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ASTRONOMY, ASTROPHYSICS, AND COSMOLOGY

Observational Constraints on the Chaplygin Gas with Inverse Power Law Potential in Braneworld Inflation[1]

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Abstract—We study Chaplygin gas as a candidate for inflation in the context of braneworld inflationary model. We investigate this model in the framework of the Randall–Sundrum type II, considering a original and generalized Chaplygin gas. We use inverse power law potential to examine the behavior of some inflationary spectrum parameters such as the spectral index n_s, the ratio r and the running of the scalar spectral index $dn_s/d\ln k$, our results are in agreement with recent observational data for a particular choice of e-folding number N and parameters space of the model.

Keywords: Braneworld inflation, Chaplygin gas, Planck data.

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1. INTRODUCTION

It is well accepted that the inflationary universe scenario [1], is the hypothesis that the early universe undergoes a brief rapid period of strong accelerated expansion following the big bang. It was first proposed by Guth and Sato in 1981 [2]. It is known that the inflation field plays an important leading role for most of models, but we do not have solid argument about his origin. Cosmic inflation has been generally accepted as a solution to solve and explain some conceptual problems as the horizon, flatness, relics problems and some other related problems of the standard big-bang theory [3]. In addition, braneworld inflation play a fundamental role in divers cosmological investigations of early universe. It is based primarily on the cosmological model Randall–Sundrum type II [4]. In this context, became a central paradigm which describes the universe in five dimensions with the presence of a brane that includes all ordinary matter dimensions. Braneworld cosmological models have been proposed in various models [5–9].

The astronomical observations indicate that our universe undergoes an accelerated expansion of the universe, an exotic component having a negative pressure is the responsible for this acceleration of the late expansion, usually is called as dark energy. There exists several models have been proposed already to describe dark energy and dark mater, one of the interesting models is Chaplygin gas [10]. It was introduced by Chaplygin who studied it in the context hydrodynamical [11]. There are several different theoretical ideas invoked to explain the dark energy, among them modifying gravity [12], Quintessence [13] and the Chaplygin gas [14].

The Chaplygin gas can be the simplest cosmological model that incorporates the dark energy and dark matter, it's generalized version attempts to unify between the two type of dark components, it plays a very crucial role in the equation of state fluid description of dark energy in cosmology. The Chaplygin gas in its original form, defined by an equation of state with negative pressure $p = -A/\rho$ [15], where p and ρ are respectively pressure and energy density, and A is a positive constant. Its has been extended to the so-called generalized Chaplygin gas by the following equation of state $p = -A/\rho^{\alpha}$ [10, 16], this model describes the entire dark sector of the Universe. For $\alpha = 1$ this generalization reduces to the original Chaplygin gas and in the case of $\alpha = 0$ is related to the ΛCDM model. The Chaplygin gas models have been intensively studied in the literature. For example, in [10] V. Gorini et al. examined two cosmological models representing the flat Friedmann Universe filled with a Chaplygin fluid in terms of the "statefinder"

[1] The article is published in the original.

parameters. They showed that these models are different and they are worth study. In [17] Saadat and Pourhassan considered the modified Chaplygin gas in FRW bulk viscous cosmology, and show that time-dependent energy density for the special case of flat space. In another work [18], the authors discussed effect of anisotropy on generalized Chaplygin gas scalar field and its interaction with other dark energy models. Also in [19] the authors examined extended model of Chaplygin gas equation of state and have found that extended Chaplygin give a best it with the observational data.

On the other hand, the Chaplygin gas inspired inflation model [20] was the subject of several cosmological and phenomenological studies. In this context, the scalar field is usually the standard inflation field, where the energy density can be extrapolated to obtain a successful inflationary period with a Chaplygin gas model. In the same context, a work has been done by R. Herrera [21] where the brane-Chaplygin inflationary model was studied in great details and considered as a viable alternative model that can provide an accelerated expansion of the early universe. In extension of the [21], the similar work was performed for the case of the tachyon-Chaplygin inflationary model by using an exponential potential in the high-energy regime [22].

