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Maslov Symbol for the Density Matrix in the Case of Quantization of Pairs

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Abstract—Ultrasecondary quantization methods are applied to the equation for a density matrix. The identity for an ultrasecondary quantized hamiltonian was obtained, which allows us to define the true symbol of the equation for a density matrix by quantization by pairs. The investigation of the corresponding system of variation equations shows that this quantum system has a Maslov superfluidity model spectrum.

Keywords: ultrasecondary quantization, Maslov character, density matrix

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Consider a system of N identical bosons in a three-dimensional torus $\mathbf{T}$ with sides of length L. The equation for the density matrix corresponding to this system has the form

$$i\frac{\partial \rho}{\partial t}(x_1, y_1; \ldots; x_N, y_N, t) = (\hat{H}_N(x) - \hat{H}_N(y))\rho(x_1, y_1; \ldots, x_N, y_N, t). \quad (1)$$

Here $\hat{H}_N(x)$ is the operator in the space of functions $L_2(\mathbf{T}^N)$, which acts on $\Phi \in L_2(\mathbf{T}^N)$ as follows:

$$\hat{H}_N(x)\Phi(x_1, \ldots, x_N) = \left(-\frac{\hbar^2}{2m}\sum_{i=1}^{N}\Delta_{x_i} + \sum_{i=1}^{N}\sum_{j=i+1}^{N} V(\sqrt[3]{N}(x_i - x_j))\right) \times \Phi(x_1, \ldots, x_N),$$

where Δ_{x_i} is the Laplace operator acting on the argument $x_i \in \mathbf{T}$ and $V(\cdot)$ is a smooth even function on the torus $\mathbf{T}$.

Function $\rho(x_1, y_1; \ldots; x_N, y_N, t)$ is symmetric under permutations of the variables x_i and x_j and also with respect to permutations of variables y_i and y_j for any i, $j = 1, \ldots, N$, and satisfies the conditions:

$$\rho(x_1, y_1; \ldots; x_N, y_N, t) = \rho^*(y_1, x_1; \ldots; y_N, x_N, t),$$
$$\int\ldots\int dx_1 \ldots dx_N \rho(x_1, x_1; \ldots; x_N, x_N, t) < \infty.$$

Moreover, following [1], we assume that the function $\rho(x_1, y_1; \ldots; x_N, y_N, t)$ is the kernel of a positive definite operator and is represented as

$$\rho(x_1, y_1; \ldots; x_N, y_N, t) = \int\ldots\int dz_1 \ldots dz_N d^*(z_1, x_1; \ldots; z_N, x_N, t) \times d(z_1, y_1; \ldots; z_N, y_N, t),$$

where $d(x_1, y_1; \ldots; x_N, y_N, t)$ is a semidensity matrix, a symmetric function of the x and y variables, which belongs to the space $L_2(\mathbf{T}^{2N})$ and satisfies an equation similar to (1):

$$i\frac{\partial d}{\partial t}(x_1, y_1; \ldots; x_N, y_N, t) = (\hat{H}_N(x) - \hat{H}_N(y))d(x_1, y_1; \ldots; x_N, y_N, t). \quad (2)$$

As a consequence of the symmetry of the function $d(x_1, y_1; \ldots; x_N, y_N, t)$ with respect to the variables x and y, it is also symmetric under permutations of pairs of variables (x_i, y_i) and (x_j, y_j). Therefore, following [2], we consider the Eq. (2) in the representation of ultrasecond quantization by pairs.

The ultrasecond quantization space [3] in the case of bosons with quantization by pairs is a boson Fock space $\mathcal{F}^{\mathrm{Symm}}$ [4]. Elements of this space are the vectors of the form

$$|\Psi\rangle = \sum_{M=0}^{\infty}\frac{1}{\sqrt{M!}} \times \int\ldots\int dx_1 \ldots dx_M dy_1 \ldots dy_M \Psi_M(x_1, y_1; \ldots; x_M, y_M) \times \hat{b}^+(x_1, y_1; x_2, y_2)\ldots\hat{b}^+(x_{M-1}, y_{M-1}; x_M, y_M)|0\rangle, \quad (3)$$

where $x_i \in \mathbf{T}$, $|0\rangle$ is the vacuum vector of the space $\mathcal{F}^{\mathrm{Symm}}$ with the property

