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DEVELOPING FUNDAMENTAL CONSTANTS IN THE SIX-DIMENSIONAL UNIVERSE

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Within the framework of the six-dimensional geometrical theory of gravi-electroweak interactions, the possible variation of fundamental physical constants in developing homogeneous isotropic cosmological models is considered. In this theory the electric charges and the masses depend on time, and the cosmological shift of light spectral lines is due to two factors: the Doppler effect and the variation of fundamental constants during the time of light propagation. Because these factors strengthen each other, the age of the Universe turns out to be greater than in the standard Friedmann model.

1. The modern investigations of many-dimensional physical theories are developing in the following three main directions: 1) the geometrization of physical interactions, which contributes to the construction of a unified geometrical theory including gravitation; 2) the description of the possible variation of physical constants (such as the explanation of the well-known Dirac hypothesis, the realization of the Mach principle and some other ideas); and 3) the investigation of theories with supersymmetries and supergravitation. In the present paper the first two of these directions are combined.

The investigation is carried out within the framework of the six-dimensional model of gravi-electroweak interactions [1], in which the ideas of the Kaluza five-dimensional classical theory are generalized [2]. In this six-dimensional theory the electromagnetic and the Z-boson fields are described by the components $G_{\mu 5}$ and $G_{\mu 6}$ of the six-dimensional metric, respectively, where $\mu = 0, 1, 2, 3$. The charged W-bosons are due to the dependence of these components on the additional dimensions. The scalar Higgs bosons are introduced via the conformal factor in the metrics. In the present theory the topology of a torus is used, i.e., the additional dimensions are compactified. The six-dimensional theory implies, as particular cases, both the Einstein general relativity theory and the well-known Weinberg–Salam model of electroweak interactions.

2. An important feature for further presentation is that in this six-dimensional theory a number of fundamental physical constants are expressed in terms of the components G_{55} and G_{66} of the metric. For instance, for the electric charge e, the constant $\overline{g}$ characterizing the interaction with the Z-boson, the Weinberg angle θ_W, and the mass m_Z of the Z-boson the following expressions are obtained:

$$e = \frac{4\sqrt{k}\hbar\alpha}{c}\frac{\lambda^5\sigma^6}{\sqrt{(\sigma^6)^2+(\lambda^5)^2}}, \qquad \overline{g} = \frac{4\sqrt{k}\hbar\alpha}{c}\sqrt{(\sigma^6)^2+(\lambda^5)^2}, \tag{1}$$

$$\sin\theta_W = \frac{\lambda^5}{\sqrt{(\sigma^6)^2+(\lambda^5)^2}}, \qquad m_Z = \frac{4\sqrt{5}}{c}\hbar(b_0\eta_0)\alpha\sqrt{(\sigma^6)^2+(\lambda^5)^2},$$

where k is the Newton constant, $\hbar$ is the Planck constant, α is a parameter characterizing the compactification radius for the additional dimensions, $b_0\eta_0$ is the mass factor, and λ^N and σ^N are the components of two orthonormalized vectors orthogonal to the metric of the four-dimensional space-time section. These components are determined by the $1+1+4$-splitting of the six-dimensional metric:

$$G_{MN} = g_{MN} - \lambda_M\lambda_N - \sigma_M\sigma_N, \tag{2}$$

where M and N assume the values 0, 1, 2, 3, 5, and 6. In a special gauging similar to the two-fold

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chronometric gauging in general relativity theory, the components λ^N and σ^N have the following form:

$$\lambda^N = \left\{0;\ 0;\ 0;\ 0;\ \frac{\sqrt{-G_{66}}}{\sqrt{G_{55}G_{66} - G_{56}^2}};\ \frac{G_{56}}{\sqrt{-G_{66}(G_{55}G_{66} - G_{56}^2)}}\right\}, \tag{3}$$

$$\sigma^N = \{0;\ 0;\ 0;\ 0;\ 0;\ 1/\sqrt{-G_{66}}\}.$$

3. In some of our previous works [3, 4], the possibility of variation of the fundamental physical constants (1) was analyzed within the framework of static, cylindrical (with respect to the additional coordinates x^5 and x^6), and spherically symmetric (with respect to the classical space coordinates) solutions to the six-dimensional Einstein equations. In particular, the possible effects of scalarism due to the ellipticity of the Earth's orbit were discussed.