The goal of present work, is to study the realization of braneworld inflationary model in the context of Chaplygin gas models as a candidate for the primordial inflation by assuming that the matter source on the brane consist of a Chaplygin gas. The Chaplygin gas emerges as a effective fluid of a generalized d-brane in a $(d + 1, 1)$ spacetime, where the action can be written as a generalized Born–Infeld action [23]. We investigate various perturbation spectrum parameters, by using inverse power law potential, such as the scalar spectral index n_s and the ratio r and the running of the scalar spectral index $\frac{dn}{d\ln(k)}$ in the high-energy limit, particularly for a suitable choice of the different parameters. We show that the inflation parameters are in good agreement with recent Planck 2015 data [24]. The paper is organized as follows, we first recall in Section 2 a brief summary of braneworld context and the perturbation spectrum expressions of braneworld scenario in the framework of Chaplygin gas models. In Section 3, we present our results concerning inverse power law potential for different parameter values of this model. The last section is devoted to conclusion.

2. BRANEWORLD INFLATIONARY MODEL WITH CHAPLYGIN GAS

2.1. Brief Review of Braneworld Context

In this section we will give a brief summary of braneworld scenario. The introduction of branes in cosmology offered another new approach to our understanding of the universe and of its evolution. It has been proposed that our universe is a three dimensional surface (3-brane) embedded in a higher dimensional space. For example the ekpyrotic model postulate that the universe did not start in a singularity, but came about from the collision of two branes colliding with each other, causing the exchange of energy at the origin of our universe [25, 26]. Generally, in braneworld scenario, the observable universe can be considered as 3-brane, embedded in $4 + d$ dimensional spacetime, particles and fields are trapped on the brane while gravity is free to access the bulk.

In this paper, we consider the Randall–Sundrum type-II model, where $d = 1$, in which the dynamics was determined with more precision in reference [27, 28]. It was originally proposed as an alternative resolution of the hierarchy problem arising from the large difference between the Planck scale and the electroweak scale. This model is based on single brane which has positive brane tension. In this theory, the metric projected on the brane is a specially flat Friedmann–Robertson–Walker model, and the Friedmann equation is then in a generalized form $H^2 = \frac{8\pi}{3M_p^2}\rho\left(1+\frac{\rho}{2\lambda}\right)$ [27], where H is the Hubble parameter, ρ is the energy density, $M_p = 1.2 \times 10^{19}$ GeV is the four-dimensional Planck mass and λ is the brane tension, such as $\lambda = \frac{3}{4\pi}\frac{M_5^6}{M_p^2}$, where M_5 is the five-dimensional Planck mass. In order to have the inflationary dynamical equations leading to an accelerated expansion one has to use slow roll approximation which are given by [27]: $\varepsilon = \frac{M_p^2}{2}\left(\frac{V'}{V\left(1+\frac{V}{2\lambda}\right)}\right)^2\left(1+\frac{V}{2\lambda}\right)$ and $\eta = \frac{M_p^2}{2}\frac{\lambda V''}{V\left(1+\frac{V}{2\lambda}\right)}$, where $V'' = \frac{d^2V}{d\phi^2}$. During inflation $\varepsilon \ll 1$ and $|\eta| \ll 1$. Inflationary phase will terminate when the universe heats up, so that the condition $\varepsilon = 1$ (or $|\eta| = 1$) is satisfied. The small quantum fluctuations in the scalar field lead to fluctuations in the energy density which is known to be related to the scalar curvature perturbation [29]. For that reason, we define the power spectrum of the curvature perturbations by [27]:

$$P_R(k) \simeq \frac{128\pi}{3M_p^6}\frac{V^3}{V'^2}\left(\frac{2\lambda+V}{2\lambda}\right)^3.$$

The number of e-folds during braneworld inflation is given by: $Ne \simeq -\frac{1}{M_p^2\lambda}\int_{\phi*}^{\phi_{\text{end}}}\frac{V}{V'}\left(1+\frac{V}{2\lambda}\right)d\phi$, where ϕ^* denotes the value of the scalar field ϕ when universe scale observed today crosses the Hubble horizon during inflation, and ϕ_{end} is the value of the scalar field when the universe exits the inflationary phase.