$$\hat{b}(x_1, y_1; x_2, y_2)|0\rangle = 0,$$

$\hat{b}^+(x_1, y_1; x_2, y_2)$, $\hat{b}(x_1, y_1; x_2, y_2)$ are operators of creation and annihilation of pairs of particles in the space $\mathcal{F}^{\mathrm{Symm}}$, and the functions $\Psi_M(x_1, y_1; ...; x_M, y_M) \in L_2(\mathbf{T}^{2M})$ are symmetric under permutations of pairs of variables that satisfy the condition

$$\sum_{M=0}^{\infty} \int\dots\int dx_1 \dots dx_M dy_1 \dots dy_M \times |\Psi_M(x_1, y_1; ...; x_M, y_M)|^2 < \infty. \tag{4}$$

The creation and annihilation operators conform to commutation relationships

$$[\hat{b}(x_1, y_1; x_2, y_2), \hat{b}^+(x_1', y_1'; x_2', y_2')]_- = \delta(x_1 - x_1')\delta(y_1 - y_1')\delta(x_2 - x_2')\delta(y_2 - y_2'),$$

$$[\hat{b}(x_1, y_1; x_2, y_2), \hat{b}(x_1', y_1'; x_2', y_2')]_- = [\hat{b}^+(x_1, y_1; x_2, y_2), \hat{b}^+(x_1', y_1'; x_2', y_2')]_- = 0,$$

where $\delta(x)$ is a Dirac delta-function. The operator of the particles pair number in the space $\mathcal{F}^{\mathrm{Symm}}$ has the form

$$\hat{N} = \iint dx_1 dx_2 dy_1 dy_2 \hat{b}^+(x_1, y_1; x_2, y_2)\hat{b}(x_1, y_1; x_2, y_2).$$

In accordance with [2] to the operator $\hat{H}_N$ corresponds to an ultrasecond quantized Hamiltonian operator $\overline{\hat{H}}$ of the following form:

$$\overline{\hat{H}} = \sum_{N=0}^{\infty} \int\dots\int dx_1 dy_1 \dots dx_{2N} dy_{2N} \overset{2}{\hat{b}}{}^+(x_1, y_1; x_2, y_2) \dots \overset{2}{\hat{b}}{}^+(x_{2N-1}, y_{2N-1}; x_{2N}, y_{2N}) \times (\hat{H}_{2N}(x) - \hat{H}_{2N}(y)) \underset{x_1, \dots, x_{2N}}{\mathrm{Symm}} (\overset{1}{\hat{b}}(x_1, y_1; x_2, y_2) \dots \overset{1}{\hat{b}}(x_{2N-1}, y_{2N-1}; x_{2N}, y_{2N})) \times \exp\left(-\int\int du_1 d\nu_1 du_2 d\nu_2 \overset{2}{\hat{b}}{}^+(u_1, \nu_1; u_2, \nu_2) \times \overset{1}{\hat{b}}(u_1, \nu_1; u_2, \nu_2)\right), \tag{5}$$

where the number over the operator denotes the operation order [5] and $\mathrm{Symm}_{x_1, \dots, x_{2M}}$ is the operator of symmetrization by the variable $x_1, ..., x_{2M}$.

The orthogonal projection operator of the space $\mathcal{F}$, which consists of all vectors (3) satisfying (4) on the subspace $\mathcal{F}^{\mathrm{Symm}}$ has the form

$$\overline{\hat{E}} = \sum_{N=0}^{\infty} \int\dots\int dx_1 dy_1 \dots dx_{2N} dy_{2N} \overset{2}{\hat{b}}{}^+(x_1, y_1; x_2, y_2) \dots \overset{2}{\hat{b}}{}^+(x_{2N-1}, y_{2N-1}; x_{2N}, y_{2N}) \times \underset{r_1, \dots, x_{2N}}{\mathrm{Symm}} (\overset{1}{\hat{b}}(x_1, y_1; x_2, y_2) \dots \overset{1}{\hat{b}}(x_{2N-1}, y_{2N-1}; x_{2N}, y_{2N})) \times \exp\left(-\int\int du_1 d\nu_1 du_2 d\nu_2 \times \overset{2}{\hat{b}}{}^+(u_1, \nu_1; u_2, \nu_2)\overset{1}{\hat{b}}(u_1, \nu_1; u_2, \nu_2)\right).$$

