ANOMALOUS HALL EFFECT IN DISORDERED ALLOYS AT HIGH TEMPERATURES

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The coherent-potential approximation is used to calculate the anomalous Hall effect (AHE) of a disordered alloy in scattering of magnetized conduction-band electrons by an "impurity" scattering potential and the scattering potential related to the thermal vibrations of the lattice. At temperatures above the Debye temperature, the AHE constant can be represented as a linear-quadratic function of temperature. The calculated results are compared with experimental data.

A theory of the residual anomalous Hall effect in disordered alloys at arbitrary concentrations of the alloys components and at arbitrary values of the scattering potential associated with the chaotic disposition of the alloy-component ions over the lattice sites (we shall refer to this potential as the "impurity" potential) was recently constructed in [1]. In this paper, the AHE is investigated in alloys at finite temperatures. The scattering of conduction electrons by both the "impurity" potential and on thermal vibrations of the lattice is taken into account. A similar problem was considered earlier in [2], but only for weak "impurity" and electron-phonon interaction.

ANOMALOUS HALL CONDUCTIVITY

The Hamiltonian characterizing the conduction band of a disordered binary ferromagnetic alloy A_xB_y (y = 1 - x) in the absence of spin-orbital interaction at finite temperatures can be written in the form [1,3]:

$$H^{(0)} = \sum_{\substack{m,m' \\ m \neq m'}} t_{mm'} a_m^+ a_{m'} + \sum_m (\Sigma_{mm}) a_m^+ a_m + \sum_m (\varepsilon_{mm}^m + \theta_{mm}^m - \Sigma_{mm}) a_m^+ a_m = \\ = \tilde{H} + \sum_m V_{mm}^{m(0)} a_m^+ a_m, \tag{1}$$

where m and m' are the numbers of the lattice sites and the hopping integral in the Wannier representation,

$$t_{mm'} = N^{-1} \sum_{\mathbf{k}} \omega(\mathbf{k}) \exp[i\mathbf{k}(\mathbf{R}_m - \mathbf{R}_{m'})]$$

is determined by the dispersion law $\omega(\mathbf{k}) = \omega \mathbf{d}(\mathbf{k})$, where ω is the half-width of the band and $-1 \leqslant \mathbf{d}(\mathbf{k}) \leqslant 1$. The lattice of the alloy contains N sites and has a unit cell of cubic symmetry and volume Ω. The self-energy part $\Sigma_{mm}(z)$ does not depend on site number and is a function of the complex energy. The electron-phonon interaction is taken into account in the harmonic approximation

$$\theta_{mm}^m = \sum_{\mathbf{q}} (\gamma_{mmq}^m b_{\mathbf{q}} + \gamma_{mmq}^{m*} b_{\mathbf{q}}^+), \tag{2}$$

where γ_{mmq}^m is the matrix element of the electron-phonon interaction (γ^m, θ^m, b_q, and $t_{mm'}$ are assumed configurationally independent), and

$$V_{mm}^{m(0)}(z) = \varepsilon_{mm}^m + \theta_{mm}^m - \Sigma_{mm}(z) \tag{3}$$

are the matrix elements of the operator of potential scattering by the m-th site.

As was shown in [1], periodic and nonperiodic spin-orbital interactions (the nonperiodic interaction is determined at finite temperatures both by the "impurity" scattering potential and by the lattice-vibration potential) result in a change in the scattering potential. To reduce the space occupied by displays, we shall not separate the effects of the periodic and nonperiodic spin-orbital interactions in this paper. Then the total increment to the scattering potential in the n-th band at the m-th site can be written in the Bloch representation and in the linear approximation in the spin-orbital interaction in the form (see (5) and (6) in [1]):

$$V^{m(1)}_{nk,nk'} = sN^{-1}\exp[i\mathbf{R}_m(\mathbf{k}'-\mathbf{k})]\,\bar{\lambda}_{so_n}(\nu+1)r_0^2[\mathbf{k}\times\mathbf{k}']\,\frac{M_c^n}{M_0^n}\,\frac{V^{m(0)}_{mm}}{\omega}. \tag{4}$$

where r_0 is the lattice parameter, $\bar{\lambda}_{so_n}$ is the spin-orbital interaction constant, M_c^n is the modulus of the projection of the magnetization of the n-th band onto the direction of complete magnetization of the alloy M_c, M_0^n is the spontaneous magnetization of the n-th band at T = 0, ν is a parameter proportional to the ratio of the interband matrix element of the scattering potential to the intraband element, and s = ±1 for a band with spin index parallel (antiparallel) to the complete magnetization of the alloy.

