USE OF THE LORENZ SELF-ACTION-FORCE METHOD TO SOLVE CERTAIN PROBLEMS IN CLASSICAL RADIATION THEORY

RADIATION OF AN ELECTRIC CHARGE IN AN UNDULATOR IN THE PRESENCE OF A TRANSPARENT ISOTROPIC FERRODIELECTRIC

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The problem of the radiation of a relativistic electron in an electric undulator in the presence of a transparent isotropic ferrodielectric is solved. The angular energy distribution of the radiation is found and analyzed.

The theory of emission by charged particles in the undulator is important from the standpoint of developing strong sources of electromagnetic energy in the millimeter and submillimeter bands [1,2] and possible uses of devices of this type in nuclear physics to measure the energies of fast charged particles [3]. A number of studies have been devoted to the theory of the emission of charged particles in undulators [2-11].

Motz [3] was the first to point out the possibility of using radiation in undulators to measure particle energies. This idea was developed in [7-9], which investigated the possibilities for use of X-ray and harder radiation in undulators to identify ultrafast charged particles and measure their energies. The spectral composition of the radiation was studied. Here we wish to find the angular distribution of the total (over the period) energy of the emission of a charged particle in an electrical undulator in the presence of a transparent isotropic medium, a ferrodielectric.

We shall assume that the charged particle moves at a constant velocity $\mathbf{v}_0 = v_0\mathbf{k}$ ($\mathbf{k}$ is the unit vector along z) in a transverse electrostatic field

$$E_x^{\text{ext}} = E_0 \sin\frac{2\pi z}{l}, \quad E_y^{\text{ext}} = E_z^{\text{ext}} = 0, \quad E_0,\ l - \text{const}. \tag{1}$$

The transparent isotropic ferrodielectric is characterized by its permittivity and permeability, $\varepsilon = \varepsilon(\omega)$ and $\mu = \mu(\omega)$.

From the equations of motion

$$\frac{d\mathbf{P}}{dt} = e\mathbf{E}^{\text{ext}}, \quad \mathbf{P} = \frac{\mathcal{E}}{c^2}\mathbf{v} \tag{2}$$

and the initial conditions $x = y = z = 0$ at $t = 0$, using the constant particle energy approximation [4,7], we find the following law of particle motion:

$$x = -x_0 \sin\omega_0 t, \quad y = 0, \quad z = v_0 t, \tag{3}$$

where

$$\omega_0 = \frac{2\pi v_0}{l}, \quad x_0 = \frac{eE_0}{\mathcal{E}}\left(\frac{c}{\omega_0}\right)^2, \quad \mathcal{E} = c\sqrt{p^2 + m_0^2c^2},$$

P is the momentum of the particle, e and m_0 are its charge and rest mass, c is the velocity of light in vacuum, and $\frac{\mathcal{E}}{m_0c^2} \gg 1$.

We determine the loss of energy by the charge in the undulator during time $T = \frac{2\pi}{\omega_0}$ from the formula

$$W = -\int_0^T dt \int \mathbf{j}(\mathbf{r}, t)\, \mathbf{E}_s(\mathbf{r}, t)\, d\mathbf{r}, \tag{4}$$

where $\mathbf{j}(\mathbf{r},t)$ is the current density and $\mathbf{E}_s(\mathbf{r},t)$ must be determined in accordance with [12] with the aid of the half-difference of the retarded and advanced potentials.

Formula (4) gives the work of the Lorenz self-action force over the period T and determines the energy losses of the particle during this time interval. Below we shall be interested only in the energy lost by the charged particle to emission. Expanding the field intensities into Fourier integrals as usual, and introducing the wave vector $\mathbf{k} = \{k\sin\theta\cos\varphi;\ k\sin\theta\sin\varphi;\ k\cos\theta\}$, we find, after the standard calculations, the following expression for the spectral-angular distribution of the total (over the period T) radiated energy:

$$W = \frac{e^2}{c^2}\left(\frac{T}{2\pi}\right)\sum_{\nu=-\infty}^{\infty}\int_0^{\infty}\frac{\mu(\omega)\,\omega^2 d\omega}{c'}\int_0^{2\pi}d\varphi\int_0^{\pi}\sin\theta d\theta\cdot J_\nu^2\left(\frac{\omega x_0\sin\theta\cos\varphi}{c'}\right)\times$$
$$\times\left\{\left(\frac{\nu\omega_0 c'}{\omega\sin\theta\cos\varphi}\right)^2+(v_0^2-c'^2)\right\}\delta\left(\omega_0\nu+\frac{\omega v_0}{c'}\cos\theta-\omega\right). \tag{5}$$