Proposition. The equation for the semidensity matrix (2) in the representation of the ultrasecond quantization has the form

$$i\frac{\partial}{\partial t}\overline{\hat{E}}|\Psi(t)\rangle = \overline{\hat{E}}|\Psi(t)\rangle, \tag{6}$$

where

$$|\Psi(t)\rangle = \sum_{M=0}^{\infty} \frac{1}{\sqrt{M!}} \times \int\dots\int dx_1 \dots dx_M dy_1 \dots dy_M d(x_1, y_1; ...; x_M, y_M, t) \times \hat{b}^+(x_1, y_1; x_2, y_2) \dots \hat{b}^+(x_{M-1}, y_{M-1}; x_M, y_M)|0\rangle$$

depending on the time t vector of the boson Fock space $\mathcal{F}^{\mathrm{Symm}}$ that satisfies the relationships

$$\hat{N}|\Psi(t)\rangle = \frac{N}{2}|\Psi(t)\rangle, \quad \overline{\hat{E}}|\Psi(t)\rangle \neq 0,$$

where N is an arbitrary even positive integer, $\overline{\hat{E}}$ and $\overline{\hat{H}}$ are the ultrasecond quantized identity operators and ultrasecond quantized Hamiltonian of the equation for the semidensity matrix in the space $\mathcal{F}^{\mathrm{Symm}}$, respectively.

Proposition. Let the vector $|\Psi(t)\rangle \in \mathcal{F}^{\mathrm{Symm}}$ be a solution of the equation

$$i\frac{\partial}{\partial t}\overline{\hat{E}}|\Psi(t)\rangle = \overline{\hat{H}}|\Psi(t)\rangle \tag{7}$$

and satisfy the conditions

$$\hat{N}|\Psi(t)\rangle = \frac{N}{2}|\Psi(t)\rangle, \quad \overline{\hat{E}}|\Psi(t)\rangle \neq 0,$$

where the N is an arbitrary even positive integer. Then the function

$$d(x_1, y_1; ...; x_N, y_N) = \frac{1}{\sqrt{N!}}\langle 0|\hat{b}(x_1, y_1; x_2, y_2) \dots \hat{b}(x_{N-1}, y_{N-1}; x_N, y_N)\Psi(t)\rangle \tag{8}$$

is symmetric under permutations of pairs of variables and satisfies Eq. (2).

Maslov's works [6, 7] obtained the identity of the ultrasecond quantized Hamiltonian corresponding to the Schrodinger equation. A similar identity is valid in the case of the equation for the density matrix.

Theorem. *The following identity holds*

$$\overline{\hat{H}} = \overline{\hat{E}}\hat{A}. \tag{9}$$

$\hat{A}$ is an operator in the ultrasecond quantization space $\mathcal{F}^{\mathrm{Symm}}$ that is dependent on the parameter set $\{a, b, \alpha, \beta\}$ and has the form

$$\hat{A} = -\frac{\hbar^2}{2m}\int dx_1 dy_1 dx_2 dy_2 \hat{b}^+(x_1, y_1; x_2, y_2)$$
$$\times (\Delta_{x_1} - \Delta_{y_1} + \Delta_{x_2} - \Delta_{y_2})\hat{b}(x_1, y_1; x_2, y_2)$$
$$+ \int dx_1 dy_1 dx_2 dy_2 \operatorname*{Symm}_{x_1, x_2}{}_{\{a\}} \operatorname*{Symm}_{y_1, y_2}{}_{\{b\}} (\hat{b}^+(x_1, y_1; x_2, y_2))$$
$$\times (V(\sqrt[3]{N}(x_1 - x_2)) - V(\sqrt[3]{N}(y_1 - y_2)))\hat{b}(x_1, y_1; x_2, y_2)$$
$$+ 2\int dx_1 dy_1 \ldots dx_4 dy_4$$
$$\times \operatorname*{Symm}_{x_1, x_2, x_3, x_4}{}_{\{\alpha\}} \operatorname*{Symm}_{y_1, y_2, y_3, y_4}{}_{\{\beta\}} (\hat{b}^+(x_1, y_1; x_2, y_2)\hat{b}^+(x_3, y_3; x_4, y_4))$$
$$\times (V(\sqrt[3]{N}(x_1 - x_2))\hat{b}(x_1, y_3; x_4, y_4)\hat{b}(x_2, y_1; x_3, y_2)$$
$$- V(\sqrt[3]{N}(y_1 - y_2))\hat{b}(x_3, y_1; x_4, y_4)\hat{b}(x_1, y_2; x_2, y_3)).$$

