APPROXIMATE CALCULATION OF THE ELECTRON SPATIAL DISTRIBUTION FUNCTION IN AN ELECTRON-PHOTON SHOWER WITHOUT CONSIDERATION OF IONIZATION LOSSES

A. K. Bakhtadze

Vestnik Moskovskogo Universiteta. Fizika,
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An approximate expression is found for the spatial distribution of the electrons and the number of particles in a circle of radius R with energy greater than E in a shower triggered by an electron or photon with an energy E_0, with the condition $E/E_0 \ll 1$ in the Landau approximation without consideration of ionization losses.

In the papers [1-8,9] on the cascade theory of electron-photon showers, the spatial distribution function of the particles in the shower was calculated by various approximate methods. In some of these studies, this problem was reduced to calculation of the first few moments with respect to the radius and to construction of the unknown function from these few moments. Unfortunately, the successive moments diverge with increasing atomic number, as was shown, for example, in [10], so that the function obtained in this way cannot be regarded as accurate. The most detailed theory of the solution of the initial equations was developed in [6,8]. Here the initial equations of the theory in the Landau approximation are transformed to differential-difference equations for the Fourier-Mellin-Laplace transforms of the unknown functions and these equations were solved only by approximate methods. In [8], equations were obtained only for the transforms of the angular and spatial-angular distributions. In [10], equations were obtained for the first time for the Mellin-Laplace transform of only the electron spatial distribution function without consideration of ionization losses, and exact analytic expressions were given for the Mellin transforms of the first four radius moments of this function under the condition $E/E_0 \ll 1$ (E is the energy of the particles in the shower and E_0 is the energy of the particle that initiated the shower). In [9], the first few moments of the angular and spatial distribution functions were calculated and an estimate was given for the behavior of the electron spatial distribution function for energies larger than E and finite values of E/E_0. A shortcoming of the method developed in [9] is the need to calculate mixed spatial-angular moments to obtain the unknown angle and radius moments.

This paper sets forth the consistent theory of the analytic solution of the initial equations, modernizing the theory developed in [8] and resulting in simpler equations for the same functions, which are determined by corresponding differential-difference equations. Having established the basic behavior of the solutions of these equations, we find an approximate analytic formula for its solution and, on the basis of this formula, calculate the approximate spatial distribution function for electrons with energies greater than E and the number of particles in a circle of radius R without consideration of ionization losses and under the condition $E/E_0 \ll 1$.

We shall also derive only approximate, but simple analytic expressions for the electron spatial distribution function and the number of particles in a circle of radius R, suitable for use in a subsequent theory of electron-photon showers and, in particular, for calculation of the fluctuations of the particle count in the shower and also for relatively quick numerical estimates. The initial equations of the theory were formulated in the Landau approximation in [1,8]. Without consideration of ionization losses they have the form

$$\left(\frac{\partial}{\partial t}+\theta\frac{\partial}{\partial r}\right)P(E_0, E, t, r, \theta)= A'P+B'\Gamma+\frac{E_k^2}{4E^2}\left(\frac{\partial^2}{\partial\theta_1^2}+\frac{\partial^2}{\partial\theta_2^2}\right)P, \qquad (1)$$

$$\left(\frac{\partial}{\partial t}+\theta\frac{\partial}{\partial \bar{r}}\right)\Gamma(E_0, E, t, \mathbf{r}, \theta)=C'P-\sigma_0\Gamma, \tag{2}$$

where P and Γ are the distribution functions of electrons and photons with energy E at depth t in cascade units with the direction of motion and the position determined by the vectors $\mathbf{r}$, θ, which lie in the plane normal to the axis of the shower, $\mathbf{r}=\{x, y\}$ determines the position of the particle in this plane at the depth t, and $t, \theta=\{\theta_1, \theta_2\}$, is the projection of the unit vector of the particle's direction of motion onto the plane normal to the shower axis. If the angle formed by the particle's vector with the shower axis is small, θ_1 and θ_2 are the angles between the shower axis and the projections of the particle's motion onto two mutually perpendicular planes that intersect along the shower axis. The form of the energy integrals of the operators A', B', and C' is given in [11].

