that each quantum particle follows along trajectory in deterministic way. Therefore, the concepts of classical mechanics are followed in this case to a great extent. Here, along with inspection of "surrealistic" trajectories, we have a golden opportunity to verify the Bohm hypothesis experimentally, at least, in the form of a thought experiment, having calculated the corresponding experimental scheme by a typical quantum-mechanical calculation. In fact, one can propose a strict proof of the residence of a photon in both arms of a Mach–Zehnder interferometer simultaneously [32], i.e., prove that ALL wave packets are not empty and there is no predetermination of the occurrence of the particle in one of them.

In the arms of the interferometer we place two identical nonlinear media that have cubic nonlinearity, in which phase self-modulation (PSM) occurs, that is, a change in the refractive index of the media under the effect of light in them, instead of, or as, phase delays. As an example, these can be quartz fibers Fig. 6. A photon passing through them must acquire an additional phase foray, which will inevitably affect the result of the interference. However, the photon

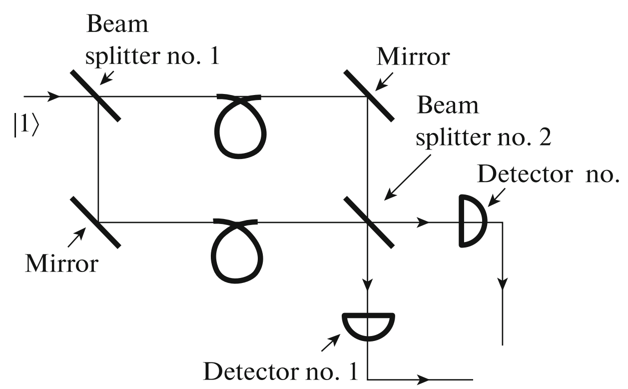

Fig. 6. The scheme of a Mach–Zehnder interferometer with identical nonlinear fibers in channels.

itself must be present at the fiber for the occurrence of this nonlinear phase foray, and not only its empty wave packet, since, in the latter no energy exists that would initiate a nonlinear PSM effect. According to Bohm, all of the energy of the particle is concentrated in the particle itself and the vector of a quantum state only directs it to one side or the other.

Let the phase forays in the arms be the same in the absence of radiation. Then, in sending a single photon to the interferometer, we have two alternatives: either the photon passes through only one arm and the phase difference changes due to the nonlinear phase foray in this arm, or the photon passes through both arms, in which the nonlinear phase forays will be the same; thus, the phase difference does not change. In the latter case, we will only see the appearance of the photon in one of the outputs of the interferometer.

We will describe the input monochromatic mode in the Fock state $|1\rangle$ by the photon annihilation operator $\hat{a}_1$ and the vacuum mode $|0\rangle$ at the second input by the operator $\hat{a}_0$. After the first 50% beam splitter, we also consider two modes described by operators $\hat{a}_2$, $\hat{a}_3$ in the Heisenberg representation:

$$\hat{a}_2 = \frac{\hat{a}_1 + \hat{a}_0}{\sqrt{2}}, \quad \hat{a}_3 = \frac{\hat{a}_1 - \hat{a}_0}{\sqrt{2}}. \tag{14}$$

Next, we take the action of the Kerr nonlinearity into account. The stable transverse intensity distribution in quartz fibers can be regarded as a mode of radiation and the four-photon process itself can be described by a single-mode Hamiltonian (see, for example, [33] and references therein):

$$\hat{H} = \frac{\hbar}{2}\chi^{(3)}\hat{a}^+\hat{a}^+\hat{a}\hat{a}, \tag{15}$$

where $\chi^{(3)}$ is a cubic nonlinearity coefficient, normalized by the number of photons. We assume the nonlinear response to be instantaneous.

The corresponding evolution operator of the quantum state in the Schrödinger representation is

$$\hat{U} = \hat{I}\exp\left(-i\frac{\overline{\chi}}{2}\hat{a}^+\hat{a}^+\hat{a}\hat{a}\right) = \hat{I}\exp\left(-i\frac{\overline{\chi}}{2}\hat{n}(\hat{n}-1)\right), \tag{16}$$

where $\overline{\chi} = \chi^{(3)}t$ and the evolution time t is related to the fiber length as $l = vt$; v is the mode propagation speed in the fiber; and $\hat{n}$ (t) is the photon number operator.