2.2. Chaplygin Gas Braneworld Inflation

In the present section, we will study Chaplygin gas in the framework of Randall–Sundrum type II braneworld model [4]. In this context the generalized Chaplygin gas model is described by a perfect fluid characterized by an exotic equation of state [30] $p = -\frac{A}{\rho^\alpha}$, where ρ and p are the energy density and pressure of the generalized Chaplygin gas, respectively, and A is a positive constant. Inserting this equation in the equation of conservation of energy $\dot{\rho} + 3H(\rho + p) = 0$, leads to an energy density as $\rho_{ch} = \left[A + \left(\rho_{ch0}^{\alpha+1} - A\right)\left(\frac{\alpha_0}{\alpha}\right)^{3(\alpha+1)}\right]^{\frac{1}{\alpha+1}}$, where a_0 and ρ_{ch0} are the current values of the scale factor and the generalized Chaplygin gas energy density, respectively. This modification is possible due to an extrapolation of the last equation so that $\rho = \left[A + \rho_m^{(\alpha+1)}\right]^{\frac{1}{\alpha+1}} \to \rho = \left[A + \rho_\phi^{(\alpha+1)}\right]^{\frac{1}{\alpha+1}}$. On the other hand we suppose that the universe is filled with a perfect fluid with energy density $\rho(t)$ and pressure $p(t)$ where $p_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$, $\rho_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$ and $V(\phi) = V$ is the scalar potential, the modified Friedmann equation can be written as following [22] $H^2 = \frac{8\pi}{3M_p^2}\left(A + \rho_\phi^{(\alpha+1)}\right)^{\frac{1}{\alpha+1}}\left[1 + \frac{\left(A + \rho_\phi^{(\alpha+1)}\right)^{\frac{1}{\alpha+1}}}{2\lambda}\right]$, where H is the Hubble parameter, $M_p = 1.2 \times 10^{19}$ GeV is the reduced Planck mass, and λ is the brane tension. The Klein–Gordon equation that describes the evolution of the scalar field is given by $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$, where $\dot{\phi} = \frac{\partial\phi}{\partial t}$, $\ddot{\phi} = \frac{\partial^2\phi}{\partial t^2}$, $V' = \frac{\partial V}{\partial\phi}$. During inflation, the scalar field comparable to the scalar potential associated to the energy density is $\rho_\phi \simeq V(\phi)$, we shall introduce the slow-roll conditions $\dot{\phi}^2 \prec\prec V(\phi)$ and $\ddot{\phi} \prec\prec 3H\dot{\phi}$, the Friedmann equation reduces to

$$H^2 = \frac{8\pi}{3M_p^2}\left(A + V_\phi^{(\alpha+1)}\right)^{\frac{1}{\alpha+1}}\left[1 + \frac{\left(A + V_\phi^{(\alpha+1)}\right)^{\frac{1}{\alpha+1}}}{2\lambda}\right]. \quad (1)$$

One can also consider the slow-roll parameters to calculate perturbation spectrum. We first defined the two parameters ε and η in braneworld model for generalized Chaplygin gas model as

$$\varepsilon = \frac{M_P^2}{16\pi}\frac{V^\alpha V'^2}{\left(A + V^{(\alpha+1)}\right)^{\frac{\alpha+2}{\alpha+1}}}\left[\frac{1 + \frac{\left(A + V^{(\alpha+1)}\right)^{\frac{1}{\alpha+1}}}{\lambda}}{\left(1 + \frac{\left(A + V^{(\alpha+1)}\right)^{\frac{1}{\alpha+1}}}{2\lambda}\right)^2}\right] \quad (2)$$

and

$$\eta = \frac{M_P^2}{16\pi}\frac{V''}{\left(A + V^{(\alpha+1)}\right)^{\frac{1}{\alpha+1}}}\left[\frac{1}{\left(1 + \frac{\left(A + V^{(\alpha+1)}\right)^{\frac{1}{\alpha+1}}}{2\lambda}\right)^2}\right]. \quad (3)$$

We recall that during inflation we have the following conditions $\varepsilon \prec\prec 1$ and $|\eta| \prec\prec 1$ and the end of inflation the condition $\varepsilon = 1$ (or $|\eta| = 1$) is satisfied. The power spectrum of the curvature perturbations defined by [29]

$$P_r = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2, \quad (4)$$

which takes the following form

$$P_r = \frac{128\pi}{3M_p^6}\frac{\left(A + V^{(\alpha+1)}\right)^{\frac{3}{\alpha+1}}}{V'^2}\left[1 + \frac{\left(A + V^{(\alpha+1)}\right)^{\frac{1}{\alpha+1}}}{2\lambda}\right]^3,$$

and the amplitude of tensor perturbations given by [31]

$$P_g = \frac{64\pi}{M_p^2}\left(\frac{H}{2\pi}\right)^2 F^2(x), \quad (5)$$

where $x = HM_p\sqrt{\frac{3}{4\pi\lambda}}$ and $F^2(x) = \left(\sqrt{1 + x^2} - x^2\sinh^{-1}\left(\frac{1}{x}\right)\right)^{-1} \simeq \frac{3}{2}x = \frac{3}{2}\frac{\left(A + V^{(\alpha+1)}\right)^{\frac{1}{\alpha+1}}}{\lambda}$. From these parameters, we can obtain the tensor-to-scalar ratio, the scalar spectral index, and the running of the scalar index as follows

