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BRIEF COMMUNICATION

ASTRONOMY

EFFECT OF THE ATMOSPHERE ROTATION ON EARTH'S NUTATION

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New normal modes were demonstrated to appear in the Earth's rotation in space, when the atmosphere dynamics is taken into account. These modes are associated with the atmosphere rotation relative to the solid Earth and must be taken into consideration when constructing the theory of nutation.

The development of the theory of the Earth's rotation in space (the study of nutation) is stimulated by greatly increased accuracies of astrometric observations and the discovered discrepancies between the theoretical and observed amplitudes of nutation terms [1, 2]. These results cannot be explained on the basis of the Earth model that consists of stratified shells in hydrostatic equilibrium: elastic mantle, fluid outer core, and elastic inner core [3, 4]. Therefore in the present article we study the effect of rotating atmosphere, regarded as a shell arbitrarily oriented with respect to the solid Earth, on nutation. Free rotation of the atmosphere gives rise to new normal modes, which may lead to additional resonance amplification of atmospheric and oceanic tides and increase the amplitude of stimulated nutation.

The calculation of Earth's nutation is carried out in the same context as in [1, 5]: the angular moment equations are solved for the entire Earth and the fluid core. In the present article, we add the angular momentum equation for the atmosphere and a kinetic equation that describes the rotation of the coordinate system related to the atmosphere. The force moments that enter into the equations are due to the Earth attraction by the Moon and by the Sun, gravitational attraction of the shells, and due to the pressure distribution at shell boundaries.

It is assumed that the vector of the Earth's instantaneous angular velocity $\mathbf{\Omega}$ is related to the angular velocity vectors of the core $\mathbf{\Omega}_f$ and of the atmosphere $\mathbf{\Omega}_a$ as

$$\mathbf{\Omega} = \mathbf{\Omega}_0 + \mathbf{w} = \Omega_0(\mathbf{i}_3 + \mathbf{m}),$$
$$\mathbf{\Omega}_f = \mathbf{\Omega}_0 + \mathbf{w}_f = \Omega_0(\mathbf{i}_3 + \mathbf{m} + \mathbf{m}_f),$$
$$\mathbf{\Omega}_a = \mathbf{\Omega}_0 + \mathbf{w}_a = \Omega_0(\mathbf{i}_3 + \mathbf{m} + \mathbf{m}_a).$$

The dimensionless vectors $\mathbf{m}$, $\mathbf{m}_f$, and $\mathbf{m}_a$ characterize the shell velocity perturbations, which are due to the lunar-solar (Φ_s), gravity (Φ_g), and centrifugal (Φ_c) potentials. The coordinate system $\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3$ is tied to the mantle and is close to the Teisserenc axes. The atmosphere arbitrary orientation can be described by introducing a new coordinate system $\mathbf{i}'_1, \mathbf{i}'_2, \mathbf{i}'_3$, which, in the general case, does not coincide with $\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3$. The "incline" of the atmosphere is defined by the unit vector $\mathbf{n}_a = \mathbf{i}'_3 - \mathbf{i}_3$.

The determination of four unknown vectors $\mathbf{m}$, $\mathbf{m}_f$, $\mathbf{m}_a$, and $\mathbf{n}_a$ as functions of the excitation potentials is a complete solution of the problem. On connection with the problem extreme complexity, only the solutions of homogeneous equations, i.e., the normal modes of the Earth with the atmosphere, are considered.

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The dynamic equations with respect to the coordinate system $\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3$ rotating with angular velocity $\boldsymbol{\Omega}$ relative to the inertial frame are

$$\begin{aligned} &\frac{d\mathbf{H}}{dt} + \boldsymbol{\Omega} \times \mathbf{H} = \mathbf{L}, \\ &\frac{d\mathbf{H}_f}{dt} - \mathbf{w}_f \times \mathbf{H}_f = 0, \\ &\frac{d\mathbf{H}_a}{dt} + \boldsymbol{\Omega} \times \mathbf{H}_a = \mathbf{L}_a \end{aligned} \tag{1}$$

for the entire Earth, the fluid core, and the atmosphere, respectively. The force acting on the Earth mass element $\rho_e dV$ on the side of the Sun and of the Moon is $\mathbf{f} = -\nabla\Phi_s \rho_e dV$, hence the moment of force is

$$\mathbf{L} = \int_V \rho_e \mathbf{r} \times \nabla\Phi_s dV = -\Omega^2 A e \mathbf{i}_3 \times \boldsymbol{\Phi},$$

where A is the Earth equatorial moment of inertia, $e = (C - A)/A$, $\boldsymbol{\Phi} = (\phi_1, \phi_2, 0)$ is the second-order tesseral harmonic of the potential Φ_s.