In the Heisenberg representation, the photon annihilation operator of the field mode obeys the equation

$$i\hbar\frac{d\hat{a}}{dt} = [\hat{a}, \hat{H}],$$

whence

$$\hat{a}(t) = e^{-i\overline{\chi}\hat{a}^+(0)\hat{a}(0)}\hat{a}(0),$$

and in our case

$$\hat{a}_2' = e^{-i\overline{\chi}\hat{a}_2^+\hat{a}_2}\hat{a}_2, \quad \hat{a}_3' = e^{-i\overline{\chi}\hat{a}_3^+\hat{a}_3}\hat{a}_3. \tag{17}$$

Accordingly, the two output modes of the interferometer are

$$\hat{a}_0' = \frac{\hat{a}_2' - \hat{a}_3'}{\sqrt{2}}, \quad \hat{a}_1' = \frac{\hat{a}_2' + \hat{a}_3'}{\sqrt{2}}. \tag{18}$$

We find the average values of the photon numbers at the outputs of the interferometer:

$$\langle\hat{n}_0\rangle \equiv \left\langle\hat{a}_0'^+\hat{a}_0'\right\rangle = 0, \quad \langle\hat{n}_1\rangle \equiv \left\langle\hat{a}_1'^+\hat{a}_1'\right\rangle = 1. \tag{19}$$

Thus, we observe interference with a zero phase difference and the photon resides in both channels simultaneously.

Everything would be fine if there was not an unfortunate issue. According to (16), a single photon in the Fock state $|1\rangle$ does not acquire a nonlinear phase foray, since $\hat{U} = \hat{I}$ at $n = 1$, i.e., at any averaging over $|1\rangle$. This is not surprising, since PSM is a type of a nonlinear four-photon process that is degenerate in frequency and direction when two pumping photons are converted into the same two photons, but with a phase foray. Therefore, the PSM process could not be performed by a single photon. What should we do? We try to direct

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|12\rangle + |21\rangle),$$

to the two arms of the interferometer, but not the superposition

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle).$$

These three-photon fields can be prepared as follows. First, a three-stage transition of the atom from the excited state into the ground state, in which two of the three emitted photons are degenerate, i.e., belong to the same mode, is used. Second, it is possible to apply a nonlinear down frequency transformation either in a medium with cubic nonlinearity $\chi^{(3)}$ [34] or as a result of a cascade process analogous to that described in [35, 36]. In the latter case, two photons are generated at the first stage in the course of a nondegenerate parametric scattering in a piezoelectric crystal, for example, of the type $3\omega \to 2\omega + \omega$, while at the second stage, splitting of one of the photons in a degenerate parametric process occurs, i.e., the appearance of the subharmonic: $2\omega \to \omega + \omega$. The modes "*a*" and "*b*" formed in this manner are then fed into the arms of a Mach–Zehnder interferometer with nonlinear fibers. The vector of state at their input is

$$|\psi\rangle_0 = \frac{1}{\sqrt{2}}(|1\rangle_a|2\rangle_b + |2\rangle_a|1\rangle_b). \tag{20}$$

After the action of the evolution operator $\hat{U}$, we have

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle_a|2\rangle_b e^{-i\overline{\chi}_b} + |2\rangle_a|1\rangle_b e^{-i\overline{\chi}_a}). \tag{21}$$

At the output of the interferometer, according to (18), the function probability of the detectors will be proportional to the average number of photons:

$$\langle \hat{n}_1 \rangle \equiv \left\langle \hat{a}_0^{'+} \hat{a}_0^{'} \right\rangle = \frac{1}{2}(1 + \cos(\overline{\chi}_a - \overline{\chi}_b)),$$
$$\langle \hat{n}_2 \rangle \equiv \left\langle \hat{a}_0^{'+} \hat{a}_0^{'} \right\rangle = \frac{1}{2}(1 - \cos(\overline{\chi}_a - \overline{\chi}_b)). \quad (22)$$

Solving this problem in the Heisenberg representation, we obtain the same result.

Thus, for the same fibers $\overline{\chi}_a = \overline{\chi}_b$ we only have photocounts of one detector; this means that interference occurs and both states of the superposition are present in both arms of the interferometer simultaneously. Consequently, as in the case of one-photon states, neither splitting of the vector of state or empty wave packets occur.

