ON THE POSSIBILITY OF DETERMINATION OF THE ELECTRON ENERGY DISTRIBUTION FUNCTION IN A PLASMA FROM THE SPECTRAL LINE INTENSITIES

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The results of solution of model problems to determine the electron energy distribution from the spectral-line intensities are set forth. The accuracy with which the solution is reconstructed is analyzed as a function of the errors of the initial data.

A new method for calculation of the plasma electron energy distribution function from experimentally measured spectral-line intensities was proposed in [1,2], and preliminary calculations were made. Formulation of this problem was required by the fact that existing experimental methods of determining the distribution functions first of all do not yield satisfactory information on high-energy electrons and, secondly, introduce certain disturbances into the plasma, whereas spectroscopic methods are free from this shortcoming.

It is known that under certain conditions, when excited levels of the atoms are populated by direct excitation and cascade transitions from higher levels and are vacated by spontaneous downward transitions, the following formula is valid for the intensity of the i-th spectral line:

$$I(x_i) = N_0 N_e' h\nu(x_i) \int_{eV_k(x_i)}^{\infty} Q(x_i, V) f(V) \sqrt{V}\, dV, \qquad (1)$$

where N_0 is the density of the atoms in the normal state, N_e is the electron density, h is Planck's constant, V is the energy of the electrons, $f(V)$ is the electron energy distribution function, $I(x_i)$ is the intensity value, $\nu(x_i)$ is the frequency, $A(x_i V)$ is the optical excitation function, and $V_k(x_i)$ is the excitation potential of the i-th spectral line.

The dependence of the variables on x_i reflects their dependence on the indices of the levels between which the radiative transition occurs.

By solving equation system (1), written for various spectral lines, we can obtain information on electrons with high energies V, i.e., energies in the range $V > \min\{eV_k(x_i)\}$. Equation system (1) represents a Volterra equation of the first kind that is given at the reference points x_i and, after simple transformations (see [1,2]), can be reduced to a Fredholm equation of the first kind:

$$U(x) = \int_a^b K(x, V) f(V)\, dV, \qquad (2)$$

where

$$U(x) = I(x)/N_0 N_e h\nu(x),$$

$K(x, V) = Q(x, V)\sqrt{V}$ is the kernel of the integral equation (2), $f(V)$ is the unknown function, $a = \min\{eV_k(x_i)\}$, and b is chosen from the condition

$$\int_b^\infty K(x, V) f(V)\, dV \ll \int_a^b K(x, V) f(V)\, dV.$$

Solution of Eq. (2), i.e., determination of the function $f(V)$ from known $K(x,V)$ and $U(x)$, is a poorly conditioned problem and the result is not unique. However, an approximate solution of (2) can be obtained by using one of the Tikhonov-regularizing algorithms of [3], which will require imposition of certain qualitative constraints on the range of variation of the function and its derivative.

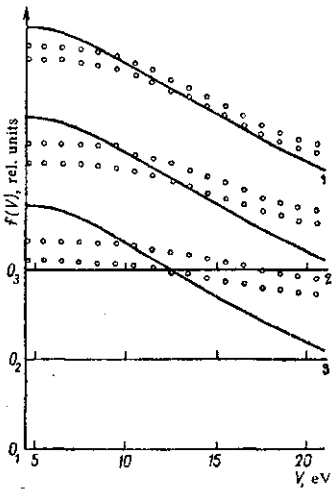

Fig. 1

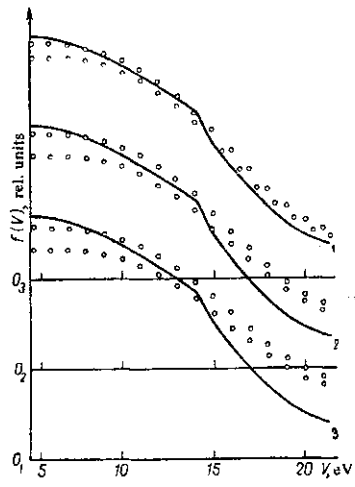

Fig. 2

Fig. 1. Assigned distribution $\exp(-V/10)\sqrt{V}$. Relative errors of the function $U(x)$: 1) 5%; 2) 10%; 3) 15%.