Here we have introduced the notation $c' = \frac{c}{n}$, $n = n(\omega) = \sqrt{\varepsilon(\omega)\,\mu(\omega)}$. Setting $E_0 = 0$ in (5), we obtain the spectral-angular energy distribution of Cerenkov radiation:

$$W^{cer} = \frac{e^2}{c^2}\left(\frac{T}{v_0}\right)\int_0^{\infty}\mu(\omega)\,\omega d\omega\int_0^{\pi}\sin\theta d\theta\,(v_0^2-c'^2)\,\delta\left(\frac{c'}{v_0}-\cos\theta\right). \tag{6}$$

Integrating over θ in (6), we find the result of [13,14], which, at μ = 1, becomes the well-known formula of I. E. Tamm and I. M. Frank.

Formula (5) can be broken up into two terms: a first term due to the zeroth harmonic, and a second term that contains the sum of all the other harmonics. After integration over the frequency ω, this second term can be written

$$W = \frac{e^2}{c^2}\left(\frac{T}{2\pi}\right)\sum_{\nu=1}^{\infty}\int_0^{2\pi}d\varphi\int_0^{\pi}\sin\theta d\theta\,\frac{\mu(\omega_\nu)\,\omega_\nu^2 J_\nu^2(x_\nu)}{c'(\omega_\nu)\left|\frac{d}{d\omega}\left(\frac{\omega v_0}{c'}\right)\cos\theta-1\right|_{\omega=\omega_\nu}}\times$$
$$\times\left\{\left(\frac{\nu\omega_0 c'(\omega_\nu)}{\omega_\nu\sin\theta\cos\varphi}\right)^2+(v_0^2-c'^2(\omega_\nu))\right\}. \tag{7}$$

Here

$$\omega_\nu = \frac{\nu\omega_0}{\left|1-\frac{v_0\cos\theta}{c'(\omega_\nu)}\right|},\qquad x_\nu = \frac{\omega_\nu x_0\sin\theta\cos\varphi}{c'(\omega_\nu)}. \tag{8}$$

From the requirement $-1\leqslant\cos\theta\leqslant 1$, we obtain for the ν-th harmonic of the radiation the frequency range $\Delta\omega_\nu$, whose boundaries are determined from the solution of the equation

$$\omega\left(1-\frac{v_0}{c'}\cos\theta\right)-\nu\omega_0 = 0. \tag{9}$$

In the case of a vacuum, we obtain conditions (2) of the first paper cited in [7].

The term in (5) due to the zeroth harmonic at $v_0 > \frac{c}{n(\omega)}$, is characterized by a continuous spectrum that is naturally cut off at frequencies at which $\frac{v_0 n(\omega)}{c} = 1$. If $v_0 < \frac{c}{n(\omega)}$ (as is the case, for example, in a vacuum), this term vanishes because of the impossibility of a zero argument of the delta function for any angle θ. When

$$n = \text{const} \quad \text{and} \quad \frac{|x_\nu|}{\nu} < 1 \tag{10}$$

formula (7) admits of summation by harmonics, since the corresponding Kaptein series converges. We obtain the result

$$W = \frac{e^2}{c^2}\left(\frac{T}{2\pi}\right)\left(\frac{\mu x_0^2\omega_0^4}{16c'}\right)\int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta d\theta \cdot \{(1-\beta_z' \cos\theta)^2 + \tag{11}$$

$$+ (\beta_x'^2 - 1)\sin^2\theta\cos^2\varphi\}\frac{4(1-\beta_z'\cos\theta)^2 + (\beta_x'\sin\theta\cos\varphi)^2}{[(1-\beta_z'\cos\theta)^2 - (\beta_x'\sin\theta\cos\varphi)^2]^{7/2}}.$$

If inequality (10) is observed, the expression under the radical sign in (11) is positive.