If we have a Mach–Zehnder interferometer in mind, the Fock state $|3\rangle$ can be fed to its input; then, the field in its arms will be described by the vector of state

$$|\psi\rangle_0 = \frac{1}{2}(|0\rangle_a|3\rangle_b + |1\rangle_a|2\rangle_b + |2\rangle_a|1\rangle_b + |3\rangle_a|0\rangle_b), \quad (23)$$

and the function probabilities 1/4 and 3/4 of the detectors will not be equal, which also proves that the interference effect and simultaneous occurrence of all components of the quantum superposition in the arms of the interferometer occur.

There is another proof of this issue, although an indirect one, based on the unquestionable occurrence of quantum superposition instead of specific values of the measured quantum observables. As well, the occurrence of the quantum superposition exactly contradicts the existence of a certain trajectory of a quantum particle or particles. This proof consists in an experiment, although only a thought one thus far, in which there is no a specific phase difference of two entangled photons, while the complete superposition of all possible values of this phase difference occurs [37].

7. HIDDEN VON NEUMANN PARAMETERS, THE BOHM AND BELL OBJECTIONS

Bohm interpreted the relationships that he obtained in the sense that they essentially mean *refutation of quantum indeterminism*. If we knew S then we would be able to determine the "individual" velocities of particles $\mathbf{v}_i$ and "individual" *trajectories* of particles using further integration over time at given initial values. Here, at least two fundamental issues arise:

The famous von Neumann theorem about hidden parameters that restricts determinism of the behavior of individual quantum particles.

Is it generally correct to speak of "individual" characteristic of particles in this case?

However, could it be possible to match the principles of quantum mechanics with a deterministic but not probabilistic description of quantum objects? The result, which was previously obtained by Robertson [38] and named "the von Neumann theorem of hidden parameters," was given in the famous paper of John von Neumann. As this theorem is based on the fundamental principles of the quantum theory it follows that it (this theory) can only give a probabilistic, but not a deterministic description of quantum objects. Because this theorem was repeatedly subjected to criticism and reinterpretation later (for example, [39] and references therein), we give its exact formulation. Let two "noncommuting" self-conjugated Hermitian operators $\hat{A}$ and $\hat{B}$, with $\hat{A}\hat{B} - \hat{B}\hat{A} = i\hat{C}$, correspond to two quantum observable quantities A and B. Then, *the Heisenberg uncertainty equation* can be strictly proven:

$$\sqrt{(\Delta A)^2(\Delta B)^2} \geq \frac{1}{2}|\overline{C}|, \quad (25)$$

where ΔA and ΔB are the standard fundamentally unavoidable deviations of the individual results of measurement of A and B from their average values. In other words, since noncommuting operators are required to describe quantum phenomena, the theory that uses them cannot be deterministic.

Bohm, while not disputing the mathematical correctness of the formal apparatus of quantum theory, insisted that this description only corresponds to an intermediate level of the reality concept but not to the final one. The logic of Bohm is that, yes, the statistical interpretation of quantum mechanics, which allows one to use noncommuting operators, is internally consistent and was confirmed by experiments. In fact, "hidden" (deterministic) parameters are not necessary and are redundant for this theory. However, does this mean that a deeper level of description is not possible that is internally consistent and allows experimental confirmation as well? As a possible illustration, Bohm made a comparison between phenomenological thermodynamics (in which macroscopic parameters, i.e., pressure, temperature, etc., occur) with the statistical physics, in which *macroscopic* parameters are not postulated but occur as a result of the action of an ensemble of the *microscopic* degrees of freedom of individual atoms and molecules. A persistent ideological struggle occurred between the supporters of these concepts; the microscopic approach won.

Another analogy, which is apparently closer to Bohm's thesis, can also be given. We consider a linear electric circuit with stationary currents and voltages. Here, one can note two levels of analysis. On a "subtle" level one can operate with instantaneous currents and voltages (and measure them); this is what Bohm had in mind. On a "rough" level, we only have to speak of "acting" (RMS, root mean square) currents and voltages, which in each loop are quadratically averaged over the period of the harmonic current, as well as the phase angles between them; in this case, we have a pragmatic and absolutely consistent and complete theory of stationary harmonic processes, which, however, can potentially be deepened to the "subtle" level. More importantly, complex quantities are also used in this theory, which leads to a number of surprising analogies with quantum mechanics, including commutation relationships as well [40].

