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Photodisintegration of Hg Isotopes

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Abstract—Yields of photonuclear reactions of several Hg isotopes are measured on a bremsstrahlung photon beam with a maximal energy of 19.5 MeV using a gamma-activation analysis. Cross sections of photonuclear reactions on stable Hg isotopes are calculated within bonds of the collective model taking isospin splitting of giant dipole resonance into account. The experimental results are compared with the results of theoretical calculations and earlier experimental studies.

Keywords: cross section, photonuclear reactions, giant resonance.

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INTRODUCTION

The study of photodisintegration of heavy nuclei is of interest since it aids in understanding the nature of the collective excitations of atomic nuclei. One characteristic peculiarity of the interactions of photons with energies of up to 20 MeV with atomic nuclei is the excitation of giant dipole resonance (GDR) in them, namely collective dipole oscillations of protons relative to neutrons [1, 2]. GDR of the in heavy nuclei is predominantly accompanied by the ejection of neutrons. In spite of a small cross section of the $(\gamma,1p)$ reaction, GDR decay with the ejection of protons makes it possible to study the excitation of the GDR isospin branch $T_> = \left|\frac{N-Z}{2}\right| + 1$, where N and Z are the number of neutrons and protons in a nucleus.

Mercury Hg ($Z = 80$) has seven stable isotopes $^{196,198-202,204}$Hg, which makes it possible to study the dependence of GDR parameters on the mass number A. Experimental data on the photodisintegration of Hg isotopes is practically absent. Cross sections of photoneutron reactions have been measured only for a natural mixture of isotopes [3]. The cross section of a photoproton reaction has been measured only for the ^{201}Hg isotope [4]. A model for describing the photodisintegration of medium and heavy nuclei based on a collective model taking isospin GDR disintegration into account is described in [5, 6].

The aim of the present study is to measure the yields of photonuclear reactions on Hg isotopes under the effect of a beam of bremsstrahlung gamma radiation with a maximal energy of 19.5 MeV and a comparison of the obtained results with the theoretical calculated data.

EXPERIMENTAL TECHNIQUE

Yields of photonuclear reactions on Hg isotopes were measured using a gamma-activation analysis. A beam of electrons from an RTM-70 accelerator [7] with an energy of 19.5 MeV hit a 2.5 mm-thick bremsstrahlung tungsten target. An Hg specimen with a mass of ~5 g was positioned just behind the bremsstrahlung target. The irradiation lasted 4 h and 20 min. Once the irradiation was completed, the Hg specimen was placed close to a Canberra GC3019 germanium detector to measure the induced activity spectra of the formed radioactive isotopes [8].

Table 1 lists the percentage of stable isotopes in a natural Hg mixture and the thresholds of photonuclear reactions of stable Hg isotopes that take place in the process of irradiation with a bremsstrahlung spectrum of gamma quanta with a maximal energy of 19.5 MeV.

The number of reactions with thresholds lower than 19.5 MeV was not observed in the present experiment. This is related to such issues as too short ($T_{1/2} < t_{\text{transfer}}$) or too long ($T_{1/2} \gg t_{\text{measur}}$) lifetimes of the formed isotopes, the formation of stable isotopes, and strong suppression of the probability of reaction product passage through a potential nucleus barrier.

EXPERIMENTAL RESULTS

We measured 343 spectra of induced activity on a natural mixture of Hg isotopes. Among them, 298 were measured for 6 days after irradiation and 45 were measured for 6 months after the irradiation.

Figure 1 shows the spectra of the gamma quanta of residual activity in the gamma-quanta energy range from 100 to 1000 keV measured at 1, 20, and 120 h after irradiation. The measurement time of each shown spectra was 1 h. The disappearance of the peaks