Applying (1)-(4), we write the Hamiltonian of the problem

$$H = \tilde{H} + \sum_m V^{m(0)}_{mm}\,a^+_m a_m + \sum_{m,m',m''} V^{m(1)}_{m'm''}\,a^+_{m'}a_{m''}. \tag{5}$$

Since we shall be dealing with the electrical conductivity of magnetized electrons, the band index is equivalent to the spin index, and the parameters of a band with spin index parallel to the magnetization ($s=1$; $\varepsilon^{A(B)\uparrow}$) differ from those of a band with the opposite spin direction ($s=-1$; $\varepsilon^{A(B)\downarrow}$). The spin index and the arguments of the complex energy of the self-energy part and the Green's functions are omitted in all expressions in which this is possible (including (5)).

A method for calculation of the anomalous Hall conductivity within the framework of the coherent-potential approximation is described in detail in [1]. Following this method, we can easily obtain a self-consistent equation for determination of the self-energy part of the alloy:

$$\left\langle\left\langle \frac{\varepsilon^m + \theta^m - \Sigma(z)}{1-(\varepsilon^m+\theta^m-\Sigma(z))F(z)}\right\rangle\right\rangle_\phi = 0, \tag{6}$$

where $\langle\ \rangle$ indicates configurational averaging and $\langle\ \rangle_\phi$ averaging over the positions of the vibrating ions, while

$$F(z) = \frac{N\Omega}{(2\pi)^3}\int d^3\mathbf{k}\,[z-\omega(\mathbf{k})-\Sigma(z)]^{-1}. \tag{7}$$

Starting from the Kubo formula, assuming validity of the Born-Oppenheimer principle for the electron-phonon system of the alloy [3], and carrying out the calculations in much the same way as in [1], Part 3, we obtain an expression for $\sigma^{(1)\uparrow\downarrow}_{\alpha\beta}$—the contribution of the zones with spin index parallel ($\sigma^{(1)\uparrow}_{\alpha\beta}$) and antiparallel ($\sigma^{(1)\downarrow}_{\alpha\beta}$) to the magnetization of the alloy:

$$\sigma^{(1)\uparrow\downarrow}_{\alpha\beta} = \frac{e^2\hbar}{\pi\Omega}\int d\eta\left(-\frac{\partial f}{\partial\eta}\right)\frac{L^{\uparrow\downarrow}(\eta)}{\Delta^2(\eta)}[Im\Phi_{\alpha\alpha}(\eta-\Sigma(n^-))]^3. \tag{8}$$

Here

$$\Phi_{\alpha\alpha}(z) = \int d\xi N^{-1}(z-\xi)^{-1}\sum_{\mathbf{k}} v_{\alpha_{\mathbf{k}}}k_\alpha\,\delta(\xi-\omega(\mathbf{k})); \tag{9}$$

$$\Delta(\eta) = Im\Sigma(\eta^-);\quad \eta^\pm = \eta\pm i0; \tag{10}$$

$$L^{\uparrow\downarrow}(\eta) = -isr_0^2\frac{\bar{\lambda}_{so}}{\omega}\,\frac{M_c^{\uparrow\downarrow}}{M_0^{\uparrow\downarrow}}\left\langle\left\langle \frac{V^{m(0)}_{mm}(\eta^+)V^{m(0)}_{mm}(\eta^-)}{1-V^{m(0)}_{mm}(\eta^+)F(\eta^+)} - \right.\right. \tag{11}$$

$$\left.\left. - \frac{V^{m(0)}_{mm}(\eta^+)V^{m(0)}_{mm}(\eta^-)}{1-V^{m(0)}_{mm}(\eta^-)F(\eta^-)}\right\rangle\right\rangle_\phi(\nu+1).$$