John Bell also joined the objections to the universal validity of the von Neumann theorem [1]. In the chapter "On the impossible pilot wave," he presented a constructive criticism of the prerequisites of this theorem and referred to the model proposed by Bohm, which moved beyond such prerequisites and, specifically introduced the parameter of a phase that explicitly serves as an example of the "hidden parameter" of von Neumann. "I saw the impossible done." In 1952 John Bell thus described his inexpressible astonishment about the David Bohm paper. In his book, Bell wondered about the silence of his mentors concerning the de Broglie–Bohm "pilot-wave" theory [1]. Therefore, he approached the analysis of the situation from the other side. In [40], while considering a thought experiment related to the famous Einstein–Podolsky–Rosen paradox, which referred to entangled particles flying apart, Bell tried to determine whether it is possible to describe the results of quantum theory and corresponding experiments using *deterministic hidden parameters*. It turned out that this cannot be done in principle unless one admits that "nonlocal" (superluminal) correlations are possible between the EPR particles that are flying apart (and/or detectors of these particles). As Bell formally proved in the 1960s [41] and Aspect et al. experimentally verified in the 1980s quantum mechanics (and the Bohm theory through the quantum potential explicitly) is characterized by exactly *nonlocal* correlations upon entanglement [42, 43].

8. ARE THE VELOCITIES AND TRAJECTORIES IN THE BOHM MODEL CHARACTERISTICS OF *INDIVIDUAL* PARTICLES?

Are the velocities and points of the trajectory that Bohm found instantaneous characteristics of exactly *individual* particles? He himself was not convinced of this and did not even try to pose the question differently, because this was the point of his interpretation. However, there are arguments in favor of the other point of view.

First, the Schrödinger equation itself, which was the starting point for Bohm, operates with a *statistical* ensemble that has some distribution. Hence, it apparently follows that any description obtained from the Schrödinger equation will also have a statistical character. This should fully apply to such objects as the Bohm velocities and Bohm trajectories.

Second, based on an analogy with a "quantum liquid" that obeys the same Bohm equations, it was shown in [44] that "quantum fluxes cannot cross in configuration space ... Therefore, two or more Bohmian trajectories cannot cross through such a point at the same time." At the same time, as noted above, it was theoretically and experimentally shown that individual quantum particles can "jump" from one Bohm trajectory to another with a non-zero probability, thus generating the so-called "surrealistic" trajectories [20]. Consequently, the "group" Bohm trajectories and "individual" particles tracks of movement are not the same thing, which was demonstrated via the example of "surrealistic" trajectories.

In fact, in 1926, Madelung saw that the equation in the form of the hydrodynamic continuity equation, in which the density and potential of velocities of a moving liquid occur, follows from the time-dependent Schrödinger equation. Elaborating these ideas, Madelung showed that each eigen function (a solution of the wave equation) depends on time but can be interpreted as a certain type of stationary flow. Since the hydrodynamic model also described other important features of the Schrödinger theory, Madelung assumed that it is possible to consider the quantum theory of atoms from this point of view. If the Bohm equations are understood as hydrodynamics equations then the trajectories that were obtained from the equations should not necessarily be regarded as the trajectories of real particles, but rather as *lines of a stream associated with a quantum liquid* (in fact, we note that the Schrödinger equation typically describes a degree of freedom rather than a "true" particle).

> This brings about another important question: are Bohmian trajectories the real paths followed by the degrees of freedom involved in our description (regardless of what these degrees of freedom may represent)?
>
> ...
>
> Consider a classical fluid. It consists of many different particles (e.g., atoms, ions, molecules, etc.), all the degrees of freedom being described by a set of differential coupled equations, with as many equations as degrees of freedom are involved, in principle (they can be reduced later on by means of different constraint conditions, but this is irrelevant here). Under some assumptions, one can pass from the microscopic description of the fluid to a macroscopic one, where equations like the Euler or Navier–Stokes ones will be rather used. These equations provide us with a phenomenological description of the evolution of a continuous fluid without paying any attention to the particular (microscopic) dynamics of its constituents. This is essentially the basis of classical hydrodynamics. Now, to understand the dynamics of such a fluid experimentally and then to compare it with the theoretical model, one usually proceeds by sprinkling the fluid with some particles. These are tracer particles that help us to visualize the flow dynamics as they move along the fluid streamlines (the lines along which the fluid current goes or, equivalently, its energy is trans-

ported). For example, if we want to observe the evolution of an air stream, we can use smoke; for a liquid like water, we can make use of another liquid, like ink, or tinny floating particles, like charcoal dust. At cosmological scales, hydrodynamical approaches can also be utilized, considering stars, galaxies or galaxy clusters as tracer particles.