Table 1. Stable Hg isotopes and the thresholds of their photonuclear reactions

Isotope	Concentration in a natural mixture	Reaction threshold, MeV				
		$(\gamma, 1n)$	$(\gamma, 2n)$	$(\gamma, 1p)$	$(\gamma, 2p)$	$(\gamma, 1p1n)$
^{196}Hg	0.15	8.90	15.78	6.55	11.64	14.92
^{198}Hg	9.97	8.48	15.27	7.10	12.88	15.17
^{199}Hg	16.87	6.66	15.15	7.25	13.70	13.77
^{200}Hg	23.1	8.03	14.69	7.70	14.17	15.28
^{201}Hg	13.18	6.23	14.26	7.68	14.85	13.93
^{202}Hg	29.86	7.75	13.98	8.23	15.32	15.44
^{204}Hg	6.87	7.49	13.49	8.84	16.69	15.65

in the spectra corresponding to the formation of short-lived isotopes and a general decrease in the radioactivity of the irradiated Hg specimen were clearly observed.

The peaks corresponding to gamma transitions in the nuclei formed during the beta decay of products of photonuclear reactions were observed in the measured spectra. The isotopes formed as the result of photonuclear reactions and reaction channels corresponding to them were reconstructed based on the energy of peaks in the spectra of gamma quanta and the period of isotope half disintegration.

The cross section of a photonuclear reaction $\sigma(E)$ is related to the yield of the reaction $Y(E^{max})$ by the relationship

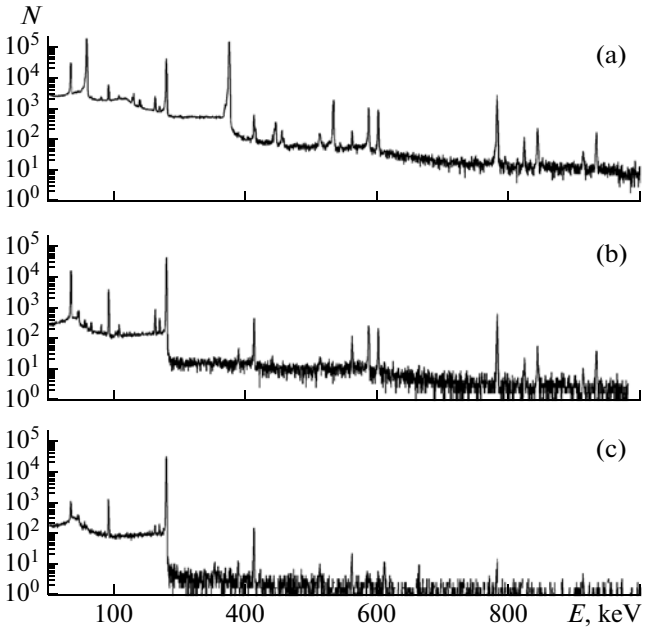

Fig. 1. Spectra of induced activity of a natural mixture of Hg isotopes irradiated by a bremsstrahlung spectrum of gamma quanta with an upper boundary of 19.5 MeV 1 (a), 20(b), and 120(c) h after irradiation (top–down). The measurement time for each spectrum is 1 h.

$$Y(E^{max}) = \int_0^{E^{max}} \sigma(E) W(E, E^{max}) dE, \quad (1)$$

where $W(E, E^{max})$ is the bremsstrahlung spectrum with the maximal energy E^{max}.

The yields of reactions in the gamma activation experiment were calculated with the equation

$$Y(E^{max}) = \frac{\lambda A}{\varepsilon I (1 - e^{-\lambda t_1}) e^{-\lambda t_2} (1 - e^{-\lambda t_3})}, \quad (2)$$

where λ is the decay constant of the formed nucleus, ε is the efficiency of registration the corresponding gamma line with a germanium detector, t_1 is the irradiation time, t_2 is the time between the irradiation end and the beginning of the spectrum measurement, t_3 is the spectrum measurement duration, A is the number of detector counts corresponding to the chosen gamma line of the spectrum of Hg specimen residual activity, and I is the line intensity if the spectrum of gamma quanta of a daughter nucleus formed in the process of beta disintegration of a reaction product. The intensities of gamma transitions [9] calculated using the interface of [10] were used for calculations. When determining the yields, self-absorption in the Hg specimen calculated using GEANT4 libraries [11] was taken into account.

The experimental technique made it possible to measure not only total yields of photonuclear reactions, but the yields of reactions with the formation of isotopes in the ground and isomeric states, separately.