It is shown in [3] that the averaging $\langle\ \rangle_\phi$ is done in accordance with the rule

$$\langle \psi(\theta^m) \rangle_\phi = \int d\xi P^m(\xi)\,\psi(\xi); \quad P^m(\xi) = (2\pi\alpha^m)^{-1/2} \exp\left[-\frac{\xi^2}{2\alpha^m}\right], \tag{12}$$

$$\alpha^m = |\gamma^m_{mmq}|^2 \coth\left(\frac{1}{2}\frac{\hbar\omega_q}{k_0 T}\right), \tag{13}$$

where ω_q is the phonon dispersion law and k_0 is Boltzmann's constant. Expression (8) describes asymmetric scattering of magnetized conduction electrons for arbitrary alloy-component concentrations, arbitrary values of both the "impurity" and electron-phonon scattering potentials, and an arbitrary temperature. L(η) characterizes the probability of asymmetric scattering, Δ(η) has the sense of the reciprocal relaxation time, and the behavior of the function $\Phi_{\alpha\alpha}(z)$ is determined by the alloy's dispersion law. Expressions (8)-(13) are investigated below to establish qualitative relationships in the concentration and temperature dependences of the AHE.

CONCENTRATION AND TEMPERATURE DEPENDENCE OF THE ANOMALOUS HALL CONDUCTIVITY OF ALLOYS AT HIGH TEMPERATURES

Since the temperature dependence of α^m is determined by the phonon dispersion law, consistent construction of a theory of the AHE in the temperature range in which the quantum nature of the lattice vibrations is manifested is not possible at the present time because the phonon spectra of alloys have not yet been studied. We shall therefore consider the case of high temperatures $T \gg T_D$ ($T_D = xT_D^A + yT_D^B$ is the Debye temperature of the alloy), when α^m is definitely proportional to T:

$$\alpha^m = g^m \frac{T}{T_D} \tag{14}$$

(the constants $g^{A(B)}$ characterize the strength of the electron-phonon interaction in pure metals, and are different for different conduction bands). Estimates of α^m [3] indicate that $\frac{\alpha}{\omega^2} < 1$. Below we shall consider only the extreme case of weak electron-phonon interaction, $\frac{\alpha}{\omega^2} \ll 1$. Then Σ and F can be expanded in power series in α/ω^2, which can be cut off at the linear terms for calculation of the AHE:

$$\Sigma = \Sigma^0 + \Sigma^\alpha; \quad F = F^0 + F^\alpha. \tag{15}$$

Here and below, the superscript 0 without parentheses indicates that the quantities are calculated for T = 0. It then follows from (6) that Σ^0 and Σ^α are determined by the equations

$$\langle (\varepsilon^m - \Sigma^0)[1 - (\varepsilon^m - \Sigma^0)F^0]^{-1} \rangle = 0, \tag{16}$$

$$-\Sigma^\alpha \langle [1 - (\varepsilon^m - \Sigma^0)F^0]^{-2} \rangle + \langle \langle \theta^{m2} F^0 [1 - (\varepsilon^m - \Sigma^0)F^0]^{-3} \rangle_\phi \rangle + F^\alpha \langle (\varepsilon^m - \Sigma^0)^2 [1 - (\varepsilon^m - \Sigma^0)F^0]^{-2} \rangle = 0. \tag{17}$$

For an arbitrary dispersion law, Σ^0, Σ^α, and $\Phi_{\alpha\alpha}$ (formulas (16), (17), and (9)) can be found only by machine computation. We shall therefore consider the following model of the alloy [3].

1. A semielliptic form of the density-of-states curves corresponds to the dispersion law ω(k):

$$\eta_0(E) = \frac{2}{\pi\omega^2}(\omega^2 - E^2)^{1/2}, \quad |E| \leqslant \omega,$$
$$\eta_0(E) = 0, \qquad\qquad |E| \geqslant \omega. \tag{18}$$

Here the Green's function corresponding to a given density of states is, in accordance with (1.36) of [1],

$$F_0(z) = \frac{2}{\omega^2}[z - (z^2 - \omega^2)^{1/2}]. \tag{19}$$

2. For a given dispersion law

$$N^{-1}\sum_{\mathbf{k}} v_{\alpha\mathbf{k}} k_\alpha \delta(\xi - \omega(\mathbf{k})) = D(\omega^2 - \xi^2)^{3/2}, \tag{20}$$

where D is a constant that characterizes the maximum velocity in the band. We shall assume $\omega = 1$.