The moment of forces acting on the atmosphere is

$$\mathbf{L}_a = \int_V \mathbf{r} \times (\nabla P + \rho_a \nabla\Phi_g - \nabla \cdot T) dV,$$

where P is the pressure at the Earth's surface, T is the stress tensor. Gravity tides in the atmosphere are disregarded, because they are small [6]. If we neglect the air friction (i.e., $T = 0$) and model the Earth's figure on a hydrostatic ellipsoid, then

$$\tilde{L}_a = L_{a_1} + iL_{a_2} = -iU\tilde{c}_3^a,$$

where $\tilde{c}_3^a = c_{13}^a + ic_{23}^a$ is the variation of the atmosphere tensor of inertia, and $U = \Omega_0^2$.

Projecting equations (1) onto the i_1- and i_2-axes, we obtain

$$\begin{aligned} &\left(\frac{d}{dt} - ie\Omega_0\right)\tilde{m} + \frac{1}{A}\left(\frac{d}{dt} + i\Omega_0\right)(\tilde{c} + A_f\tilde{m}_f + A_a\tilde{m}_a) = -ie\Omega_0\tilde{\phi}, \\ &\frac{d}{dt}\left(\tilde{m} + \frac{\tilde{c}_3^f}{A_f}\right) + \left[\frac{d}{dt} + i\Omega_0(1 + e_f)\right]\tilde{m}_f = 0, \\ &\left(\frac{d}{dt} - ie_a\Omega_0\right)\tilde{m} + \left(\frac{d}{dt} + i\Omega_0\right)\left(\tilde{m}_a + e_a\tilde{n}_a + \frac{\tilde{c}_3^a}{A_a}\right) = -\frac{iU}{A_a\Omega_0}(\tilde{c}_3^a + A_a e_a \tilde{n}_a), \\ &\frac{d\tilde{n}_a}{dt} + i\Omega_0\tilde{m}_a = 0, \end{aligned} \tag{2}$$

where $e_f = (C_f - A_f)/A_f$, $e_a = (C_a - A_a)/A_a$ The parameter $\tilde{m}$ is $\tilde{m} = m_1 + im_2$. The other unknowns are determined in a similar way. The fourth equation is derived from

$$\frac{d\mathbf{n}_a}{dt} = \Omega_0(\mathbf{m}_a \times \mathbf{i}_3).$$

Added to these four equations, adhering to [5], must be two more equations that relate the tensors of inertia of the Earth $\tilde{c}$, of the mantle $\tilde{c}_3^m$, and of the core $\tilde{c}_3^f$, produced by deformations, to the potential of perturbations

$$\begin{aligned} \tilde{c} &= A[\varkappa(\tilde{m} - \tilde{\phi}) + \xi\tilde{m}_f] + \tilde{c}_3^m(1 + k'), \\ \tilde{c}_3^f &= A_f[\gamma(\tilde{m} - \tilde{\phi}) + \beta\tilde{m}_f] + \tilde{c}_3^m(\xi/\tau + h_c). \end{aligned} \tag{3}$$

Equations (3) involve the load on the Earth's surface ($k' = -0.30$ is the second-order load number, $h_c = -0.128$ is the dynamic number, $\tau = \Omega_0^2 a^5/3GA$, a is the Earth radius, G is the gravity constant). The parameters $\varkappa, \xi, \gamma$, and β were produced in [2, 5] for several Earth models.

Explicit equations for the unknown quantities $\widetilde{m}, \widetilde{m}_f, \widetilde{m}_a$, and $\widetilde{n}_a$ can easily be produced by assuming them to be the periodic functions, i.e., $\widetilde{m} \sim \widetilde{m}\,e^{i\sigma t}$, etc. Therefore $d\widetilde{m}/dt = i\sigma\widetilde{m}$. Replacing the derivatives d/dt by $i\sigma$ and substituting (3) for $\widetilde{c}$ and $\widetilde{c}_3^f$ in (2), we obtain