Operator $\operatorname*{Symm}_{x_1, \ldots, x_N}{}_{\{\alpha\}}$ acts on a function that depends on the arguments $x_1, \ldots, x_N$ as a symmetrizer with coefficients:

$$\operatorname*{Symm}_{x_1, \ldots, x_N}{}_{\{\alpha\}} f(x_1, \ldots, x_N) = \sum a_{k_1 \ldots k_N} f(x_{k_1}, \ldots, x_{k_N}),$$

where the summation is over of all permutations k_i taking values from 1 to N, and $\sum a_{k_1 \ldots k_N} = 1$.

Proof. Equation (9) follows from the commutation relationships for the creation and annihilation operators and the form of the operators $\overline{\overline{H}}$, $\overline{\overline{E}}$, and $\hat{A}$.

Operator $\hat{A}$ has the form

$$NA\left[\frac{\hat{b}^+(\cdot)}{\sqrt{N}}, \frac{\hat{b}(\cdot)}{\sqrt{N}}\right],$$

where $A[\Phi^+(\cdot), \Phi(\cdot)]$ is the following functional (the lower arguments of this functional will be omitted):

$$A = -\frac{\hbar^2}{2m}\int dx_1 dy_1 dx_2 dy_2 \Phi^+(x_1, y_1; x_2, y_2)$$
$$\times (\Delta_{x_1} - \Delta_{y_1} + \Delta_{x_2} - \Delta_{y_2})\Phi(x_1, y_1; x_2, y_2)$$
$$+ \int dx_1 dy_1 dx_2 dy_2 \operatorname*{Symm}_{x_1, x_2}{}_{\{a\}} \operatorname*{Symm}_{y_1, y_2}{}_{\{b\}} (\Phi^+(x_1, y_1; x_2, y_2))$$
$$\times (V(\sqrt[3]{N}(x_1 - x_2)) - V(\sqrt[3]{N}(y_1 - y_2)))\Phi(x_1, y_1; x_2, y_2)$$
$$+ 2N\int dx_1 dy_1 \ldots dx_4 dy_4$$
$$\times \operatorname*{Symm}_{x_1, x_2, x_3, x_4}{}_{\{\alpha\}} \operatorname*{Symm}_{y_1, y_2, y_3, y_4}{}_{\{\beta\}} (\Phi^+(x_1, y_1; x_2, y_2)$$
$$\times \Phi^+(x_3, y_3; x_4, y_4))$$
$$\times (V(\sqrt[3]{N}(x_1 - x_2))\Phi(x_1, y_3; x_4, y_4)\Phi(x_2, y_1; x_3, y_2)$$
$$- V(\sqrt[3]{N}(y_1 - y_2))\Phi(x_3, y_1; x_4, y_4)\Phi(x_1, y_2; x_2, y_3)).$$

The functions $\Phi^+(x_1, y_1; x_2, y_2)$, $\Phi(x_1, y_1; x_2, y_2) \in L_2(\mathbf{T}^4)$ are symbols of the creation operator $\hat{b}^+(x_1, y_1; x_2, y_2)$ and the annihilation operator $\hat{b}(x_1, y_1; x_2, y_2)$, respectively, and satisfy the condition

$$\int dx_1 dy_1 dx_2 dy_2 \Phi^+(x_1, y_1; x_2, y_2)$$
$$\times \Phi(x_1, y_1; x_2, y_2) = \frac{1}{2}. \tag{10}$$

Further, we assume they are symmetric under permutations of the variables x_1, x_2 and the variables y_1, y_2.

The choice of operator $\hat{A}$, as well as its corresponding symbol is ambiguous. Below, we consider the case $\alpha_{1 \ldots N} = 1$, $\beta_{1 \ldots N} = 1$; the other $\alpha_{k_1 \ldots k_N}$ and $\beta_{k_1 \ldots k_N}$ are equal to zero. The interaction potential depends on N so that the radius of the interaction potential decreases with increasing particle number N, so that the value of $NV(\sqrt[3]{N}(x - y))$ in the weak sense converges to the Dirac delta function $\delta(x - y)$. In this case, when $N \longrightarrow \infty$ the symbol takes the form