Within the framework of the model considered (18)-(20),

$$F(z) = F_0(z - \Sigma(z)); \quad F^0(z) = F_0(z - \Sigma^0(z)); \Sigma(z) = z - \frac{1}{F(z)} - \frac{1}{4}F(z), \tag{21}$$

$$F^\alpha = \Sigma^\alpha \frac{4(F^0)^2}{4 - (F^0)^2} = -2\Sigma^\alpha \left[1 - \frac{z - \Sigma^0}{\sqrt{(z-\Sigma^0)^2 - 1}}\right], \tag{22}$$

$$Im\Phi_{\alpha\alpha}(\eta - \Sigma(\eta^-)) = D\frac{\pi}{32}\{\pi^3\eta^3(\eta) + 3\pi\eta(\eta)[4 - (ReF(\eta))^2]\}, \tag{23}$$

where $\eta(\eta) = \pi^{-1} ImF(\eta^-)$ is the density of states of the alloy with consideration of the electron-phonon interaction.

To find $\sigma^{(1)}_{\alpha\beta}$, it is necessary to solve the equation system (16)-(17), (21)-(22) and to use the resulting Σ^0, Σ^α, F^0, and F^α to calculate $\Delta(\eta)$; $L(\eta)$ and $Im\Phi_{\alpha\alpha}(\eta - \Sigma(\eta^-))$ (see (10), (11), and (23)). Equation (16) can be solved analytically only in the various limiting cases considered below. We denote $\varepsilon^A - \varepsilon^B = \delta$ (for the sake of argument, $\delta > 0$), $\bar{\alpha} = x\alpha^A + y\alpha^B$, $\overline{\in} = x\varepsilon^A + y\varepsilon^B$ and neglect the temperature spreading of the Fermi function, i.e., $-\frac{\partial f}{\partial \eta} = \delta(E_F - \eta)$.

Weak "impurity" scattering ($\delta \ll 1$). From Eq. (16) with $\delta \ll 1$, we have accurate to terms of order δ^2

$$\Sigma^0(z) = \overline{\in} + xy\delta^2 F_0(z - \overline{\in}).$$

Then, solving Eq. (17) and applying (22), we obtain

$$\Sigma(z) = \overline{\in} + (xy\delta^2 + \bar{\alpha})F_0(z - \overline{\in}); \; F(z) = F^0(z) + 4\bar{\alpha}(F^0(z))^3[4 - (F^0(z))^2]^{-1}.$$

From this

$$\Delta_1(\eta) = Im\Sigma(\eta^-) = (xy\delta^2 + \bar{\alpha})\pi\eta^0(\eta), \tag{24}$$

$$\pi\eta^0(\eta) = ImF_0(\overline{\eta} - \overline{\in}) = 2\sqrt{1 - (\eta - \overline{\in})^2}; \tag{25}$$

$$\pi\eta(\eta) = \pi\eta^0(\mu) - 2\bar{\alpha}\pi\mu^0(\eta)\left[1 - \frac{(\eta - \overline{\in})^2}{1 - (\eta - \overline{\in})^2}\right]. \tag{26}$$

Calculating $L(\eta)$ from (11), we represent it in the form

$$L^{\uparrow\downarrow}(\eta) = L^{0\uparrow\downarrow}_{(\eta)} + L^{\alpha\uparrow\downarrow}_{(\eta)} + L^{\alpha^2\uparrow\downarrow}_{(\eta)} = -sr_0^2(\nu + 1)\frac{M_\phi^{\uparrow\downarrow}}{M_0^{\uparrow\downarrow}}\bar{\lambda}_{s0}2\pi\eta^0(\eta) \times \tag{27}$$

$$\times [B_1^0(\eta) + B_1^\alpha(\eta) + B_1^{\alpha^2}(\eta)],$$

$$B_1^0(\eta) = xy(y - x)\delta^3 + 4xy(1 - 3xy)\delta^4(\eta - \overline{\in}) - 8x^2y^2\delta^4(\eta - \overline{\in}), \tag{28}$$

$$B_1^\alpha(\eta) = 3xy\delta(\alpha^A - \alpha^B) + 24xy(y - x)(\alpha^A - \alpha^B)\delta^2(\eta - \overline{\in}) + 8xy\bar{\alpha}\delta^2(\eta - \overline{\in}), \tag{29}$$

$$B_1^{\alpha^2}(\eta) = 4[3(x\alpha^{A^2} + y\alpha^{B^2}) - 2\bar{\alpha}^2](\eta - \overline{\in}), \tag{30}$$