4. In the present paper we analyze the possible consequences of the variability of physical constants within the framework of six-dimensional homogeneous isotropic (in the three-dimensional sense) cosmological models [5] obtained from solutions to the six-dimensional Einstein equations

$${}^6R_{MN} - (1/2)G_{MN}{}^6R = \varkappa T_{MN}, \tag{4}$$

where T_{MN} is the six-dimensional energy-momentum tensor of nongeometric matter. The metric was sought in the form

$$dI^2 = \varphi^2\{a^2[dx^{0^2} - dx'^2 - F^2(d\theta^2 + \sin^2\theta\, d\varphi^2)] - dx^{5^2} - \psi^2\, dx^{6^2}\}, \tag{5}$$

where $G_{55} = -\varphi^2$ and $G_{66} = -\varphi^2\psi^2$ are the additional diagonal components of the metric; the function $F^2(x^1)$ can have three forms: $F^2 = \sinh^2 x^1$ for a hyperbolic spatial section, $F^2 = x'^2$ for the Euclidean space, and $F^2 = \sin^2 x^1$ for the three-dimensional Riemann spatial section. Four functions of the time-like coordinate x^0 were determined from the equations: $a(x^0)$, $\varphi(x^0)$, $\psi(x^0)$, and the density $\mu(x^0)$ of pulverized matter. A number of exact solutions were obtained for open cosmological models. We particularly stress that the components G_{55} and G_{66} were interpreted as squares of the potentials of additional (hypothetical) scalar fields. On this basis, in [5] the vacuum solutions (for $C_{(2)} = 2$) were interpreted as unrealistic and the third and fourth solutions (for $C_{(2)} = 8/3$) in formulas (25) were considered physically unacceptable because they corresponded to a negative density of matter.

However, a fundamentally different view on the physical meaning of the additional components G_{55} and G_{66} of the six-dimensional metric is possible: these can be regarded as a geometrical image of the substance filling the Universe. In this case the above-mentioned solutions become realistic, whereas the first and second solutions in formulas (25) in [5] turn out to be false.

In particular, let us consider the third of the solutions indicated in formulas (25) in [5], which corresponds to an open expanding cosmological model:

$$a = a_0 e^{\sqrt{3}x^0};\quad \varphi = \varphi_0 e^{-x^0/\sqrt{3}};\quad \psi = \psi_0 = \text{const}, \tag{6}$$

where a_0, φ_0, and ψ_0 are integration constants. Transferring the terms with G_{55} and G_{66} in the Einstein six-dimensional equations to the right-hand side and interpreting the whole right-hand side via the effective density and pressure of the substance filling the Universe, we find a positive value for the substance density:

$$\mu_{\text{ef}} = 26/3\varkappa a^2\varphi^2, \tag{7}$$

where $\varkappa$ is the Einstein gravitational constant. Now the condition $\dot\psi/\psi = 0$ used in [5] corresponds in fact to the setting of an equation of state for the medium (a perfect fluid) filling the Universe. In the particular case under consideration we have $p < 0$. However, a number of other exact solutions are also obtained for which the equations of state of the medium are related to the conditions $(\overline{\alpha} - 1)\dot\varphi/\varphi = \dot\psi/\psi$, where $\overline{\alpha}$ is a parameter.

5. Within the entire class of the above cosmological models, Eqs. (1) take the form

$$e = \frac{4\sqrt{k}\hbar\alpha}{c\sqrt{1+\psi_0^2}}\,\frac{1}{\varphi} \equiv \frac{e_0}{\varphi};\qquad \overline{g} = \frac{4\sqrt{k}\hbar\alpha}{c}\,\frac{\sqrt{1+\psi_0^2}}{\psi_0}\,\frac{1}{\varphi} \equiv \frac{\overline{g}_0}{\varphi}; \tag{8}$$

$$\sin\theta_W = \frac{\psi_0}{\sqrt{1+\psi_0^2}} = \text{const};\qquad m_Z = m_{Z_0}/\varphi.$$

It follows that the Weinberg angle θ_W remains constant, whereas e, $\overline{g}$, and m_Z become functions of the time-like coordinate x^0, i. e., they increase in the expanding cosmological model (6).