$$r = \left(\frac{P_g(k)}{P_r(k)}\right)_{k=k^*}$$

$$= \frac{M_P^2}{\pi} \frac{V'^2 F^2(x)}{\left(A + V^{(\alpha+1)}\right)^{\frac{2}{\alpha+1}} \left(1 + \frac{\left(A + V^{(\alpha+1)}\right)^{\frac{1}{\alpha+1}}}{2\lambda}\right)^2}, \quad (6)$$

$$n_s - 1 = \frac{d \ln P_R(k)}{d \ln k}$$

$$= \frac{M_P^2}{8\pi \left(A + V^{(\alpha+1)}\right)^{\frac{1}{\alpha+1}} \left(1 + \frac{\left(A + V^{(\alpha+1)}\right)^{\frac{1}{\alpha+1}}}{2\lambda}\right)} \quad (7)$$

$$\times \left(-\frac{3V^{\alpha} V'^2}{\left(A + V^{(\alpha+1)}\right)} \frac{\left(1 + \frac{\left(A + V^{(\alpha+1)}\right)^{\frac{1}{\alpha+1}}}{\lambda}\right)}{\left(1 + \frac{\left(A + V^{(\alpha+1)}\right)^{\frac{1}{\alpha+1}}}{2\lambda}\right)} + 2V'' \right),$$

$$\frac{dn_s}{d \ln k}$$

$$= \frac{M_P^2}{4\pi} \frac{V'}{\left(A + V^{(\alpha+1)}\right)^{\frac{1}{\alpha+1}}} \quad (8)$$

$$\times \frac{1}{\left(1 + \frac{\left(A + V^{(\alpha+1)}\right)^{\frac{1}{\alpha+1}}}{2\lambda}\right)} \left(3\frac{\partial \varepsilon}{\partial \phi} - \frac{\partial \eta}{\partial \phi}\right).$$

Finally, the expression of the number of e-folds during inflation takes the following form

$$N = \frac{-8\pi}{M_P^2} \int_{V_*}^{V_{\text{end}}} \frac{\left(A + V^{(\alpha+1)}\right)^{\frac{1}{\alpha+1}}}{V'^2} \left[1 + \frac{\left(A + V^{(\alpha+1)}\right)^{\frac{1}{\alpha+1}}}{2\lambda}\right] dV \quad (9)$$

where V_{end} and V^* are the values of the potentials at the horizon exit and the end of inflation. Note that in the limit $A \to 0$, the perturbation spectrum parameters coincides with brane-inflation [27] in particular for $\alpha = 1$. Also, in the low-energy limit, $\left(A + V^{(\alpha+1)}\right)^{\frac{1}{\alpha+1}} \ll \lambda$, the slow-parameters reduce to the standard form [20]. In the following, we study in the high-energy limit $\left(A + V^{(\alpha+1)}\right)^{\frac{1}{\alpha+1}} \prec\prec \lambda$, the evolution of various inflationary perturbation spectrum and we give our results concerning an inverse power law potential with original and generalized Chaplygin gas models.

3. INVERSE POWER LAW POTENTIAL WITH CHAPLYGIN GAS

3.1. Original Chaplygin Gas

In this subsection we will focus on the original Chaplygin gas model, it has well described the early universe. This model has been proposed and studied widely in the literature. For instance, in [32] it has been suggested a correspondence between Chaplygin gas energy density in Friedmann–Robertson–Walker universe and the holographic dark energy density, to reconstruct the potential and the dynamics of the scalar field which describe the Chaplygin cosmology. In [33] Kamenshchik et al. have studied Chaplygin gas in Friedmann–Robertson–Walker cosmological model, and found that the resulting evolution of the universe is not in agreement with the observation of cosmic acceleration. In [34], the authors have focused to study a Chaplygin gas model in Braneworld inflation with an exponential potential, and have obtained for negligible and small running of the scalar spectral index, the inflationary parameters are in good agreement with observation data.

We are investigate in this subsection the perturbation spectrum parameters in relation with recent Planck data. We shall apply the above braneworld formalism with inflationary inverse power law in the case of original Chaplygin gas ($\alpha = 1$), this model described by the following equation of state $p = -\frac{A}{\rho}$ where A is a positive constant.