Fig. 2. Assigned distribution: $\exp(-V/10)\sqrt{V}$ for $V < 13.5$, $7.6\exp(-V/4)\sqrt{V}$ for $V \geq 13.5$. Legend as for Fig. 1.

Before applying the regularization method to calculation of the distribution function from experimental data, it is necessary to determine the effects of the inevitable errors of the initial data, i.e., of the quantities $I(x_i)$, N_0, N_e, and $Q(x_i,V)$, on the accuracy of the reconstructed distribution, or, in other words, to establish the possibilities of the method as applied to our specific problem. For this purpose, we solved a series of model problems the idea of which consisted in calculation of values of the function $U(x)$ from a given distribution and given excitation functions and then solving the inverse problem, with the $U(x)$ and $Q(x,V)$ put in with certain predetermined errors. The errors of the solution obtained can be judged from the deviation of the calculated function from the given function. The calculations were performed on a BESM-4 computer using the program of [4]. In our calculations, the regularizing functional had the form

$$\Omega[f(V)] = \int_a^b (K_1(V) f'(V)^2)\, dV, \qquad (3)$$

with $K_0(V) \equiv 0$, $K_1(V) \equiv 1$, i.e., limitations were imposed only on the derivative of the unknown function.

For initial distribution functions, we used 1) the Maxwell distribution with $T_e = 10$ eV, 2) a distribution with a fast-electron deficiency, and 3) the distribution $\exp(-V/10)(1 + 0{,}5\sin 0{,}628\,V)\sqrt{V}$.

This last distribution was necessary to establish the sensitivity of the method to the appearance of "fine" structure in the distribution. The calculations were made from the intensities of 7 lines, and the electron distribution was obtained at 36 points. The values of the $U(x)$ were put into the machine with various relative errors. The inverse problem was solved twice for each series of values of $U(x)$: once with the kernel overstated and again with the kernel understated by the same amount. Figures 1-3 present the

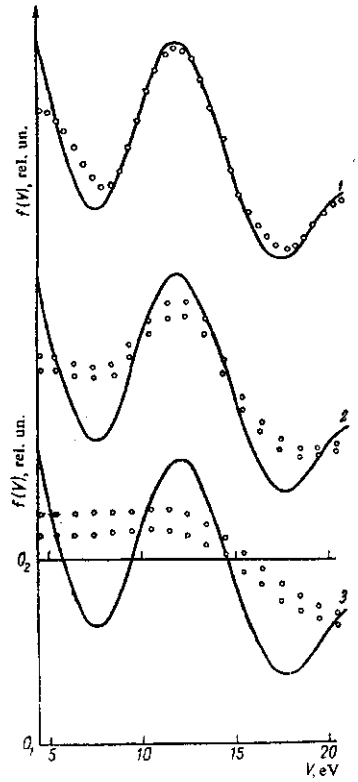

Fig. 3. Assigned distribution $\exp(-V/10)(1+0{,}5\sin\times 0{,}628V)\sqrt{V}$. 1) relative errors of the function U(x): 1) 1%; 2) 5%; 3) 15%. Curve 1 was calculated with the exact kernel.

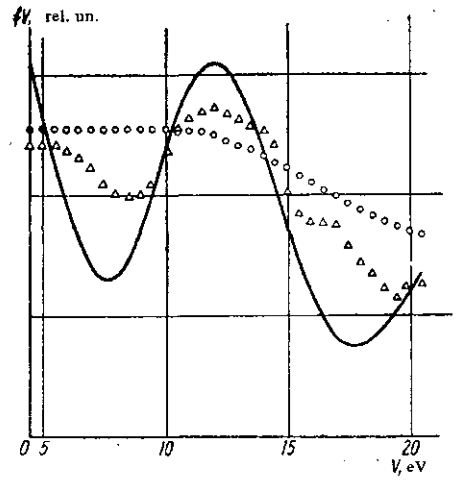

Fig. 4. Assigned distribution $\exp(-V/10)(1+0{,}5\sin 0{,}628\ V)\sqrt{V}$. Relative error of the function U(x) 15%.