$L^0(\eta)$ characterizes the probability of "impurity" scattering. The quantity $L^{\alpha^2}(\eta)$ relates to scattering by phonons in the alloy, and $L^\alpha(\eta)$ describes the probability of intereference scattering. Since α has the dimensions of energy squared ($\alpha \ll 1$), and δ those of energy, all terms of the orders δ^5; $\alpha^2\delta$; $\delta^2\alpha$ and higher were dropped in the present case of $\delta \ll 1$ to calculate $\sigma^{(1)}_{\alpha\beta}$ with the same accuracy:

$$Im\Phi_{\alpha\alpha}(\eta - \Sigma(\eta^-)) = \frac{\pi^4}{8}D[\eta^0(\eta)]^3. \tag{31}$$

Substituting (24) and (27)-(31) into (8), we obtain the following expression for the anomalous Hall conductivity at $\delta \ll 1$:

$$\sigma_{\alpha\beta}^{(1)\uparrow\downarrow} = -D' s \bar{\lambda}_{s0} \frac{M_c^{\uparrow\downarrow}}{M_0^{\uparrow\downarrow}} [\eta_{\uparrow\downarrow}^0(E_F)]^7 \frac{B_1^{0\uparrow\downarrow}(E_F) + B_1^{\alpha\uparrow\downarrow}(E_F) + B_1^{\alpha^2\uparrow\downarrow}(E_F)}{[\Delta^{\uparrow\downarrow}(E_F)]^3}, \tag{32}$$

where

$$D' = \frac{e^2\hbar}{\pi\Omega} D^2 \frac{\pi^7}{32} (\nu+1) r_0^2.$$

Strong "impurity" scattering ($\delta \gg 1$) (Fermi level in the neighborhood of ε^A). In this case, $|(\varepsilon^B - \Sigma^0(z)) F^0(z)| \gg 1$ for z near ε^A, and we have from (16) and (21), accurate to terms of order δ^{-1}

$$\Sigma^0(z) = \varepsilon^A - y(F^0(z))^{-1}; \qquad F^0(z) = 2[z - \varepsilon^A - \sqrt{(z-\varepsilon^A)^2 - x}].$$

Substituting Σ^0 and F^0 into (17) and applying (22), we obtain for $\alpha\delta \ll 1$

$$F^\alpha(z) = \frac{a^A}{x} F^0(z) \frac{4(F^0(z))^2}{4x - (F^0(z))^2}; \qquad \Sigma^\alpha(z) = \frac{a^A}{x} F^0(z) \frac{4 - (F^0(z))^2}{4x - (F^0(z))^2}. \tag{33}$$

Therefore,

$$\Delta_2(\eta) = \pi\eta^0(\eta) \left[-\frac{y}{y_x} + \frac{\alpha^A}{x} \left(1 + \frac{2y}{[\pi\eta^0(\eta)]^2} \right) \right]; \tag{34}$$

$$\pi\eta^0(\eta) = 2\sqrt{x - (\eta - \varepsilon^A)^2}; \tag{35}$$

$$\pi\eta(\eta) = \pi\eta^0(\eta) - \pi\eta^0(\eta) \frac{2\alpha^A}{x} \left[1 - \frac{(\eta-\varepsilon^A)^2}{x - (\eta-\varepsilon^A)^2} \right]; \tag{36}$$

$$Im\Phi_{\alpha\alpha}(\eta - \Sigma(\eta^-)) = D \frac{\pi^4}{8} [\eta^0(\eta)]^3 \left[1 + \frac{3y}{[\pi\eta^0(\eta)]^2} - \right. \tag{37}$$

$$\left. - \frac{3\alpha^A}{2x} \left(1 - \frac{2(\eta-\varepsilon^A)^2}{x - (\eta-\varepsilon^A)^2} \right) \right];$$

$$L^{\uparrow\downarrow}(\eta) = -sr_0^2(\nu+1) \frac{M_c^{\uparrow\downarrow}}{M_0^{\uparrow\downarrow}} \bar{\lambda}_{s0} 2\pi\eta^0(\eta) [B_2^0 + B_2^\alpha(\eta) + B_2^{\alpha^2}(\eta)]; \tag{38}$$

$$B_2^0 = -\frac{y}{4x}\delta; \qquad B_2^\alpha(\eta) = -\frac{y\delta\alpha^A}{2x[x - (\eta-\varepsilon^A)^2]}; \tag{39}$$

$$B_2^{\alpha^2}(\eta) = \left(\frac{\alpha^A}{x} \right)^2 4(\eta - \varepsilon^A).$$