To solve Eqs. (1) and (2), we multiply them by $E^s\exp[-\lambda t+i(\varkappa r)+i(\xi\theta)]$ and integrate over E from 0 to E_0, over t from 0 to ∞, and over Σ and $\bar{\theta}$ from $-\infty$ to $+\infty$. We obtain the equations

$$\left(\lambda-\varkappa\frac{\partial}{\partial\zeta}\right)P(E_0, s, \lambda, \varkappa, \zeta)=-A(s)P+B(s)\Gamma-$$
$$-\frac{E_n^2\zeta^2}{4}P(E_0, s-2, \lambda, \varkappa, \zeta)+f_1(E_0, s, \varkappa, \zeta), \tag{3}$$

$$\left(\lambda+\sigma_0-\varkappa\frac{\partial}{\partial\zeta}\right)\Gamma(E_0, s, \lambda, \varkappa, \zeta)=C(s)P+f_2(E_0, s, \varkappa, \zeta). \tag{4}$$

The functions $A(s)$, $B(s)$, $C(s)$ were defined in [1,12]. Here we have denoted

$$P(E_0, s, \lambda, \varkappa, \zeta)=\int\limits_{-\infty}^{\infty}\int dr\, d\theta\int\limits_0^{\infty}dt\int\limits_0^{E_0}dE\,E^s\exp[-\lambda t+i(\varkappa r)+i(\zeta\theta)]\times$$
$$\times P(E_0, E, t, \mathbf{r}, \bar{\theta}). \tag{5}$$

The function Γ is treated similarly.

The functions $f_{1,2}$ are the transforms of P and Γ at t = 0. We determine the electron spatial distribution function in the form

$$P(E_0, E, t, r)=\int\limits_{-\infty}^{\infty}d\theta\,P(E_0, E, t, \mathbf{r}\theta). \tag{6}$$

Setting ξ = 0 in (5), we obtain the transform for the spatial distribution function. Generalizing it, we obtain

$$P(E_0, s, \lambda, \mathbf{r})=\frac{1}{(2\pi)^2}\int\limits_{-\infty}^{\infty}d\varkappa e^{(-\varkappa r)}P(E_0, s, \lambda, \varkappa, \zeta=0). \tag{7}$$

Below we shall calculate only the spatial distribution function and assume ζ = 0; we can therefore regard the vectors $\varkappa$, ξ as collinear and substitute scalars for them in the term $\bar{\varkappa}\frac{\partial}{\partial\zeta}$ in Eqs. (3) and (4). We multiply (3) by the operator $\left(\lambda+\sigma_0-\varkappa\frac{\partial}{\partial\zeta}\right)$ and substitute (4) into (3):

$$\left[\left(\lambda+\sigma_0-\varkappa\frac{\partial}{\partial\zeta}\right)\left(\lambda-\varkappa\frac{\partial}{\partial\zeta}+A(s)\right)-B(s)C(s)\right]P(E_0, s, \lambda, \varkappa, \zeta)=$$
$$=\left(\lambda+\sigma_0-\varkappa\frac{\partial}{\partial\zeta}\right)\left[-\frac{E_n^2\xi^2}{4}P(E_0, s-2, \lambda, \varkappa, \zeta)+f_1(E_0, s, \varkappa, \zeta)\right]+ \tag{8}$$
$$+B(s)f_2(E_0, s, \varkappa, \zeta).$$

We shall assume that the particles are vertically incident on the boundary of the substance, i.e., that the boundary conditions depend on θ in the form of delta-functions; then $f_{1,2}$ will not depend on ζ. It follows from (8) that

$$\left(\lambda-\lambda_1(s)-\varkappa\frac{\partial}{\partial\zeta}\right)\left(\lambda-\lambda_2(s)-\varkappa\frac{\partial}{\partial\zeta}\right)P(E_0,s,\lambda,\varkappa,\zeta)=$$
$$=\left(\lambda+\sigma_0-\varkappa\frac{\partial}{\partial\zeta}\right)\left[-\frac{E_k^2\zeta^2}{4}P(E_0,s-2,\lambda,\varkappa,\zeta)\right]+ \tag{9}$$
$$+(\lambda+\sigma_0)\,\Phi(E_0,s,\lambda,\varkappa),$$

$$\Phi(E_0,s,\lambda,\varkappa)=f_1(E_0,s,\varkappa)+\frac{B(s)}{\lambda+\sigma_0}f_2(E_0,s,\varkappa). \tag{10}$$