results obtained in sim

results of the calculations. The solid curves represent the assigned distributions and the circles the computed results. The lower curve corresponds to the calculations made with the high kernel and the upper curve to those with the low kernel. The figures show that "smoothing" is a common shortcoming of the solutions. This is especially conspicuous at the ends of the curves, since we used the natural boundary conditions $f'(V)|_{a,b}=0$. It is therefore reasonable to reject the values calculated at the first and last few points.

The figures show that distributions 1 and 2 are reconstructed quite well. Thus, with a 15% error, the accuracy of the solutions is no worse than 35-45%, i.e., of the order of the accuracy obtainable by the sonde method. Distribution 3 is not reconstructed nearly as well with the same input-data errors. However, it should not be concluded that "oscillations" cannot be detected at all by the regularization method. The solution obtained with a 1% error in U(x) is quite good, especially in the center (Fig. 3).

It is seen from Figs. 1-3 that, owing to the strong dependence of the error of the computed distributions on the error of the measurements, the experimental accuracy requirements become very stringent. It is therefore legitimate to inquire as to the possibilities for improving the accuracy of solution reconstruction. The regularizing algorithm (3) that we used is based on selection, from among all of the possible solutions of (2), of a solution with a certain degree of smoothness, i.e., some qualitative information on the solution sought is used.

We usually know much more concerning the sought distribution function, e.g.: it becomes a decreasing function beginning at a certain energy. All known information on the solution can be used if the regularizing functional $\Omega[f(V)]$ is represented in the form [5]

$$\Omega[f(V)]=\int_a^b K_0(V)f(V)^2+K_1(V)f'(V)^2\,dV, \qquad (4)$$

$$K_0(V)=\int_c^d K(x,v)U(x)/f_0(V)\,dx,\ K_1(V)=\text{const},\ c\leqslant x\leqslant d.$$

For the zeroth approximation of the unknown distribution function $f_0(V)$, we can use a Maxwell distribution with the electron temperature obtained by the sonde method or a distribution measured by the second-derivative method and extrapolated to large energies, or results obtained in similar experiments.

If, on the other hand, there is no reliable information on the zeroth approximation of the distribution function, or it is necessary to use only the results of spectroscopic investigations, the following scheme can be used to calculate the distribution function. First, the distribution is calculated with the regularizing functional (3), and then it is used as the zeroth approximation for subsequent calculations with the functional (4).

However, in using the method described above to calculate the zeroth approximation of the unknown solution, much attention must be paid to the choice of the regularization parameter α. If a solution obtained with $\alpha \leq \alpha_{opt}$ is chosen as the zeroth approximation, it may possess a structure that has nothing in common with the sought solution, so that the solution may be distorted instead of improved. It is more advantageous to use a solution obtained with $\alpha \geq \alpha_{opt}$ as the zeroth approximation. The resulting solution will be somewhat smoothed in this case. On this basis, we may assume with confidence that if the solution obtained exhibits any structure, the structure is definitely present in the actual distribution as well, i.e., the solution obtained conveys the basic features of the true solution in spite of the smoothing. Figure 4 shows the results of calculations made by the above scheme for a model problem with an initial distribution function of the form 3. The solid curve represents the assigned distribution, the open circles the solution obtained with the regularizing functional in the form (3), and the ∇ the solution with Ω in the form (4). The solution indicated by the circles was used as the zeroth approximation. Figure 4 shows the result of solution of the model problem with the low kernel. The curve calculated with the high kernel is of similar shape. It is seen from comparison of the solutions (Figs. 3 and 4) that the distribution obtained with the zeroth approximation is much closer to the assigned distribution.

From analysis of the results, therefore, we may conclude that the described method for calculation of the electron energy distribution function from the spectral line intensities is a promising one.

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