We consider the following inverse power law potential

$$V = \frac{\mu}{\phi^m} \quad (10)$$

μ is a parameter of dimension $[E]^{4-m}$ and m is constant. Note that this potential has been extensively studied [3, 35, 36]. The slow-roll parameters became in terms of the scalar field

$$\varepsilon = \frac{M_p^2 \lambda}{4\pi} \frac{m^2 \mu^3 \phi^{-3m-2}}{(A + (\mu\phi^{-m})^2)^2}, \quad (11)$$

$$\eta = \frac{M_p^2 \lambda}{4\pi} \frac{m(m-1)\mu\phi^{-m-2}}{A + (\mu\phi^{-m})^2}. \quad (12)$$

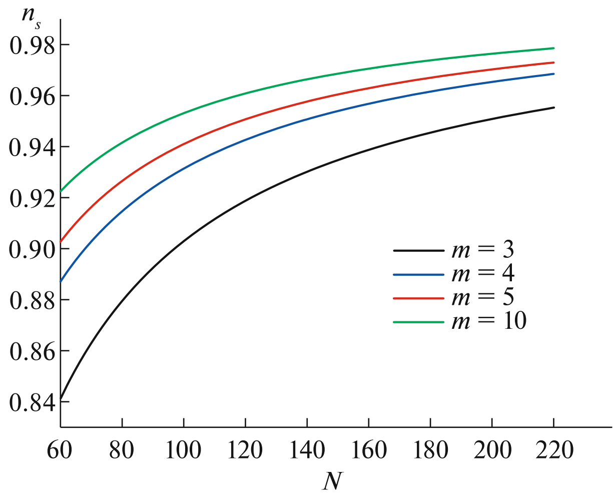

Fig. 1. n_s vs. N with $m = 3, 4, 5, 10$.

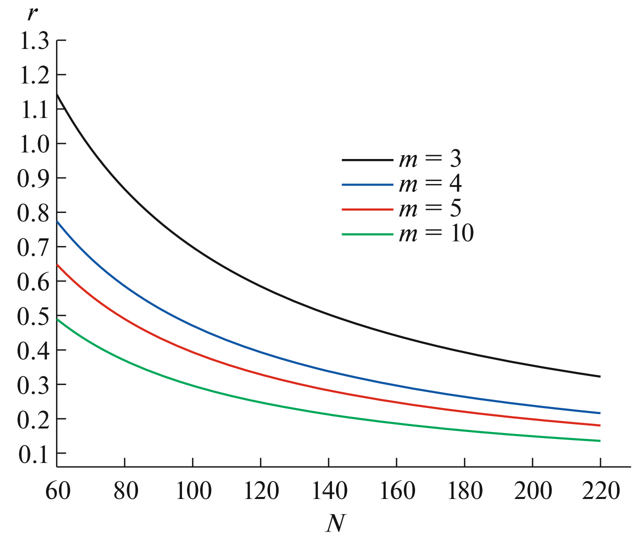

Fig. 2. r vs. N with $m = 3, 4, 5, 10$.

The expressions of the inflationary parameters n_s, r and $\frac{dn_s}{d\ln(k)}$ can be written analytically as

$$n_s - 1 = \frac{M_p^2\lambda}{2\pi(A + (\mu\phi_*^{-m})^2)} \times \left(-\frac{3m^2\mu^3\phi_*^{-3m-2}}{(A + (\mu\phi_*^{-m})^2)} + 2m(m+1)\mu\phi_*^{-m-2}\right), \tag{13}$$

$$r = \frac{6M_p^2\lambda\mu^2 m^2\phi_*^{-2m-2}}{\pi(A + \mu^2\phi_*^{-m})^{3/2}}, \tag{14}$$

and

$$\frac{dn_s}{d\ln k} = \frac{M_p^2\lambda^2 m^2(-m+2)(2m-1)}{8\pi^2\phi_*^4(A + \mu^2\phi_*^{-2m})}. \tag{15}$$

Although we have analytic results for slow-roll parameters and the number of e-folding, it is not easy to solve them to obtain ϕ^* at which the observables n_s, r and $\frac{dn_s}{d\ln(k)}$ should be evaluated. For it, we proceed numerically by finding ϕ_{end} and using the Eq. (9) to obtain ϕ^* while making sure that the slow-roll parameters remain small in this range of ϕ. Note that in the limit $A \to 0$, the scalar spectral index n_s, the ratio r and $\frac{dn_s}{d\ln(k)}$ coincides with [35].