Since $B_2^{\alpha^2} \ll B_2^\alpha$ at all concentrations except $y \ll 1$, the terms linear in y are omitted in the expression for $B_2^{\alpha\,2}$. Substituting (34)-(39) into (8), we obtain for the anomalous Hall conductivity at $\delta \gg 1$

$$\sigma_{\alpha\beta}^{(1)\uparrow\downarrow} = -D' s \bar{\lambda}_{s0} \frac{M_c^{\uparrow\downarrow}}{M_0^{\uparrow\downarrow}} [\eta_{\uparrow\downarrow}^0(E_F)]^7 \frac{B_3^{0\uparrow\downarrow}(E_F) + B_3^{\alpha\uparrow\downarrow}(E_F) + B_3^{\alpha^2\uparrow\downarrow}(E_F)}{[\Delta_3^{\uparrow\downarrow}(E_F)]^3}, \tag{40}$$

where

$$B_3^0(E_F) = -\frac{y}{4x}\delta \left(1 + \frac{3y}{[\pi\eta^0(E_F)]^2} \right),$$

$$B_3^\alpha(E_F) = \frac{y\alpha^A\delta}{4x^2} \left(1 + \frac{8(E_F - \varepsilon^A)^2}{[\pi\eta^0(E_F)]^2} \right), \tag{41}$$

$$B_3^{\alpha^2}(E_F) = \left(\frac{\alpha^A}{x} \right)^2 4(E_F - \varepsilon^A), \qquad \Delta_3(E_F) = \pi\eta^0(E_F) \left[\frac{y}{4x} + \frac{\alpha^A}{x} \right]. \tag{42}$$

Terms linear in y have been omitted in the expressions bearing exclusively on scattering by phonons.

Strong impurity scattering ($\delta \gg 1$). (Fermi level in the neighborhood of ε^B.) This case is equivalent to the one considered above with B substituted for A and y for x in all expressions of (40). As a result of this substitution, B_3^0 and B_3^α change sign.

Pure ferromagnetic metal A. In this case with $\delta=0$, $\varepsilon^B=\varepsilon^A$ and $\overline{\in}=\varepsilon^A$, it is easy to obtain in analogy to the case of weak "impurity" scattering

$$\sigma_{\alpha\beta}^{(1)\uparrow\downarrow} = -D' s\bar{\lambda}_{s0} \frac{M_c^{\uparrow\downarrow}}{M_0^{\uparrow\downarrow}} [\eta_{\uparrow\downarrow}^0(E_F)]^7 \frac{B_4^{\alpha^2\uparrow\downarrow}(E_F)}{[\Delta_4^{\uparrow\downarrow}(E_F)]^3}, \tag{43}$$

where

$$\Delta_4(E_F) = a^A \pi \eta^0(E_F); \qquad B_4^{\alpha^2}(E_F) = a^{A^2} 4 (E_F - \varepsilon^A). \tag{44}$$

In conclusion, we present two well-known expressions that are needed for discussion of the results obtained. In our nomenclature, the constant of the AHE is

$$R = -\frac{\sigma_{\alpha\beta}^{(1)\uparrow} + \sigma_{\alpha\beta}^{(1)\downarrow}}{4\pi M_c} \rho^2, \tag{45}$$

where ρ is the total resistivity of the alloy and the contribution to the ordinary conductivity of the conduction band $\sigma_{\alpha\alpha}^{\uparrow\downarrow} = \sigma^{\uparrow\uparrow}$ [3]:

$$\sigma^{\uparrow\downarrow} = \frac{\pi e^2 \hbar^2}{12\Omega} D \left(\frac{\partial^2 \omega(\mathbf{k})}{\partial \mathbf{k}^2}\right)_{E_F}^{-1} \frac{\eta_{\uparrow\downarrow}^3(E_F)}{\Delta^{\uparrow\downarrow}(E_F)} \left(1 + \frac{6\Delta^{\uparrow\downarrow}(E_F)}{\pi \eta_{\uparrow\downarrow}(E_F)}\right). \tag{46}$$