We introduce the new variable $\xi=\zeta/\varkappa$. We represent the solution of Eq. (9) in the form

$$P(E_0,s,\lambda,\varkappa,\xi)=\sum_{n=0}^{\infty}\left(-\frac{E_k^2\varkappa^2}{4}\right)^n f(s,n,\lambda,\xi)\,\Phi(E_0,s-2n,\lambda,\varkappa)=$$
$$=\frac{1}{2\pi i}\int_{\beta-i\infty}^{\beta+i\infty}dq\left(\frac{E_k\varkappa}{4}\right)^{2q}\Gamma(-q)\,\Gamma(q+1)\,f(s,q,\lambda,\xi)\,\Phi(E_0,s-2q,\lambda,\varkappa), \tag{11}$$

$$-1<\beta<0$$

We substitute (11) into (9), equate terms with equal powers of $(-E_k^2\varkappa^2/4)$, and obtain

$$\left[\lambda-\lambda_1(s)-\frac{\partial}{\partial\xi}\right]\left[\lambda-\lambda_2(s)-\frac{\partial}{\partial\xi}\right]f(s,0,\lambda,\xi)=\lambda+\sigma_0, \tag{12}$$

$$\left[\lambda-\lambda_1(s)-\frac{\partial}{\partial\xi}\right]\left[\lambda-\lambda_2(s)-\frac{\partial}{\partial\xi}\right]f(s,n,\lambda,\xi)=$$
$$=\left(\lambda+\sigma_0-\frac{\partial}{\partial\xi}\right)[\xi^2 f(s-2,n-1,\lambda,\xi)],\quad n=1,2,\ldots \tag{13}$$

In the absence of scattering, i.e., at $E_k = 0$, we obtain the one-dimensional problem of shower development. Its solution is discussed, for example, in [1]. With arbitrary boundary conditions, this solution has the form

$$P(E_0,s,\lambda)=\Phi(E_0,s,\lambda)/\Psi(s,\lambda). \tag{14}$$

The function $\Psi(s,\lambda)$ is defined in [1].

At $E_k = 0$, we have from (11) and (14)

$$f(s,0,\lambda,\xi)=1/\Psi(s,\lambda). \tag{15}$$

This solution satisfies Eq. (12). We carry out the inverse Mellin transformation with respect to s in (11) in accordance with (5). We replace the variable s - 2q in the integrand by the variable s and obtain

$$P(E_0,E,\lambda,\bar{\varkappa},\xi)=\frac{1}{(2\pi i)^2}\int_{\alpha-i\infty}^{\alpha+i\infty}ds\times$$
$$\times\int_{\beta-i\infty}^{\beta+i\infty}dq\,E^{-s-1}\left(\frac{E_H\varkappa}{2E_1}\right)^{2q}\Gamma(-q)\,\Gamma(q+1)\,f(s+2q,q,\lambda,\xi)\times \tag{16}$$
$$\times\,\Phi(E_0,s,\lambda,\bar{\varkappa}),\ \alpha>0,\ -1<\beta<0,\ E\leqslant E_0.$$

The general solution of (13) can be written in integral form:

$$f(s+2n,n,\lambda,\xi)=-\sum_{i=1}^{2}H_i(s+2n)\exp\{[\lambda-\lambda_i(s+2n)]\,\xi\}\times$$
$$\times\left\{\int_0^{\xi}dv\exp\{-[\lambda-\lambda_i(s+2n)]\,v\}\,v^2 f(s+2n-2,n-1,\lambda,v)+C_{in}(s,\lambda)\right\}, \tag{17}$$

where the C_{in} are arbitrary constants that are chosen with the condition that the solution (17) does not depend on the exponential $\exp\{[\lambda-\lambda_i(s+2n)]\xi\}$.

