ON THE MOTION OF ASTEROIDS OF THE HECUBA TYPE

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The group of Hecuba-type asteroids is interesting in that their mean motions are nearly 1:2 commensurable with the mean motion of Jupiter. This commensurability corresponds to a gap in the mean-motion distribution of the asteroids. N. D. Moiseev's averaged scheme is usually used to investigate the motion of asteroids in the case of resonance because application of the classical perturbation theory is impossible here. However, such schemes have been justified only for a time interval on the order of a thousand years, so that it appears reasonable to use numerical integration methods to study the motion of resonant asteroids over long time intervals. The present paper investigates the evolution of the orbits of asteroids of this group over a time span on the order of 2500 years within the framework of the plane circular limited three-body problem.

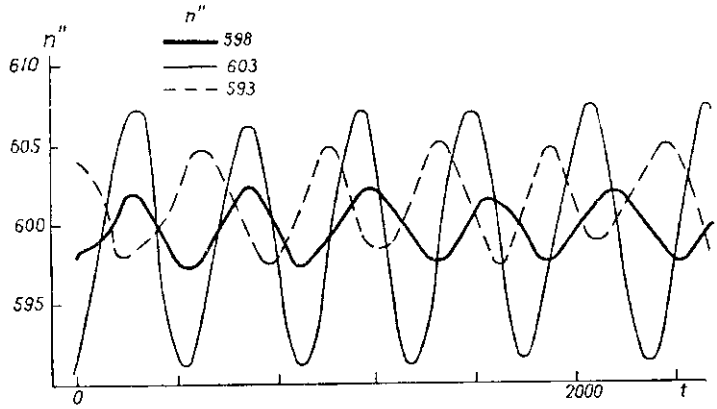

Fig. 1

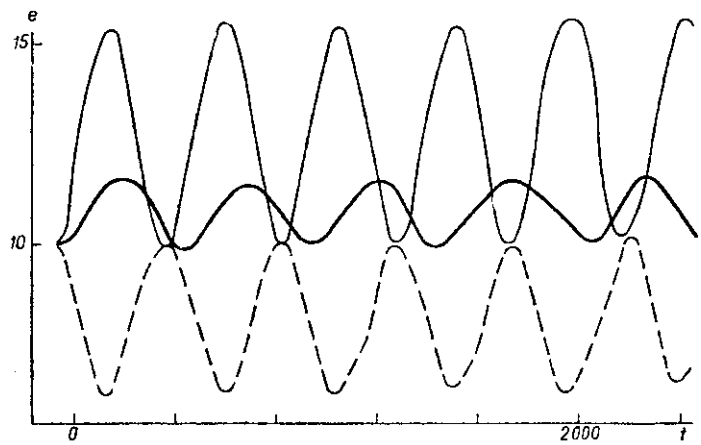

Fig. 2

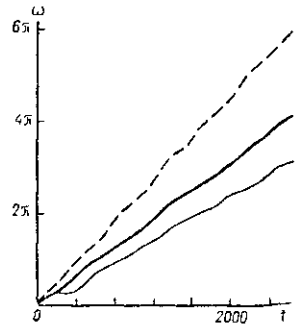

Fig. 3

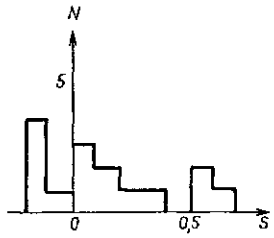

Fig. 4

The equations of perturbed motion of an asteroid in the heliocentric coordinate system can be written in the form [1]

$$\frac{d^2x}{dt^2} = -k^2\left\{m_0\frac{x}{r^3} - m_j\left(\frac{a\cos n_j t - x}{\Delta^3} - \frac{\cos n_j t}{a^2}\right)\right\},$$
$$\frac{d^2y}{dt^2} = -k^2\left\{m_0\frac{y}{r^3} - m_j\left(\frac{a\sin n_j t - y}{\Delta^3} - \frac{\sin n_j t}{a^2}\right)\right\}, \qquad (1)$$

where $\Delta = \sqrt{(x - a\cos n_j t)^2 + (y - a\sin n_j t)^2}$ is the distance between Jupiter and the asteroid, $r = \sqrt{x^2 + y^2}$ is the length of the asteroid's radius vector, m_0 and m_j are the masses of the sun and Jupiter, a is the radius of Jupiter's orbit, n_j is its mean motion, k^2 is the Gauss constant, and t is time.

The following system of units was adopted: the astronomical unit was the unit of length, the sun's mass the mass unit, and the mean solar day the unit of time. In this system [2]:

$$k = 0{,}0172099, \quad m_j = \frac{1}{1047{,}39},$$
$$a = 5{,}2032354, \quad n_j = 0{,}00145003$$

System (1) was integrated by the Runge-Kutta method on a BESM-4M computer accurate to the fourth power of the step. The step was chosen as follows: in system (1), we put $m_j = 0$ and integrated the equations of the unperturbed motion over a time interval of 200 years with initial conditions corresponding to circular motion. The difference between the exact solution found from the Keplerian formulas and the solution obtained by integration was printed out at periodic intervals. This operation was continued for step values from two to twenty days. The minimum was then found on the difference vs. step plot; in our case, it corresponded to a five-day step. Further integrations of the equations of perturbed motion were carried out with this step and checked with a Jacobi constant computed to nine places, seven of which remained constant.

The initial conditions chosen were

$$t = 0, \quad y = 0, \quad x = A(1 - e),$$
$$\frac{dx}{dt} = 0, \quad \frac{dy}{dt} = k\sqrt{\frac{1+e}{x}},$$

where e is the initial eccentricity and equals 0.1, A is the initial value of the semimajor axis, which was chosen as follows: $A_1 = 3.2767903$ for variant 1, which corresponds to exact commensurability, $A_2 = A_1 + 0.02$ for variant 2, and $A_3 = A_1 - 0.02$ for variant 3.

The results of integration appear in Figs. 1-3. The results indicate that the eccentricity and mean motion of the asteroid vary periodically, and that a secular term is present in the variation of the longitude of perigee.

The eccentricity in variant 2 is larger than that in variant 3, an apparent reflection of Braun's rule [3].

The mean motions vary in such a way that the mean value exceeds the resonant value for all variants. Such effects have long been known in the form of Schütte's rule, which

states that asteroids tend to accumulate on the inner (the value of the mean motion exceeds the resonant value) sides of gaps.

We considered Schütte's rule for all commensurabilities up to and including that of order 9 [4,5]. For each value n of the mean motion corresponding to exact resonance, we chose Δn such that the number of asteroids in the interval $(n-\Delta n, n+\Delta n)$ would exceed 20. We then computed the number of asteroids in the interval $n + \Delta n$ (denoted by N_1) and in the interval $n - \Delta n$ (N_2). We then calculated the coefficient $S=\frac{N_1-N_2}{N_1+N_2}$, the distribution of which is represented in Fig. 4; as we see, Schütte's rule is satisfied in the mean for all asteroids. The average value is S = 0.15, and the average for asteroids of the Hecuba type is S = 0.56.

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