In all of the cases that we have considered, the second parenthesis in (46) can be neglected and if $\eta(E_F) \approx \eta^0(E_F)$,

$$\rho_{\uparrow\downarrow} = (\sigma^{\uparrow\downarrow})^{-1} = \frac{12\Omega}{\pi^2 e^2 \hbar^2 D} \left(\frac{\partial^2 \omega(\mathbf{k})}{\partial \mathbf{k}^2}\right)_{E_F} \frac{\Delta^{(\uparrow\downarrow)}(E_F)}{[\eta_{\uparrow\downarrow}^0(E_F)]^3}. \tag{47}$$

DISCUSSION OF RESULTS

It follows from formulas (32), (40), (43), and (45) that the AHE constant of an alloy in which the effect is carried simultaneously by conduction-band electrons with the spin index parallel (s = +1) and antiparallel (s = -1) to the magnetization can be written in the form

$$R = \frac{D'}{4\pi} \frac{\bar{\lambda}_{s0}}{M_c} \left\{ \frac{M_c^{\uparrow}}{M_0^{\uparrow}} [\eta_{\uparrow}^0(E_F)]^7 \frac{B_{\uparrow}^0 + B_{\uparrow}^{\alpha} + B_{\uparrow}^{\alpha^2}}{(\Delta^{\uparrow})^2} - \right.$$
$$\left. - \frac{M_c^{\downarrow}}{M_0^{\downarrow}} [\eta_{\downarrow}^0(E_F)]^7 \frac{B_{\downarrow}^0 + B_{\downarrow}^{\alpha} + B_{\downarrow}^{\alpha^2}}{(\Delta^{\downarrow})^2} \right\} \rho^2, \tag{48}$$

and the concentration dependences of all of the quantities in (48) differ considerably in the cases of weak and strong "impurity" scattering, while the temperature dependences remain unchanged.

At T = 0, $R \to R^0 \sim \frac{B^0}{(\Delta^0)^2} (\rho^0)^2$ in agreement with [1], and in the case of the pure metal (see (43)) $R^A \sim \rho^2 \sim T^2$ (see [4]). Therefore relation (48) correctly describes the limiting cases of the residual AHE and the AHE in pure metals, and Matthiessen's rule is not valid for the AHE constant for arbitrary impurity concentrations.

It is easily verified that the concentration dependences of B^0 and B^α for $\delta \ll 1$ (formulas (28)-(30)) agree with the corresponding ones obtained in [2] (with $\rho \sim \Delta$ and $\rho^0 \sim xy$). However, the concentration dependences of B^{α^2} differ slightly (last term of formula (35) in [2] and formula (30) of the present study). This is because the term B^{α_2}, which describes the probability of electron scattering on phonons, is determined in [2] by two-phonon processes ($B_{II}^{\alpha^2}$) and here by single-phonon processes ($B_I^{\alpha^2}$) (see (2)). Since two-phonon processes do not contribute to B^α [2] and do not affect the temperature dependence or the order of magnitude of B^{α^2} [4], exclusive consideration of single-phonon or two-phonon processes in the determination of B^{α^2} will not result in errors in a qualitative

theory. Nevertheless, both $B_I^{\alpha^2}$ and $B_{II}^{\alpha^2}$ must be taken into account in construction of a quantitative theory.

The probability of interference scattering is determined by the term B^{α}. At $\delta \ll 1$, by virtue of (29), interference arises basically due to the different ways in which the A and B ions vibrate ($B_I^{\alpha} \sim a^A - a^B$); then B^{α} is of the order of B^0. At $\delta \gg 1$, B^{α} (see (41)) appears as a result of renormalization of the Green's function under the influence of the electron-phonon interaction (see (36)-(39) and (41)-(42)) and $B^{\alpha} \ll B^0$.