$$A_{\mathfrak{Z}} = -\frac{\hbar^2}{2m}\int dx_1 dy_1 dx_2 dy_2 \Phi^+(x_1, y_1; x_2, y_2)$$
$$\times (\Delta_{x_1} - \Delta_{y_1} + \Delta_{x_2} - \Delta_{y_2})\Phi(x_1, y_1; x_2, y_2)$$
$$+ 2N\int dx_1 dy_1 \ldots dx_4 dy_4$$
$$\times \Phi^+(x_1, y_1; x_2, y_2)\Phi^+(x_3, y_3; x_4, y_4)$$
$$\times (V(\sqrt[3]{N}(x_1 - x_2))\Phi(x_1, y_3; x_4, y_4)\Phi(x_2, y_1; x_3, y_2)$$
$$- V(\sqrt[3]{(y_1 - y_2)}))\Phi(x_3, y_1; x_4, y_4)\Phi(x_1, y_2; x_2, y_3)).$$

The functional $A_{\mathfrak{Z}}$ analogue that appeared during ultrasecond quantization of the Schrodinger equation was called the *true* symbol of the ultrasecond quantized problem in [7]. The functional $A_{\mathfrak{Z}}$ will be called the *true* symbol of the ultrasecond quantized Hamiltonian (5) of the equation for the semydensity matrix, or *Maslov character*.

Consider the problem of finding the asymptote of the solutions of Eq. (7) in the limit at $N \longrightarrow \infty$. According to [1, 6, 7], this asymptote is determined by the periodic solutions corresponding to the *true* character of the Hamilton equation system and the system of equations in variations.

A system of Hamilton equations corresponds to the *true* character $A_{\mathfrak{Z}}$:

$$i\dot{\Phi}(x_1, y_1; x_2, y_2, t) = \frac{\delta A_{\mathfrak{Z}}}{\delta \Phi^+(x_1, y_1; x_2, y_2, t)},$$
$$-i\dot{\Phi}(x_1, y_1; x_2, y_2, t) = \frac{\delta A_{\mathfrak{Z}}}{\delta \Phi(x_1, y_1; x_2, y_2, t)}. \tag{11}$$

We will search for the periodic solutions of the Hamilton equation system in the class of functions

that are symmetric relative to the permutations of pairs of variables of the form

$$\Phi(x_1, y_1; x_2, y_2, t) = \sqrt{2}e^{-i\omega t}\Phi(x_1, x_2)\Phi^*(y_1, y_2),$$
$$\Phi^+(x_1, y_1; x_2, y_2, t) = \sqrt{2}e^{i\omega t}\Phi^+(x_1, x_2)\Phi^{+*}(y_1, y_2), \quad (12)$$

where $\Phi(x, y)$, $\Phi^+(x, y)$ are symmetric

$$\Phi(x, y) = \Phi(y, x), \quad \Phi^+(x, y) = \Phi^+(y, x)$$

and satisfy

$$\iint dxdy\Phi^+(x, y)\Phi(x, y) = \frac{1}{2}. \quad (13)$$

Theorem. *Let the functions $\Phi(x, y)$, $\Phi^+(x, y)$ satisfy the Hamilton equations system*

$$\Omega\Phi(x, y) = -\frac{\hbar^2}{2m}(\Delta_x + \Delta_y)\Phi(x, y)$$
$$+ 2N\iint dx'dy'(V(\sqrt[3]{N}(x-y)) + V(\sqrt[3]{N}(x'-y')))$$
$$\times \Phi^+(x', y')\Phi(x, x')\Phi(y, y'),$$
$$\Omega\Phi^+(x, y) = -\frac{\hbar^2}{2m}(\Delta_x + \Delta_y)\Phi^+(x, y) \quad (14)$$
$$+ 2N\iint dx'dy'(V(\sqrt[3]{N}(x-x')) + V(\sqrt[3]{N}(y-y')))$$
$$\times \Phi(x', y')\Phi^+(x, x')\Phi^+(y, y').$$

Then the functions (12) *are the solutions of the system* (11) *at* $\omega = 0$.

Proof. The theorem is verified by direct substitution of (12) into the system (11).