The measured yields of photonuclear reactions were normalized to the yield of the reaction ^{196}Hg$(\gamma, 1n)^{195}$Hg. The normalized yields are listed in Table 2.

DISCUSSION OF THE RESULTS

Based on a collective model of GDR excitation that was that was developed in [5, 6] taking isospin splitting of a giant resonance into account, the total cross sections of photoabsorption $\sigma(\gamma, abs)$, partial cross sections of photoneutron $\sigma(\gamma, 1n)$ $\sigma(\gamma, 2n)$, and

Table 2. Yields of photonuclear reactions on Hg isotopes normalized to the total yield of the ^{196}Hg(γ, 1*n*)^{195}Hg reaction

Primary nucleus	Reaction	Finite nucleus	Spin, parity J^P			Yield	Yield calculated based on [5, 6]
			Primary nucleus	Finite nucleus			
				Ground state	Isomeric state		
^{204}Hg	(γ, 1*n*)	^{203}Hg	0^+	$5/2^-$		1.57	1.05
^{200}Hg	(γ, 1*n*)	^{199}Hg	0^+	$1/2^-$			1.06
		^{199m}Hg			$13/2^+$	0.09	
^{198}Hg	(γ, 1*n*)	^{197}Hg	0^+	$1/2^-$		1.26	1.02
		^{197m}Hg			$13/2^+$	0.1	
^{196}Hg	(γ, 1*n*)	^{195}Hg	0^+	$1/2^-$		0.9	1
		^{195m}Hg			$13/2^+$	0.1	
^{201}Hg	(γ, 1*p*)	^{200}Au	$3/2^-$	$1(^-)$		2.3×10^{-4}	0.7×10^{-4}
		^{200m}Au			12^-	$<4 \times 10^{-6}$	
^{200}Hg	(γ, 1*p*)	^{199}Au	0^+	$3/2^+$		4.6×10^{-4}	1.0×10^{-4}
^{199}Hg	(γ, 1*p*)	^{198}Au	$1/2^-$	2^-		3.9×10^{-4}	1.5×10^{-4}
		^{198m}Au			(12^-)	$<1 \times 10^{-6}$	

Table 3. Results of calculations of the photodisintegration of stable Hg isotopes

Mass number A	196	198	199	200	201	202	204
Position of GDR maximum E^m(γ, *abs*), MeV	13.5	13.5	13.5	13.5	13.5	13.5	13.5
GDR cross section in maximum σ^m(γ, *abs*), mb	561	564	548	552	558	562	580
GDR width Γ(γ, *abs*), MeV	4.3	4.3	4.3	4.5	4.3	4.6	4.2
Integral GDR cross section σ_{int}(γ, *abs*), MeV mb	3421	3462	3563	3518	3603	3555	3580
Position of maximum σ(γ, 1*n*) E^m(γ, 1*n*), MeV	13.5	13.5	13.5	13.5	13.5	13.5	13.5
Integral cross section σ_{int}(γ, 1*n*), MeV mb	2585	2560	2714	2586	2551	2430	2395
Position of maximum σ(γ, 2*n*) E^m(γ, 2*n*), MeV	17.5	16.9	17.1	16.9	16.1	15.9	15.7
Integral cross section σ_{int}(γ, 2*n*), MeV mb	606	663	562	697	748	855	907
Difference of threshold of (γ, 2*n*) reaction and position of GDR maximum E_{thres}(γ, 2*n*) − E^m(γ, *abs*), MeV	2.3	1.8	1.7	1.2	0.8	0.5	0.0
Position of maximum of GDR $T_<$-component $E^m(T_<)$(γ, *abs*), MeV	13.5	13.5	13.5	13.5	13.5	13.5	13.5
Integral cross section of GDR $T_<$-component $\sigma_{int}(T_<)$(γ, *abs*), MeV	3223	3270	3373	3331	3415	3371	3396
Position of maximum of GDR $T_>$-component $E^m(T_>)$(γ, *abs*), MeV	23.3	24.5	24.9	25.3	25.3	26.2	26.8
Integral cross section of GDR $T_>$-component $\sigma_{int}(T_>)$(γ, *abs*), MeV	28	20	18	14	14	10	8
Difference of positions of cross section maxima of isospin components $T_>$ and $T_<$ $E_m(T_>)$(γ, *abs*) − $E_m(T_<)$(γ, *abs*), MeV	9.8	11	11.4	11.8	11.8	12.7	13.3
Position of maximum σ(γ, 1*p*) E^m(γ, 1*p*), MeV	22.5	23.7	24.1	24.3	24.7	25.1	26.2
Integral cross section σ_{int}(γ, 1*p*), MeV mb	34	26	21	21	16	16	14