Using the initial condition (15), we obtain successive solutions of (13) from (17) for n = 1,2,... . It can be shown by induction that these solutions satisfy the equation

$$f(s+2n, n, \lambda, \xi) = \varphi_0(s, \lambda)\left(\frac{\partial}{\partial\lambda} - \xi\right)^2 f(s+2n, n-1, \lambda, \xi) \quad n = 1, 2, \ldots \tag{18}$$

$$\varphi_m(s, \lambda) = \sum_{i=1}^{2} \frac{H_i(s)}{[\lambda - \lambda_i(s)]^{m+1}}. \tag{19}$$

The solution of (18) can be presented in the form

$$f(s+2n, n, \lambda, \xi) = \sum_{m=0}^{2n} \xi^m a_m(s, n, \lambda). \tag{20}$$

Substituting (20) into (17), we obtain the system of finite-difference equations for $a_m(s, n, \lambda)$ that was given and solved in [11].

We introduce the function

$$G(s, z, \lambda, \xi) = \frac{1}{2\pi i}\int_{\beta-i\infty}^{\beta+i\infty} dq\, z^{2q}\Gamma(-q)\Gamma(q+1) f(s+2q, q, \lambda, \xi). \tag{21}$$

$$-1 < \beta < 0$$

We write Eq. (13) in a form analogous to (8), replace s by s + 2q, and represent the functions $A(s+2q)$, $B(s+2q)$, $C(s+2q)$ by their integrals from [10]. We substitute q for q^{-1} in the integrand, transfer the contour of integration over q from the point β - 1 to point β, and consider the pole of the integrand at the point q = -1. Solving Eq. (13), we obtain for n = 0

$$f(s-2, -1, \lambda, \xi) = \frac{1}{\xi^2}(1 + Ce^{(\alpha+\sigma_0)\xi}). \tag{22}$$

We reject the exponential solution. As a result, we obtain the following equation for the function G:

$$\left(\lambda + \sigma_0 - \frac{\partial}{\partial\xi}\right)\left\{\left(\lambda - \frac{\partial}{\partial\xi} + \xi^2 z^2\right) G(s, z, \lambda, \xi) + \right.$$

$$\left. + \int_0^1 dx\, \varphi_1(x)\,[g(s, z, \lambda, \xi) - (1-x)^s G(s, z(1-x), \lambda, \xi)]\right\} - \tag{23}$$

$$- 2\int_0^1\int_0^1 dx\, dy\, \varphi_1(x)\,\varphi_2(y)\,(xy)^s G(s, z, xy, \lambda, \xi) = \lambda + \sigma_0.$$

We require

$$\lim_{\xi\to\infty} G(s, z, \lambda, \xi) = 0. \tag{24}$$

Under these conditions, the solution of (23) can be written

$$G(s, z, \lambda, \xi) = \exp\left[\frac{\xi^3 z^2}{3} + \lambda\xi\right] \times$$

$$\times \int_{\infty}^{\xi} d\zeta \exp\left[-\frac{\zeta^3 z^2}{3} - \lambda\zeta\right]\left\{\int_0^1 dx\, \varphi_1(x)\,[G(s, z, \lambda, \zeta) - \right. \tag{25}$$

$$\left. - (1-x)^s G(s, z(1-x), \lambda, \zeta)] - e^{(\lambda+\sigma_0)\zeta}\int_{\zeta}^{\infty} dv\, e^{-(\lambda+\sigma_0)v}\, 2\int_0^1\int_0^1 dx dy\,(xy)^s \times\right.$$

$$\left. \times \varphi_1(x)\,\varphi_2(y)\, dx dy\, G(s, zxy, v) - 1\right\}.$$

The functions $\varphi_1(x)$, $\varphi_2(x)$ were determined in [10]. It follows from (20) and (21) that the function $G(s, z, \lambda, \xi=0)$ can be expressed with the inverse Mellin transformation in terms of the function $a_0(s, q, \lambda)$. We obtain the equation for the latter from (18) by putting ξ = 0:

$$a_0(s, q, \lambda) = \varphi_0(s, \lambda)\frac{\partial^2}{\partial\lambda^2}a_0(s+2, q-1, \lambda),\ a_0(s, 0, \lambda) = \varphi_0(s, \lambda). \tag{26}$$