According to Planck data [24], we will present our results concerning this model and study the variations of the perturbation spectrum parameters with respect to N for various values of $m = 3, 4, 5, 10$. We observe that these observables depends on different parameters. For this, we take the brane tension value $\lambda \sim O$ (10^{68} GeV^4), the inflationary scale $\mu \sim O$ (10^{15} GeV), $A \sim 10^{-13}M_p^8$ and $\alpha = 1$ [21], in order to plot the scalar spectral index n_s, the ratio r and the running of the scalar spectral index by varying the e-folding number N with different values of the parameter m.

Figure 1 present the evolution of n_s with respect to e-foldings number N for various values of parameter m. The trajectories of this figure show the increasing behavior of n_s with respect to N. We also note that the values of n_s are found to be consistent with the Planck data for large domain of N and the central value of $n_s = 0.965$ is obtained in particular for large values of m.

Figure 2 plots ratio r versus e-foldings number N for $m = 3, 4, 5, 10$. This figure show that ratio r is a decreasing function with respect to N, and in order to confront r with Planck data we must have very large value of $N \geqslant 220$ and $m \geqslant 10$.

Figure 3 shows the variation of the running of the scalar spectral index with respect to e-foldings number N for different values of m, we observe that $\frac{dn_s}{d\ln(k)}$ increases as N increases. We can remark that $\frac{dn_s}{d\ln(k)}$ is consistent with the Planck data for different values of m.

To summarize this subsection, we have found that the results reviewed in the context of the original Chaplygin gas model are compatible with the latest observational measurements for a particular choice of e-folding number N and constant values of n.

In the following, we will study the case of generalized Chaplygin gas with $\alpha \neq 1$. We will discuss the effect of introducing the constant α on the perturba-

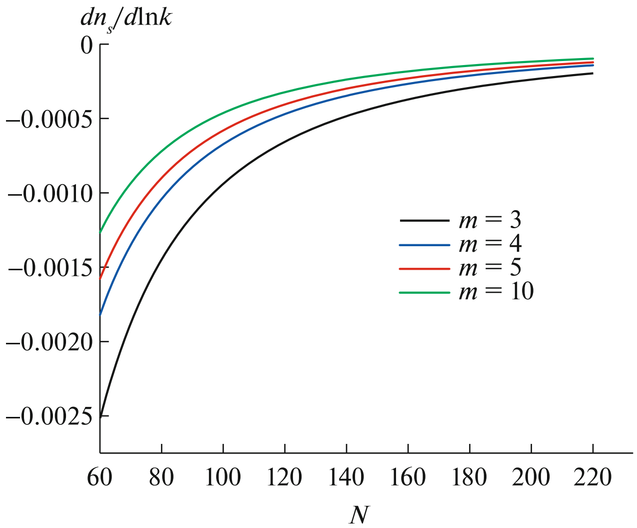

Fig. 3. $dn_s/d\ln k$ vs. N with $m = 3, 4, 5, 10$.

tion spectrum of the model. Our results will be compared to observations and we will show that the inflation can occur successfully in relation to recent observations.

3.2. Generalized Chaplygin Gas

The generalized Chaplygin gas can describe some primitive epochs of the Universe [37], it was studied in the literature by various authors. For instance, the paper [38] investigate the warm inflationary universe models in the context of generalized Chaplygin gas, and show that the dependence of tensor scalar ratio r on spectral index n_s and observe that the range of tensor scalar ratio is $r < 0.05$. In addition, in work [39] the authors have studied the dependence of the location of the Cosmic Microwave Background Radiation peaks on the parameters of the generalized Chaplygin gas model. Also shown that the emerging model is clearly distinguishable from the $\alpha = 1$ Chaplygin case and the ΛCDM model. Moreover, according to the work [40] L. Amendola et al. compare the WMAP temperature power spectrum and SNIa data to models with a GCG as dark energy, they show that a Chaplygin gas $\alpha = 1$ as a candidate for dark energy is ruled out by there analysis at more than the 99.99% CL. In other models [41], the authors have analyzed the dynamical evolution of viscous generalized Chaplygin gas for different parameters and initial conditions, and have included a linear barotropic fluid that could be used to mimic the matter sector of our universe and thus generalized Chaplygin gas can be considered only as dark energy.