It should be stressed that x^0 has the meaning of only a time-like parameter from which the physical time τ is obtained, as in the Friedmann model, with consideration for the scale factor. However, in the six-dimensional theory there are two versions of choice of time. In the first version the quantity a is taken as the scale factor, and then we have

$$d\tau = a(x^0)\,dx^0 \to \tau = \int a(x^0)\,dx^0 = \frac{1}{\sqrt{3}}a_0(e^{\sqrt{3}x^0} - 1), \tag{9}$$

where the integration constant is chosen so that $\tau = 0$ for $x^0 = 0$. Substituting (9) into (6) we obtain

$$a = a_0 + \sqrt{3}\tau, \quad \varphi = \varphi_0(1 + \sqrt{3}\tau/a_0)^{-1/3}. \tag{10}$$

In the other version the expression $a(x^0)\varphi(x^0)$ is taken as the scale factor, and then we have

$$d\widetilde{\tau} = a(x^0)\varphi(x^0)\,dx^0 \to \widetilde{\tau} = \int a\varphi\,dx^0 = \frac{\sqrt{3}}{2}a_0\varphi_0(e^{\frac{2}{\sqrt{3}}x^0} - 1) \tag{11}$$

for the same concept of choice of the integration constant. In this case

$$a = a_0\left(\frac{2\widetilde{\tau}}{\sqrt{3}a_0\varphi_0} + 1\right)^{3/2}; \quad \varphi = \varphi_0\left(\frac{2\widetilde{\tau}}{\sqrt{3}a_0\varphi_0} + 1\right)^{-1/2}. \tag{12}$$

We shall analyze the two versions.

6. Recall that in the four-dimensional general relativity theory the evolution of the Universe is described by solutions to the Friedmann equations. Proceeding from the condition of correspondence to the experimental data on the Hubble red shift of spectral lines, only the solutions describing the expanding Universe were chosen. As is known, the line shift is determined by the formula

$$\frac{\Delta\nu}{\nu} = \frac{l}{c}H \simeq \frac{\Delta\tau}{\tau}, \tag{13}$$

where

$$H = c\,\frac{1}{a^2}\,\frac{da}{dx^0} \tag{14}$$

is the Hubble constant, $\Delta\nu = \nu_1 - \nu_2$ is the difference between the frequencies of the emitted (ν_1) and received (ν_2) radiation, and $l = ca\Delta x^0$ is the distance from the receiver to the source. Here it is implicitly postulated that the physical constants do not change with time. As to the case of the six-dimensional theory we are dealing with, this postulate does not take place, and some corrections to the interpretation of the observed red shift must be introduced. Two factors now give contributions to the red shift: first, as in the Einstein theory, the Doppler effect of scattering galaxies and, second, the effect of change of spectral line frequencies owing to the different values of the physical constants at times of photon emission and absorption.

7. To estimate the latter effect we use the well-known formula for energy levels in hydrogen-like atoms:

$$E_n = -\frac{mZ^2e^4}{2n^2\hbar^2}, \tag{15}$$

where $n = 1, 2, 3, \ldots$ is the principal quantum number, m is the reduced mass, and Z is the nuclear charge. Suppose that the mass m varies according to the law (8) obtained for the mass of the Z-boson. We shall analyze separately the effect for the two above-mentioned versions of determination of physical time.