...

Individual real quantum particles thus behave like individual pointlike particles, although their distributions display wavelike behavior, in accordance with the Schrödinger's equation or its Bohmian equivalents. Hence, these ensemble properties must be dealt with in terms of statistical descriptors, namely a density distribution function, with its role being played in quantum mechanics by the probability density (or, equivalently, the wave function). This is in agreement with Born's statistical interpretation of quantum mechanics [50].

9.THE GRÖSSING MODEL

We recall the quotation from [4], given at the beginning: "Recognizing that [this] equation has the structure of a diffusion equation with an imaginary diffusion coefficient, Schrödinger relaxed his original requirement concerning the reality of ψ and allowed complex-valued functions for what he called the "mechanical field scalar ψ..." Two key words, that is, "diffusion" and "complexity" are encountered in it. The Austrian theoretician Gerhard Grössing invoked these two concepts in a number of his papers, constructing the so-called "emergent quantum mechanics." By this term, Grössing means quantum mechanics *emerging at the subquantum* level from essentially classical concepts.

In [45], he proposed the following model, describing propagation of a quantum particle in a medium of "zero-point oscillations" of a vacuum. The energy of this quantum particle is considered to consist of two components. The first of these is a typical (constant) energy of a quantum oscillator (proportional to the frequency of oscillations), and the second one is *an additional kinetic component, caused by a fluctuating (harmonically varied) momentum of the particle*, which is continuously acquired and lost due to energy exchange with a vacuum. As a result, the motion of the particle acquires a Brownian character. Assuming that the kinetic energy of the "vacuum thermostat" is $kT/2$ per degree of freedom and the average kinetic energy of the particle-oscillator is $\hbar\omega/2$, then equating them, introducing the probability for the particle and expressing it in a typical manner through the real amplitude and phase of the wave function, Grössing obtained the macroscopic equation of *diffusion* that exactly match the Bohm equations, where the above-mentioned specific quantum potential occurs automatically. This potential does not have a significant effect for a single free particle; however a new understanding of the fundamental properties of the quantum potential immediately follows for diffusion fields when there are several particles.

The particle eigen frequency ω, which (as far as we know, (the author)) resonates with a frequency of vacuum oscillations in the entire volume, plays an important role in this respect.[2] It is due to this that *diffusion* waves, whose properties drastically differ from those for the typical ("running") waves, occur. In particular, attention should be drawn to *nonlocal properties of diffusion waves*. Since the "velocity of their propagation" is not restricted, the initial equation does not lead to running waves or wave fronts or the phase velocity. Rather, the entire region appears to be "breathing" in phase with the oscillating source. In the world of diffusion waves only spatially correlated phase lags occur, as determined by the diffusion length.

Therefore, instead of the common analysis of the behavior of a single quantum particle (a photon or electron) by itself, we have to consider its "diffusion" propagation in medium comprised of zero-point oscillations of a vacuum. In fact, this thought is not as unexpected as it might seem. Concepts of interaction of a quantum particle with a medium, when this medium "measures" the particle and entangles with it, are actively used in the decoherence theory. As an example [46],

> D.I. Blokhintsev considered that the features of quantum mechanics stem from the impossibility of isolating a particle from the environment. Bodies emit and absorb electromagnetic waves at any temperature above absolute zero. From the standpoint of quantum mechanics, this means that their position is continuously "measured," causing a collapse of wave functions. Blokhintsev wrote: "From this point of view, there are no isolated "free" particles by themselves." It is possible that the nature of this impossibility to isolate a particle, which is manifested in the apparatus quantum mechanics, is hidden in this relationship between particles and the medium.