We apply Inverse power law potential using the expression of slow-roll parameters with generalized Chaplygin gas in braneworld inflation, the slow-roll parameters became in terms of the scalar field

$$\varepsilon = \frac{M_p^2\lambda}{4\pi}\frac{m^2\mu^3\phi^{-3m-2}}{(A + (\mu\phi^{-m})^{\alpha+1})^{\frac{4}{\alpha+1}}}, \tag{16}$$

$$\eta = \frac{M_p^2\lambda}{4\pi}\frac{m(m-1)\mu\phi^{-m-2}}{(A + (\mu\phi^{-m})^{\alpha+1})^{\frac{2}{\alpha+1}}}. \tag{17}$$

The scalar spectral index, the ratio and the running of the scalar spectral index takes the following form

$$n_s - 1 = \frac{M_p^2\lambda}{2\pi(A + (\mu\phi_*^{-m})^{\alpha+1})^{\frac{2}{\alpha+1}}} \times \left(-\frac{3m^2\mu^3\phi_*^{-3m-2}}{(A + (\mu\phi_*^{-m})^{\alpha+1})} + 2m(m+1)\mu\phi_*^{-m-2}\right), \tag{18}$$

$$r = \frac{6M_p^2\lambda\mu^2 m^2\phi_*^{-2m-2}}{\pi(A + (\mu\phi_*^{-m})^{\alpha+1})^{\frac{3}{\alpha+1}}}, \tag{19}$$

$$\frac{dn_s}{d\ln k} = \frac{M_p^4\lambda^2 m^2(2m-1)(-m+2)}{8\pi^2\phi_*^4(A + (\mu\phi_*^{-m})^{\alpha+1})^{\frac{2}{\alpha+1}}}. \tag{20}$$

We note that, as in the previous case, the inflation value before the end of inflation ϕ^*, can be obtained numerically from Eq. (9).

Based on the above formulas and to confront simultaneously the observables n_s, r, and $\frac{dn_s}{d\ln(k)}$ with observations, we study the relative variation of these parameters. We can see that these observables depends on several parameters. Therefore, as the previous case we take the inflationary scale $M \sim O$ (10^{15} GeV), the brane tension value $\lambda \sim O$ (10^{68} GeV^4) and $A \sim 10^{-13}M_p^8$ [21], and $m = 4$.

We will discuss some values of the above inflationary parameters in relation with the e-folds number N and the parameter α. The central region, given by the Planck data, of the spectral index $n_s \in [0.959; 0.969]$ gives $115 < N < 155$ for the values of a in order of $\alpha \sim O(10^{-4}) - O(10^{-2})$. Regarding the ratio of scalar to tensor curvature perturbation, the Planck constraint $r < 0.11$, which requires large values of the folding number, that is, $N > 200$, and for $\frac{dn_s}{d\ln(k)} \in [-0.0166; -0.0039]$, we get $115 < N < 155$ for large domain of value of a between $0 < \alpha < 1$. This shows that parameter α has a

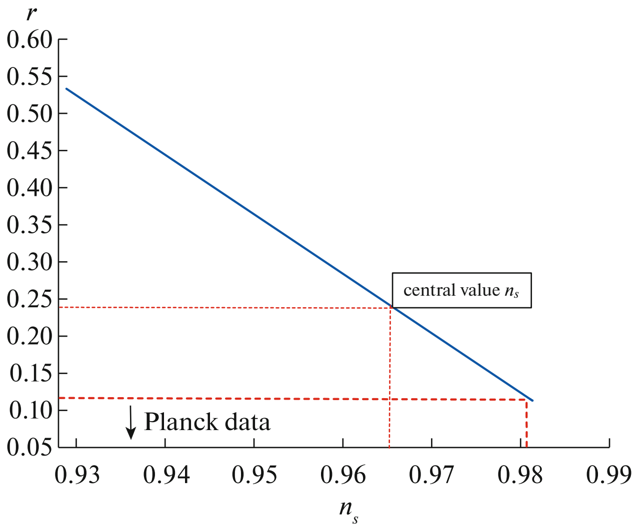

Fig. 4. r versus n_s for different values of N.

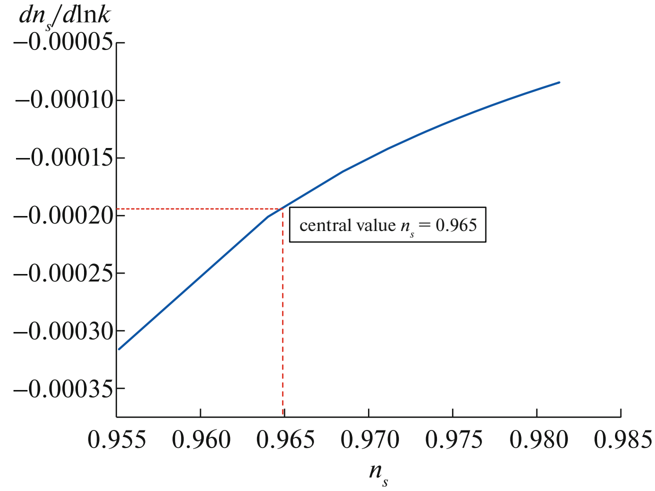

Fig. 5. $dn_s/d\ln k$ versus n_s for different values of N.

small influence on the inflationary observables compared to the original Chaplygin gas.