$$Mx = y,$$

where

$$M = \begin{pmatrix} \sigma(1+\varkappa) + \Omega_0(\varkappa - e) & (\sigma+\Omega_0)\left(\dfrac{A_f}{A} + \xi\right) & (\sigma+\Omega_0)\dfrac{A_a}{A} & (\sigma+\Omega_0)\dfrac{A_a}{A}e_a(1+k') \\ \sigma(1+\gamma) & \sigma + \Omega_0(1+e_f) + \sigma\beta & 0 & \left(\dfrac{\xi}{\tau} + h_c\right)\dfrac{A_a}{A_f}e_a \\ \sigma - \Omega_0 e_a & 0 & \sigma + \Omega_0 & \left(\sigma + \Omega_0 + \dfrac{U}{\Omega_0}\right)e_a \\ 0 & 0 & \Omega_0 & \sigma \end{pmatrix},$$

$$x = \begin{pmatrix} \widetilde{m} \\ \widetilde{m}_f \\ \widetilde{m}_a \\ \widetilde{n}_a \end{pmatrix}, \qquad y = \begin{pmatrix} [-\Omega_0 e + (\sigma+\Omega_0)\varkappa]\widetilde{\phi} - (\sigma+\Omega_0)(1+k')\dfrac{\widetilde{c}_3^a}{A} \\ \sigma\gamma\widetilde{\phi} - \left(\dfrac{\xi}{\tau} + h_c\right)\dfrac{\widetilde{c}_3^a}{A_f} \\ -\left(\sigma + \Omega_0 - \dfrac{U}{\Omega_0}\right)\dfrac{\widetilde{c}_3^a}{A_a} \\ 0 \end{pmatrix}.$$

To determine the frequencies σ_1, σ_2, σ_3, and σ_4 of normal modes, we employ an artificial techniques rather than solve the equation $\det M = 0$ directly. We note that in the absence of the atmosphere ($\widetilde{m}_a = \widetilde{n}_a = 0$) the problem is reduced to Molodensky's problem of finding the modes for the elastic Earth with a liquid core. In this case, the equations determine the two well-known modes: Chandler's oscillation and nearly-the-diurnal nutation with the frequencies σ_1 and σ_2, respectively:

$$\sigma_1 = \Omega_0(e - \varkappa)\frac{A}{A_m}, \quad \sigma_2 = -\Omega_0\left[1 + (e_f - \beta)\frac{A}{A_m}\right],$$

where $A_m = A - A_f$ is the mantle's moment of inertia. Limiting oneself to the accuracy $O(me)$, one can represent $\det M$ in the form

$$\left(1 - \frac{A_f}{A}\right)(\sigma - \sigma_1)(\sigma - \sigma_2)(\sigma - \sigma_3)(\sigma - \sigma_4) = \det M.$$

Let us substitute the above expressions for σ_1 and σ_2. Expanding M in powers of σ, we can equate the coefficients for the same powers of σ. As a result, solving a quadratic equation, we obtain the frequencies of the new modes

$$\sigma_3 = \Omega_0\left(1 + \frac{\alpha}{\Omega_0}\right)e_a, \quad \sigma_4 = -\Omega_0\left(1 + \frac{\alpha}{\Omega_0}e_a + \delta_1\frac{A}{A_m}\right),$$

where $\alpha = U/\Omega_0$, $\delta_1 = \varkappa + \beta - 2\xi$. For the Earth's figure under consideration, $\alpha/\Omega_0 = 1$, $\delta_1 A/A_m = (1.370 \pm 0.006) \times 10^{-3}$. The atmosphere rocking frequency σ_3 is determined by the compression e_a. Given C_a and A_a [7], we obtain $e_a = 0.01476$ and hence $\sigma_3 = 0.0295\Omega_0$, $\sigma_4 = -1.0161\Omega_0$. We note that in this article we disregarded the friction of air against the surface, as well as the Earth's topography. Under these assumptions the frequency σ_4 is far from the frequencies of diurnal tides. The real Earth surface differs

significantly from a hydrostatic ellipsoid. Therefore the effect of topography is expressed as a change in the parameter α/Ω_0, i.e., a shift of the frequencies σ_3 and σ_4. Accurate evaluation of this shift is difficult due to insufficient knowledge of the pressure field and the coefficients of air friction against oceanic and continental surfaces. Approximate estimation of the shifts of σ_3 and σ_4, and therefore, the possibility of tide amplification and their contribution into the amplitude of backward annual nutation will be the subject of our next article.

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