For positive integer q = n, the solution of (26) is a sum of exponentials $\exp[\lambda; (s+2m)t]$ (if we convert from the variable λ to the variable t by the inverse Mellin transformation with respect to λ). Leaving only the first exponential $e^{\lambda_j(s)t}$, which is equivalent to the condition $\frac{E}{E_0} \ll 1$ (see (16)), we obtain at $\xi = 0$

$$a_0(s, q, t) \simeq \sum_{i=1}^{2} H_i(s) M_{i0}(s, q) \exp[\lambda_i(s) t], \tag{27}$$

$$M_{i0}(s, q) = \frac{\partial^2}{\partial\lambda_i^2(s)}[\varphi_0(s+2, \lambda_i(s)) M_{i0}(s+2, q-1)],\ M_{i0}(s, 0) = 1. \tag{28}$$

The functions $M_{i0}(s,n)$ for n = 1, 2, 3, 4 were calculated in [11]. An approximate solution of (28) can be indicated. We make the substitution

$$\varphi_0(s+2n, \lambda_i(s)) \simeq \frac{1}{\lambda_i(s) + A(s)},$$

then

$$M_{i0}(s, n) \simeq \frac{(3n)!}{3^n n!\,(\lambda_i(s) + A(s))^{3n}}. \tag{29}$$

The dependence on s is not represented as well in this formula as in the exact formulas for integer n. Setting $\xi = 0$ in (20), (21), and (25), it can be shown that $a_0(s, q, \lambda)$ has simple poles at the points

$$q = -\frac{n+1}{3} - \frac{s}{2} + k,\ n, k = 0, 1, \ldots.$$

The following approximation for M_{i0} is therefore more accurate:

$$M_{i0}(s, q) \simeq T_{i0}(s, q) = \frac{1}{\Gamma\left(\frac{3}{2}s+1\right)} \frac{\Gamma\left(3q + \frac{3}{2}s + 1\right)}{5^q \Gamma(q+2)\,[\lambda_i(s) + A(s)]^{3q}}. \tag{30}$$

Table 1

Numerical Values of the Function $f_1(s,z)$ Calculated from Formula (32)

s \ E_0/E_k	0	10^{-3}	0,00316	10^{-2}	0,0316	0,1	0,316	1,0	3,16	10
0,5	0,552	0,514	0,477	0,412	0,311	0,185	0,0728	0,0140	$7{,}86\cdot10^{-4}$	$3{,}65\cdot10^{-6}$
1,0	0,968	0,872	0,795	0,672	0,501	0,303	0,130	0,0311	$2{,}83\cdot10^{-3}$	$4{,}13\cdot10^{-5}$
1,5	2,77	1,90	1,57	1,18	0,768	0,401	0,148	0,0313	$2{,}58\cdot10^{-3}$	$3{,}86\cdot10^{-5}$
2,0	∞	8,02	5,14	2,97	1,49	0,594	0,167	0,0266	$1{,}60\cdot10^{-3}$	$1{,}69\cdot10^{-9}$

The integral over q in (21) can be evaluated by putting $\xi = 0$ and applying (27), (30). We choose boundary conditions in the form of a single electron incident vertically on the boundary of the substance at the point $\bar{r}_0$.

We then obtain for the number of electrons with energy greater than E after integration over E in (16) and over $\bar{\varkappa}$ in (7)

$$N_p(E_0, E, t, \rho) = \frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty} ds \left(\frac{E_0}{E}\right)^s \left(\frac{E}{E_k}\right)^2 \sum_{j=1}^{2} H_i(s) F_j(s, z) e^{\lambda_j(s)t}, \tag{31}$$

$$F_j(s, z) = \frac{5}{2\pi} \frac{[\lambda_j(s) + A(s)]^3}{\Gamma\left(\frac{3}{2}s+1\right)} z^{\frac{s}{2}-1} \int_0^\infty dx\, e^{-\left(\frac{z}{x}\right)^{1/3}} x^{-\frac{s}{2}} \int_x^\infty \frac{dt}{t} e^{-t}, \tag{32}$$

$$z = 5\,[\lambda_i(s) + A(s)]^3 \left(\frac{E\rho}{E_k}\right)^2 = \left(\frac{E\rho}{E_k}\right)^{s-2} f_i(s, z), \qquad \rho = |\bar{r} - \bar{r}_0|. \tag{32}$$