Separating the temperature dependence explicitly in B^{α} and B^{α^2} (see (14), $B^{\alpha} = C^{\alpha} \frac{T}{T_D}$; $B^{\alpha^2} = C^{\alpha^2}\left(\frac{T}{T_D}\right)^2$) and applying (47), we can rewrite (48) in the equivalent form

$$R = \frac{D^* \bar{\lambda}_{s0}}{M_c}\left\{\frac{M_c^{\uparrow}}{M_0^{\uparrow}}\eta_{\uparrow}^0(E_F)\left[B_{\uparrow}^0 + C_{\uparrow}^{\alpha}\frac{T}{T_D} + C_{\uparrow}^{\alpha^2}\left(\frac{T}{T_D}\right)^2\right]\left(\frac{\rho}{\rho_{\uparrow}}\right)^2 - \right.$$
$$\left. - \frac{M_c^{\downarrow}}{M_0^{\downarrow}}\eta_{\downarrow}^0(E_F)\left[B_{\downarrow}^0 + C_{\downarrow}^{\alpha}\frac{T}{T_D} + C_{\downarrow}^{\alpha^2}\left(\frac{T}{T_D}\right)^2\right]\left(\frac{\rho}{\rho_{\downarrow}}\right)^2\right\}, \tag{49}$$

where D" is a constant that does not depend on the temperature or on the composition of the alloy. We begin by establishing the temperature dependence of the AHE. We assume that

$$\frac{\rho(T)}{\rho_{\uparrow\downarrow}(T)} = \frac{\rho(T_D)}{\rho_{\uparrow\downarrow}(T_D)}, \quad T \gg T_D, \tag{50}$$

and that, by virtue of the weakness of the electron-phonon interaction, the shift of the Fermi level under the influence of temperature spreading of the density of states of the alloy (see (20), (36)). We then obtain from (49)

$$R = a + b\frac{T}{T_D} + c\left(\frac{T}{T_D}\right)^2, \qquad T \gg T_D, \tag{51}$$

where a, b, and c do not depend on the temperature. It is shown in [5,6] that the linear-quadratic temperature function gives a good description of the temperature behavior of R in various alloys at $T < T_C$ (T_C is the Curie point).

Two remarks are in order concerning relation (51). Firstly, comparison of formula (51) for ferromagnetic alloys at $T < T_C$ (concerning the paramagnetic region, see below) is possible only for alloys with $T_C \gg T_D$. However, scattering on magnetic inhomogeneities may become significant in alloys with high Curie points. Since no theory of the AHE that takes account of scattering on magnetic inhomogeneities and the associated interference processes in alloys at $T_D < T < T_C$ has as yet been constructed, agreement of the temperature dependence expressed by (51) with experimental data in this temperature range does not constitute proof that these scattering processes are insignificant in determination of the AHE. Secondly, although a is proportional to R^0, relations (49)-(51) indicate that it does not coincide exactly with it.

Formula (51) remains in force in the parametric region, whence it follows that the temperature dependence in alloys at $T > T_C$ is much weaker than in pure metals above T_C and that a number of experimental difficulties will be encountered in detecting the temperature dependence of R at $T > T_C$, chief among which is the smallness of the AHE emf in the paramagnetic region. At the present accuracy of measurement of the emf's ($\sim 10^{-3}$ V), the temperature dependence of the AHE can be observed $T > T_C$ only in an alloy with a certain concentration composition at which $T_C \gg T_D$, so that it becomes possible to neglect scattering on magnetic inhomogeneities at $T \gg T_D$ and $a \approx 0$ and hence to obtain a sharper temperature dependence of R in this alloy as compared to the pure ferromagnetic metal. (In the pure ferromagnetic metal at $T > T_C$, $R \approx a^m + cT^2$, where the first term is related to scattering on magnetic inhomogeneities and the order of the second term, which is related to the scattering on phonons.) Since $a \sim R^0$ (R^0 can change sign at a certain concentration composition of the alloy outside of the range in which the concentration of one of its components is low [1]), these requirements are not contradictory, and experimental study of alloys possessing these properties may provide direct confirmation of the proposed theory. The temperature dependence of R at $T > T_C$ was observed in [7] in a Ni_xCu_y alloy containing

61.87% Ni (T_C = 230°K). Since R^0 was not separated in this study, it remains unclear whether $a \approx 0$ in fact in this alloy.

Since the extreme cases of weak ($\delta \ll 1$) and strong "impurity" scattering that we have considered definitely do not correspond to the actual values of δ in transition-metal alloys, great caution should be exercised in comparing the concentration dependences obtained in this study for B^0, B^a, B^{a^2} (see (28)-(30) and (41)-(42)), to which a, b, and c are, respectively, proportional, with experimental data.

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