Modern physics focuses on theoretical models of decoherence and their experimental verification at the quantitative level. In particular [48],

> A tractable model of the environment is afforded by a collection of harmonic oscillators ... or, equivalently, by a quantum field ... If a particle is present, excitations of the field will scatter off the particle. The resulting "ripples" will constitute a record of its position, shape, orientation, and so on, and most important, its instantaneous location and hence its trajectory.
>
> A boat traveling on a quiet lake or a stone that fell into water will leave such an imprint on the water surface. Our eyesight relies on the perturbations left

[2] Compare this with the constancy of the oscillation frequency (i.e., energy feeding) for the entire liquid volume in the experimental tray in the macroscopic experiments of Couder's group with "walking droplets" [47], which have become well known due to the strong analogy between them and fundamental quantum motion effects. Waves that occur there have a diffusion character as well.

by these objects on the preexisting state of the electromagnetic field.

In this respect, it is important to understand that a purely classical description of quantum objects, where a vacuum continuum of modes occurs, gives erroneous results. As an example, an attempt to numerically simulate a hydrogen atom in a vacuum white noise field within the Rutherford planetary model certainly does not give stationary Bohr orbits [49].

CONCLUSIONS

We will draw some conclusions.

First of all, we note that formally Bohm did not extend the quantum mechanics apparatus itself. He “simply” proposed to replace the Schrödinger equation with a single *complex-valued* wave function with an equivalent system of two equations for two *real* functions, i.e., the amplitude and phase of the wave function.

This representation of the mathematical apparatus enables one to observe quantum theory from new positions.

Before Bohm, as far as one can tell, the “absolute” phase of the wave function was regarded as a formal parameter, without physical sense. In the new representation, as known from classical physics, the continuity equation (for the probability density) and the Hamilton–Jacobi equation, which allows one to fix the phase of the wave function for an individual trajectory (or a group of trajectories), should replace the Schrödinger equation. A new term occurs in the quantum analog of the Hamilton–Jacobi equation, that is, the quantum potential; it explicitly causes *nonlocal* correlations of quantum particles.

Bohm interpreted separate trajectories fixed by a certain phase of the wave function, as the tracks of the movement of individual quantum particles. However, theoretical arguments and experimental facts allow one to believe that this is an issue of classes of trajectories, which are averaged over a given value of the phase, whereas the concept of the trajectory of an individual particle is very disputable, especially since “surrealism” is added here. The Bohm theory apparently refers to mass/energy transfer lines and not to individual particles and their trajectories.

Bohm gave a number of arguments, refuting the universal validity of the von Neumann theorem about hidden parameters. The “absolute” phase of the wave function he introduced is a good example of a hidden (nonlocal) parameter.

Bohm expressed his strong belief that although quantum mechanics is a complete and self-consistent theory, other physical theories, which are also complete and self-consistent but operate at the more subtle level of the concepts of space, time, and physical interactions, are possible.

Thus, despite the unquestionable usefulness of the hypothesis of David Bohm, the determinism of quantum processes can hardly be justified by using it; splitting the wave function into empty and nonempty wave packets appears to contradict the results of possible experiments and their interpretations.

ACKNOWLEDGMENTS

I am very grateful to M.H. Shulman for fruitful discussions and cooperation. This work was supported by the Russian Foundation for Basic Research, project no. 18-01-00598.