Table 2

The Function $f_1(s,0)$ Calculated from Formula (33)

s	$f_1(s, o)$	Result from [8]	s	$f_1(s, o)$	Result from [8]
0	0,1592		1,1	1,112	
0,1	0,2402		1,2	1,312	
0,2	0,3239		1,3	1,599	
0,3	0,4047		1,4	2,040	1,8
0,4	0,4805				
0,5	0,5524				
0,6	0,6216	0,63	1,5	2,769	
			1,6	4,102	
			1,7	6,968	
0,7	0,6923		1,8	15,08	
0,8	0,7700		1,9	58,48	
0,9	0,8581		2,0	∞	∞
1,0	0,9683	1,01			

Table 3

Numerical Values of the Function $\Phi_1(s,v)$
Calculated from Formula (34)

s	ER/E_k								
	10^{-3}	0,00316	10^{-2}	0,0316	0,1	0,316	1,0	3,16	10
0,5	0,742	0,748	0,791	0,983	1,38	1,76	1,95	2,0	2,0
1,0	0,106	0,107	0,114	0,150	0,292	0,557	0,832	0,975	0,999
1,5	0,0121	0,0124	0,0144	0,0269	0,0811	0,221	0,442	0,617	0,664
2,0	0,00142	0,00152	0,00225	0,00703	0,0304	0,109	0,272	0,440	0,492

We can assume E = 0 in (31); we then obtain the number of particles with energy greater than 0, expressed in terms of $f_1(s,0)$. The so-called "axial approximation" for the function N_p, which is valid for small ρ, is expressed by the same formula with the function $f_1(s,0)$. It does not depend on the energy E:

$$f_i(s, 0) = \frac{5^{s/2}\,[\lambda_i(s) + A(s)]^{\frac{3}{2}s}\,\Gamma\left(1 - \frac{s}{2}\right)}{2\pi\,\Gamma\left(\frac{3}{2}s + 1\right)\left(1 - \frac{s}{2}\right)}. \tag{33}$$

We obtain the number of electrons in a circle of radius R by integrating (31) over ρ. On evaluation of the integral, we obtain

$$N_p(E_0, E, t, < R) = \left(\frac{E_0}{E}\right)^s H_1(s)\,\Phi_1(s, \nu)\,e^{\lambda_1(s)t} \times \left[2\pi\left(\lambda_1''(s)\,t + \frac{\partial^2}{\partial s^2}\ln\Phi_1(s, \nu)\right)\right]^{-1/2},$$

$$\Phi_1(s, \nu) = \frac{\nu^{s/2}}{2\Gamma\left(\frac{3}{2}s + 1\right)} \int_0^\infty dx\, e^{-\left(\frac{\nu}{x}\right)^{1/s}} x^{-\frac{s}{2}} \times \left[\int_x^\infty \frac{dt}{t} e^{-t} - \frac{e^{-x}}{x}\right] + \frac{1}{s}, \tag{34}$$

$$\nu = 5\,[\lambda_1(s) + A(s)]^3\,\frac{E^2R^2}{E_k^2}.$$

$$t = -\frac{1}{\lambda_1'(s)}\left[\ln\frac{E_0}{E} + \frac{\partial}{\partial s}\ln\Phi(s, \nu)\right].$$

In the axial approximation, which is valid for $\frac{ER}{E_k} < 10^{-2}$, we obtain the following expression for the number of electrons in a circle of radius R:

$$N_p(E_0, E, t, <R)_{oc} = \left(\frac{E_0 R}{E_k}\right)^s \frac{H_1(s)}{s} f_1(s, 0) \times$$

$$\times 2\pi e^{\lambda_1(s)t} \left[2\pi \left(\lambda_1''(s)\, t + \frac{\partial^2}{\partial s^2} \ln \frac{f_1(s, 0)}{s}\right)\right]^{-1/2},$$

$$t = -\frac{1}{\lambda_1'(s)} \left[\ln \frac{E_0 R}{E_k} + \frac{\partial}{\partial s} \ln \frac{f_1(s, 0)}{s}\right].$$

Numerical values of the functions $f_1(s,z)$ and $\Phi_1(s,v)$ are given in Tables 1-3. Table 2 shows that in the axial approximation our results agree closely with those of [8]. To compare Table 1 with the corresponding table of [8], $f_1(s,z)$ must be multiplied by s or the results of [8] must be divided by s, since the function in [8] was normalized to 1, and our function to 1/s.

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Scientific-Research Institute for Nuclear Physics