Integrating (11) over the angle θ, we obtain

$$W = \frac{e^2}{c^2}\frac{\mu x_0^2\omega_0^4}{12c'(1-\beta_z'^2)}\left(\frac{T}{2\pi}\right)\int_0^{2\pi}\frac{M(\varphi)\,d\varphi}{(1-\beta_z'^2-\beta_x'^2\cos^2\varphi)^3}. \tag{12}$$

$$M(\varphi) = \beta_x'^4\cos^4\varphi - [4(1-\beta_z'^2)^2 + 3(1-\beta_z'^2)\beta_x'^2]\cos^2\varphi + 6(1-\beta_z'^2)^2.$$

$$\beta_x' = \frac{x_0\omega_0}{c'}, \quad \beta_z' = \frac{v_0}{c'} \tag{13}$$

It is evident from formulas (12) and (13) that to investigate the energy distribution over the angle φ it is sufficient to investigate it in the angle range $0 \leqslant \varphi \leqslant \frac{\pi}{2}$. We have, in particular,

$$\left(\frac{dW(\varphi)}{d\varphi}\right)_{\varphi=\frac{\pi}{2}} \sim \frac{6}{(1-\beta_z'^2)^2}, \quad \left(\frac{dW(\varphi)}{d\varphi}\right)_{\varphi=0} \sim \frac{(1-\beta_z'^2)+(1-\beta_x'^2-\beta_z'^2)}{(1-\beta_z'^2)(1-\beta_x'^2-\beta_z'^2)^2}.$$

We find the distribution of the radiated energy over the angle θ:

$$W = \frac{e^2}{c^2}\frac{\mu x_0^2\omega_0^4}{12c'}\left(\frac{T}{2\pi}\right)\int_0^{\pi}\frac{\sin\theta d\theta}{a_0^{3/2}}\{E(8\chi^3 + 5\chi^2 + 6\chi) + \tag{14}$$

$$+ K(-4\chi^2 - 3\chi) + q_0[E(-8\chi^3 + 5\chi^2 + \chi) + K(4\chi^2 - 2\chi)]\}.$$

$$K = F\left(\frac{\pi}{2}, k\right) = \int_0^{\pi/2}\frac{d\varphi}{\sqrt{1-k^2\cos^2\varphi}}, \quad E = E\left(\frac{\pi}{2}, k\right) = \int_0^{\pi/2}\sqrt{1-k^2\cos^2\varphi}\,d\varphi \tag{15}$$

are complete elliptic integrals of the first and second kinds:

$$k^2 = \frac{b_0}{a_0}, \quad \chi^{-1} = k'^2 = 1 - k^2, \quad a_0 = (1-\beta_z'\cos\theta)^2, \quad b_0 = (\beta_x'\sin\theta)^2, \tag{16}$$

$$q_0 = \frac{1-\beta_z'^2}{\beta_x'^2}. \tag{17}$$

It is seen from (14)-(17) that the emission of a relativistic particle occurs for the most part forward in the direction of its motion, with

$$\frac{\left(\frac{dW(\theta)}{d\cos\theta}\right)_{\theta=0}}{\left(\frac{dW(\theta)}{d\cos\theta}\right)_{\theta=\pi}} = \frac{(1+\beta_z')^3}{(1-\beta_z')^3}.$$

For the global (over the period T) energy of the radiation we obtain

$$W = \frac{T}{4}\frac{\mu e^2 x_0^2\omega_0^4}{c^2c'}(1-\beta_z'^2)^{-\frac{1}{2}}\left\{(1-\beta_z'^2)^{-1}\times\right. \tag{18}$$

$$\left.\times(1-\beta_x'^2-\beta_z'^2)^{-\frac{1}{2}} + \frac{1}{3}(1-\beta_x'^2-\beta_z'^2)^{-\frac{3}{2}}\right\}.$$

We see that expression (18) is real if

$$\beta_x'^2 + \beta_z'^2 < 1. \tag{19}$$

We also obtained the results (11)-(19) by generalizing formulas (73) and (11) of the monograph [15] to the case of a medium.

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