FIRST VERSION

Substituting the values of the charge and the mass from (8) in the determination of τ and φ from (9) and (10) into (15), we obtain

$$E_n = E_{n0}(1 + \sqrt{3}\tau/a_0)^{5/3}, \tag{16}$$

where E_{n0} does not depend on τ, whence we find the change of the atomic energy levels over the time $\Delta\tau$ of light propagation from the source to the absorber:

$$\Delta E_n = E_{n0}\frac{5\Delta\tau}{\sqrt{3}a_0}(1 + \sqrt{3}\tau/a_0)^{2/3}. \tag{17}$$

The relative frequency shift is given by the expression

$$\frac{\Delta E_n}{E_n} = \left(\frac{\Delta\nu}{\nu}\right)_{\text{var. c}} = \frac{5\Delta\tau}{\sqrt{3}a_0(1 + \sqrt{3}\tau/a_0)}. \tag{18}$$

We confine ourselves to the case when $\tau/a_0 \gg 1$ and then obtain

$$\left(\frac{\Delta\nu}{\nu}\right)_{\text{var. c}} \approx \frac{5}{3}\frac{\Delta\tau}{\tau}, \tag{19}$$

and the total effect of frequency variation due to the two factors is described by the formula

$$\left(\frac{\Delta\nu}{\nu}\right)_{\Sigma} = \left(\frac{\Delta\nu}{\nu}\right)_{D} + \left(\frac{\Delta\nu}{\nu}\right)_{\text{var. c}} \approx \frac{\Delta\tau}{\tau} + \frac{5}{3}\frac{\Delta\tau}{\tau}, \tag{20}$$

i.e., the two contributions are of the same sign (and thus strengthen each other). Note that for contracting cosmological models a similar situation of strengthening of the violet shift takes place.

SECOND VERSION

We substitute into (15) the charge and the mass as functions of $\tilde{\tau}$ given by Eqs. (11) and (12), which results in the following expressions for the energy levels and the relative variation of frequency:

$$E_n = E_{n0}\left(1 + \frac{2\tilde{\tau}}{\sqrt{3}a_0\varphi_0}\right)^{5/2}; \quad \left(\frac{\Delta\nu}{\nu}\right)_{\text{var. c}} = \frac{5\Delta\tilde{\tau}}{\sqrt{3}a_0\varphi_0(1 + 2\tilde{\tau}/\sqrt{3}a_0\varphi_0)}. \tag{21}$$

Let again $\tilde{\tau}/a_0\varphi_0 \gg 1$, then

$$\left(\frac{\Delta\nu}{\nu}\right)_{\text{var. c}} \approx \frac{5}{2}\frac{\Delta\tilde{\tau}}{\tilde{\tau}} \tag{22}$$

and the total red shift is determined by the formula

$$\left(\frac{\Delta\nu}{\nu}\right)_{\Sigma} \approx \frac{\Delta\tilde{\tau}}{\tilde{\tau}} + \frac{5}{2}\frac{\Delta\tilde{\tau}}{\tilde{\tau}}. \tag{23}$$

Thus, for the two versions of determination of the observed physical time in the expanding open cosmological model considered here the contributions to the red shift of spectral lines produced by the two effects (the Doppler effect due to the expansion of the Universe and the variation of the fundamental physical constants) are of the same sign, i.e., they strengthen each other. At the present time the age of the Universe is estimated proceeding from the assumption that the entire red shift is due exclusively to the Doppler effect alone. As is known, this leads to an underestimation of the order of the time of existence of the Earth's crust. However, if the effect of variation of physical constants in the six-dimensional theory is taken into account, then the age of the Universe must be increased. It can be shown that a similar relationship takes place in the other versions of six-dimensional isotropic cosmological solutions.

REFERENCES

1. Yu. S. Vladimirov, *The Dimension of the Physical Space-Time and the Unification of Interactions* (in Russian), Moscow, 1987.
2. T. Kaluza, in: *Albert Einstein and Gravitation Theory* (Russian translation), p. 529, Moscow, 1979.
3. Yu. S. Vladimirov and A. V. Mishakov, in: *Abstracts of International Symposium on Motion of Test Bodies in Relativistic Gravitation Theory,* p. 16, Vilnius, 1990.
4. Yu. S. Vladimirov, A. O. Miroshnik, and A. V. Mishakov, *Wissenschaftliche Zeitschrift Universität Jena,* vol. 39, no. 1, p. 128, 1990.
5. Yu. S. Vladimirov and Americo Peraza Alvarez, *Vest. Mosk. Univ. Fiz. Astron.,* vol. 33, no. 6, p. 17, 1992.

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