Note that Eqs. (14) are the Hamiltonian system for the true symbol

$$\mathcal{H} = -\frac{\hbar^2}{2m}\iint dxdy\Phi^+(x, y)(\Delta_x + \Delta_y)\Phi(x, y)$$
$$+ 2N\iiiint dx_1dy_1dx_2dy_2\Phi^+(x_1, y_1)\Phi^+(x_2, y_2) \quad (15)$$
$$\times V(\sqrt[3]{N}(x_1 - y_1))\Phi(x_1, x_2)\Phi(y_1, y_2),$$

considered in [7].

The system of equations in variations that correspond to the solutions (12) of the Hamiltonian system (11), according to [1], has the form

$$-\lambda F(x_1, y_1; x_2, y_2) = \iint du_1 dv_1 du_2 dv_2$$
$$\times\left(\frac{\delta^2 A_\Im}{\delta\Phi^+(x_1, y_1; x_2, y_2)\delta\Phi(u_1, v_1; u_2, v_2)}\right.$$
$$\times F(u_1, v_1; u_2, v_2)$$
$$+ \frac{\delta^2 A_\Im}{\delta\Phi^+(x_1, y_1; x_2, y_2)\delta\Phi^+(u_1, v_1; u_2, v_2)}$$
$$\left.\times G(u_1, v_1; u_2, v_2)\right),$$
$$\lambda G(x_1, y_1; x_2, y_2) = \iint du_1 dv_1 du_2 dv_2 \quad (16)$$
$$\times\left(\frac{\delta^2 A_\Im}{\delta\Phi(x_1, y_1; x_2, y_2)\delta\Phi(u_1, v_1; u_2, v_2)}\right.$$
$$\times F(u_1, v_1; u_2, v_2)$$
$$+ \frac{\delta^2 A_\Im}{\delta\Phi(x_1, y_1; x_2, y_2)\delta\Phi^+(u_1, v_1; u_2, v_2)}$$
$$\left.\times G(u_1, v_1; u_2, v_2)\right).$$

Theorem. *Let the functions $F_\lambda(x_1, y_1; x_2, y_2)$, $G_\lambda(x_1, y_1; x_2, y_2)$ satisfy the system of equations in variations*

$$(\Omega - \lambda)F_\lambda(x, y) = -\frac{\hbar^2}{2m}(\Delta_x + \Delta_y)F_\lambda(x, y)$$
$$+ N\iint dx'dy'(V(\sqrt[3]{N}(x-y)) + V(\sqrt[3]{N}(x'-y')))$$
$$\times (G_\lambda(x', y')\Phi(x, x')\Phi(y, y')$$
$$+ \Phi^+(x', y')F_\lambda(x, x')\Phi(y, y')$$
$$+ \Phi(x', y')\Phi(x, x')F_\lambda(y, y')),$$
$$(\Omega + \lambda)G_\lambda(x, y) = -\frac{\hbar^2}{2m}(\Delta_x + \Delta_y)G_\lambda(x, y)$$
$$+ N\iint dx'dy'(V(\sqrt[3]{N}(x-y)) + V(\sqrt[3]{N}(x'-y')))$$
$$\times (F_\lambda(x', y')\Phi^+(x, x')\Phi^+(y, y')$$
$$+ \Phi^+(x', y')G_\lambda(x, x')\Phi^+(y, y')$$
$$+ \Phi(x', y')\Phi^+(x, x')G_\lambda(y, y')),$$

corresponding to the symbol $\mathcal{H}$. Then the functions

$$F_\lambda(x_1, y_1; x_2, y_2) = F_\lambda(x_1, x_2)\Phi^*(y_1, y_2)$$
$$+ \Phi(x_1, x_2)F^*_{-\lambda}(y_1, y_2),$$
$$G_\lambda(x_1, y_1; x_2, y_2) = G_\lambda(x_1, x_2)\Phi^{+*}(y_1, y_2)$$
$$+ \Phi^+(x_1, x_2)G^*_{-\lambda}(y_1, y_2)$$

are solutions of equations system in variations (16).

Proof. The theorem is verified by direct substitution of the functions $F_\lambda(x_1, y_1; x_2, y_2)$, $G_\lambda(x_1, y_1; x_2, y_2)$ in the equations in variations.

Thus, we studied a new model for ultrasecond quantization by pairs of equation for a density matrix. We established an important mathematical fact: for one of the symbols of the Hamiltonian system of equations the system of equations in variations are expressed in terms of the solutions of equations of a simpler model that was discussed in [7]. The investi-

gated system contains a spectrum corresponding to the true character H that was obtained in [7].

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