Figure 4 shows the variation of the scalar spectral index n_s with respect to ratio r, we note that n_s is an decreasing linear function with r. We remark that the ratio r is compatible with Planck data where $r < 0.11$ corresponds to $N > 200$ and $n_s \geqslant 0.979$. The central value $n_s \simeq 0.965$, where $m = 4$ corresponds to the ratio $r \sim 0.23$.

Figure 5 presents the variations of the running of the scalar spectral index $\frac{dn_s}{d\ln(k)}$ according to n_s. We see that the running of the scalar index $\frac{dn_s}{d\ln(k)}$ is an increasing function with respect to n_s, the central value of the scalar spectral index $2n_s \simeq 0.965$ corresponds to $N = 120$, gives $\frac{dn_s}{d\ln(k)} \simeq -0.00019$, which is in agreement with the Planck observations.

In Fig. 6 we present the variations of the ratio r as a function of the running of the scalar spectral index $\frac{dn_s}{d\ln(k)}$. The ratio r has a decreasing behavior with respect to N. For the ratio tensor-to-scalar given by Planck corresponds to $N > 200$ which gives a domain of the running $\frac{dn_s}{d\ln(k)} \geqslant -0.00008$.

To resume, we have found that the parameter α has a negligible effect on inflationary parameters. Also, the results reviewed in the context of the generalized Chaplygin gas model are compatible with the latest

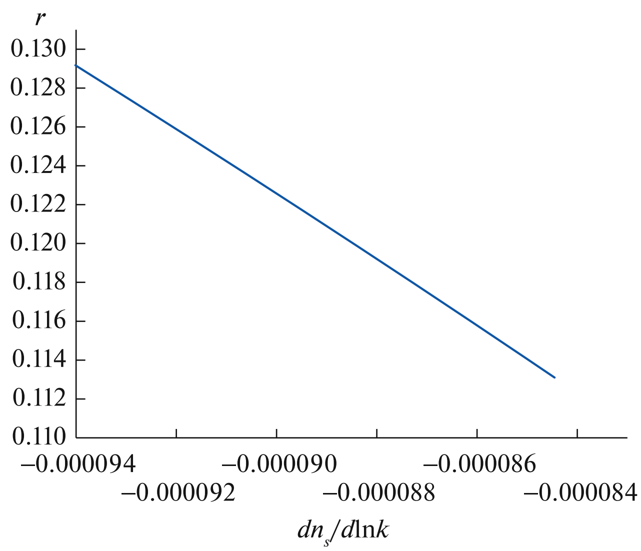

Fig. 6. $dn_s/d\ln k$ versus r for different values of N.

observational measurements for a particular choice of e-folding number N.

4. CONCLUSIONS

In this work, we have studied original and generalized Chaplygin gas model, in framework of the Randall–Sundrum braneworld type II model. The slow-roll and inflationary spectrum perturbation parameters are reformulated and evaluated in the high-energy limit. We have used Inverse power law potential to evaluate various inflationary spectrum parameters such as the spectral index n_s, the ratio r and the running of the scalar spectral index $\frac{dn_s}{d\ln(k)}$ and we found that they depend on several parameters. In the original Chaplygin gas case, a desirable results of $(n_s; r; \frac{dn_s}{d\ln(k)})$ with recent observational data are reproduced. In particular, the best values of n_s, where $m > 4$, corresponds a $N > 120$. Also we must have very large value of $N > 220$, in order to confront r with Planck data. The running of the spectral index $\frac{dn_s}{d\ln(k)}$ is consistent with observations for large interval of N. In the case of generalized Chaplygin gas, we have shown that for $115 < N < 155$ the central value of the spectral index is reproduced especially for the values of α in order of $O(10^{-4})$–$O(10^{-2})$. The ratio r is consistent with Planck 2015 for $N > 200$ which gives a domain of the running $\frac{dn_s}{d\ln(k)} \gtrsim -0.00008$. This shows that the introduction of parameter α has a small influence on the inflationary parameters. The results obtained in the context of Chaplygin gas model are compatible with the latest observational measurements for a particular choice of the parameter space of the model.

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