REFERENCES

1. J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge Univ. Press, 1987).
2. L. de Broglie, C. R. Acad. Sci. **177**, 507 (1923).
3. E. Schrödinger, Ann. Phys. **79**, 361 (1026).
4. M. Jammer, *The Conceptual Development of Quantum Mechanics* (McGraw-Hill, 1967).
5. D. Bohm, Phys. Rev. **85**, 166 (1952).
6. D. Bohm, Phys. Rev. **85**, 180 (1952).
7. D. F. Styer, M. S. Balkin, K. M. Becker, et al., Am. J. Phys. **70**, 288 (2002). doi 10.1119/1.1445404
8. D. H. Mahler, L. Rozema, K. Fisher, et al., Sci. Adv. **2**, e1501466 (2016).
9. B. Hensen, H. Bernien, A. E. Dréau, et al., Nature **526**, 682 (2015). doi 10.1038/nature15759
10. B. Hessmo, P. Usachev, K. Heydar, and G. Bjork, Phys. Rev. Lett. **92**, 180401 (2004).
11. S. A. Babichev, J. Appel, and A. I. Lvovsky, Phys. Rev. Lett. **92**, 193601 (2004).
12. M. Fuwa, S. Takeda, M. Zwierz, et al., Nat. Commun. **6**, 6665 (2015).
13. A. V. Belinsky and A. K. Zhukovskiy, J. Russ. Laser Res. **37**, 521 (2016).
14. A. V. Belinsky and M. H. Shulman, Prostranstvo, Vremya Fundam. Vzaimodeistviya, No. 1, 38 (2017).
15. A. V. Belinsky and M. H. Shulman, J. Russ. Laser Res. **38**, 230 (2018).
16. X. Oriols and J. Mompart, *Applied Bohmian Mechanics: From Nanoscale Systems to Cosmology* (CRC Press, 2012), p. 15.
17. V. Jacques, E. Wu, F. Grosshans, et al., Phys. Rev. Lett. **100**, 220402 (2008).
18. C. Philippidis, C. Dewdney, and B. J. Hiley, Nuovo Cimento B **52**, 15 (1979).
19. M. Gondran and A. Gondran, Am. J. Phys. **73**, 6 (2005).
20. B.-G. Englert, M. O. Scully, G. Sussmann, and H. Walther, Z. Naturforsch., A **47**, 175 (1992).
21. N. Katz, M. Neeley, M. Ansmann, et al., Phys. Rev. Lett. **101**, 200401 (2008).
22. Y.-H. Kim, R. Yu, S. P. Kulik, Y. Shih, and M. O. Scully, Phys. Rev. Lett. **84**, 1 (2000).
23. J. von Neumann, *Mathematische Grundlagen der Quantenmechanik* (Springer, 1932).

24. D. Dürr, S. Goldstein, and N. Zanghi, J. Stat. Phys. **116**, 9595 (2004).
25. D. Bohm, *Quantum Theory* (Dover, New York, 1951).
26. H. Everett III, Rev. Mod. Phys. **29**, 454 (1957).
27. M. B. Menskii, Phys.-Usp. **43**, 585 (2000).
28. M. B. Menskii, Phys.-Usp. **48**, 389 (2005).
29. A. A. Grib, Phys.-Usp. **56**, 1230 (2013).
30. M. B. Menskii, *Quantum Measurements and Decoherence. Models and Phenomenology* (Fizmatlit, Moscow, 2001).
31. R. P. Feynman, Rev. Mod. Phys. **20**, 367 (1948).
32. A. V. Belinsky, Moscow Univ. Phys. Bull. **72**, 224 (2017).
33. A. V. Belinsky, *Quantum Measurements* (BINOM, Moscow, 2008).
34. P. V. Elyutin and D. N. Klyshko, Phys. Lett. A **149**, 241 (1990).
35. A. V. Belinskii, JETP Lett. **54**, 11 (1991).
36. P. H. Eberhard, Phys. Rev. A **47**, 747 (1993).
37. A. V. Belinsky and A. A. Klevtsov, Phys.-Usp. **61**, 313 (2018).
38. H. P. Robertson, Phys. Rev. **34**, 163 (1929).
39. A. V. Belinsky and V. B. Lapshin, Phys.-Usp. **60**, 325 (2017).
40. M. H. Shulman, *Variations on the Quantum Theory* (Editorial URSS, Moscow, 2004).
41. J. S. Bell, Physics **1**, 195 (1964).
42. A. Aspect, in *Quantum (Un)speakables: From Bell to Quantum Information*, Ed. by R. A. Bertlmann and A. Zeilinger (Springer, 2002), p. 119.
43. M. Gondran and A. Gondran, arXiv:1309.4757 [quant-ph].
44. A. S. Sanz, J. Phys.: Conf. Ser. **361**, 012016 (2012).
45. G. Grossing, Phys. A **388**, 811 (2009).
46. A. Shishlova, Nauka Zhizn', No. 8 (1998).
47. Y. Couder et al., Europhys. News **41**, 14 (2010).
48. W. H. Zurek, arXiv:quant-ph/0306072.
49. A. V. Belinsky, Vestn. Mosk. Univ. Fiz. Astron., No. 3, 20 (1999).
50. A. S. Sanz and S. Miret-Artes, Am. J. Phys. **80**, 525 (2012).

Translated by D. Churochkin