the photoproton σ(γ, 1*p*) reactions of stable Hg isotopes were calculated:

$$\sigma(\gamma, abs) = \sigma(\gamma, 1n) + \sigma(\gamma, 2n) + \sigma(\gamma, 1p).$$

Figure 2 shows the obtained energy dependences of cross sections of photonuclear reactions (γ, *abs*), (γ, 1*n*), (γ, 2*n*), and (γ, *p*) on stable Hg isotopes.

Integral cross sections of the above reactions (namely, σ_{int}(γ, *abs*) σ_{int}(γ, 1*n*), σ_{int}(γ, 2*n*), and σ_{int}(γ, 1*p*) were also calculated. The results of these calculations are listed in Table 3.

The position of the GDR maximum E^m(γ, *abs*) practically did not change with the variations in the atomic number *A* and is 13.5 MeV. The obtained result

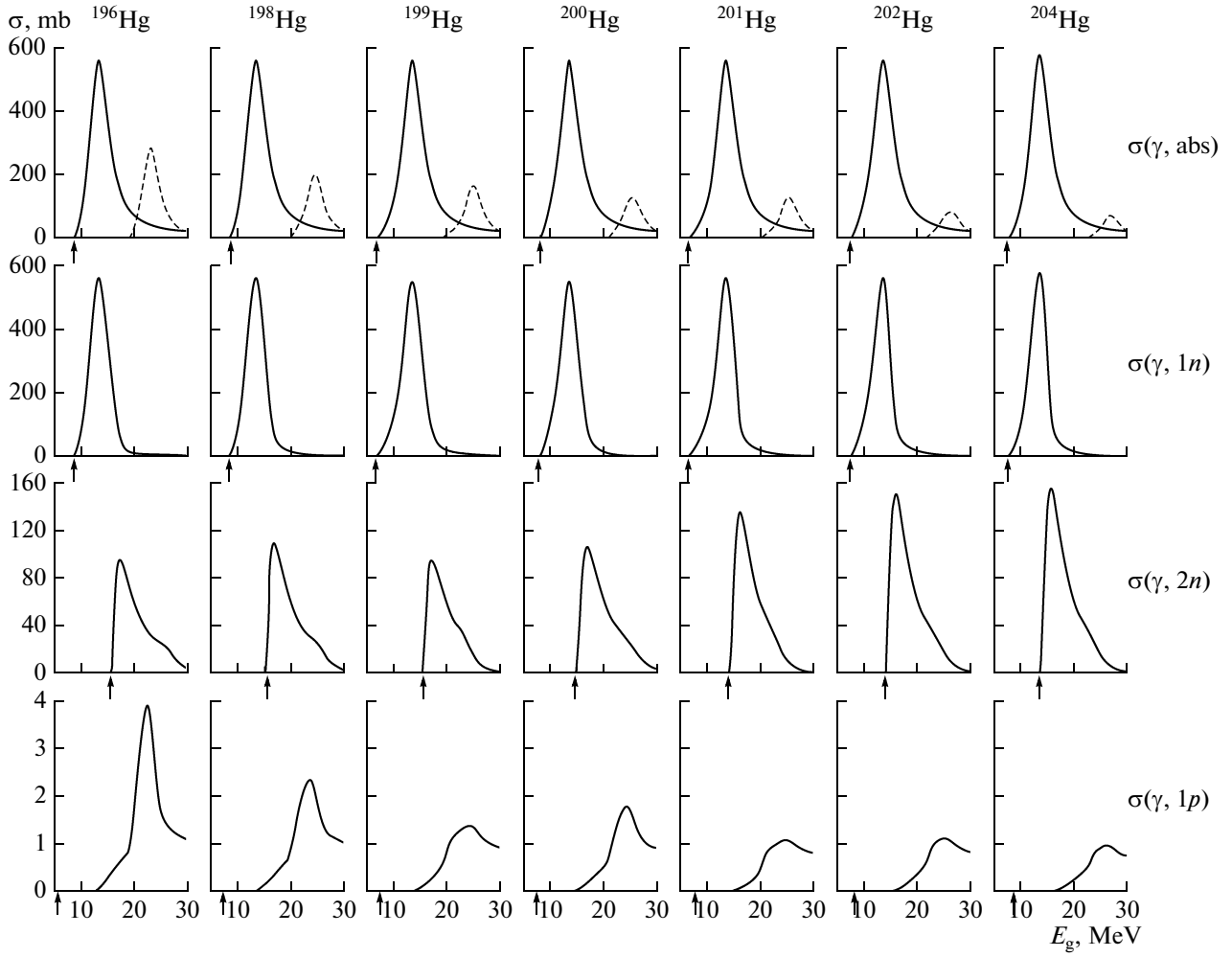

Fig. 2. Cross sections of photonuclear reactions on stable Hg isotopes. The contribution of the $T_>$-component into the cross section of a photoabsorption reaction (γ, *abs*) is marked with a dashed line (for obviousness, it is enlarged 50 times). Arrows show reaction thresholds.

is in agreement with the GDR calculations in the case of heavy nuclei [12]. Since the number of protons and neutrons is close to the magic numbers and the shape of a nucleus for different isotopes is almost spherical, the giant resonance manifests itself as one relatively narrow maximum of the width $\Gamma(\gamma, abs) \approx 4.2$–4.6 MeV. The width of the cross sections on Hg isotopes coincides with that of the absorption cross section on a doubly magic nucleus ^{208}Pb($Z = 82$, $N = 126$) [12]. Integral cross sections of absorption also change only slightly with variations in the mass number A from $\sigma_{int}(\gamma, abs) = 3421$ MeV mb for the lightest isotope ^{196}Hg up to $\sigma_{int}(\gamma, abs) = 3580$ MeV mb for the heaviest isotope ^{204}Hg and are in agreement with the integral cross section calculated based on the dipole summation rule [12]

$$\sigma_{int}(\gamma, abs) = 60\frac{NZ}{A} \text{ MeB mb.}$$

GDR states decay occurs with the predominant ejection of one neutron. On the whole, the cross section of the reaction σ(γ, 1*n*) decreases with an increase in the mass number A and is 65–75% of the total absorption cross section. The maximum of the cross section of σ(γ, 1*n*) reaction on all stable Hg isotopes coincides with the GDR maximum.

The cross section of the σ(γ, 2*n*) reaction increases with an increase in the mass number A from 606 MeV mb (^{196}Hg) to 907 MeV mb (^{204}Hg), which is related to a shift of the σ(γ, 2*n*) reaction threshold toward lower energies. The threshold of the σ(γ, 2*n*) reaction on a ^{204}Hg isotope coincides with the position of the GDR maximum that yields an increased probability of GDR decay with the ejection of two neutrons.

In Table 4, the results of an experiment [3], where the total cross sections of a photoneutron reaction σ(γ, *sn*) and cross sections of σ(γ, 1*n*) and σ(γ, 2*n*) on a natural mixture of Hg were measured, are compared with the results of the theoretical studies shown in Table 3.

Table 4. GDR characteristics obtained in [3] for a natural mixture of Hg isotopes

Isotope	$E^m(\gamma, sn)$, MeV	$\sigma_m(\gamma, sn)$, mb	$\Gamma(\gamma, sn)$, MeV	$\sigma_{int}(\gamma, sn)$, MeV mb	$\sigma_{int}(\gamma, 1n)$, MeV mb	$\sigma_{int}(\gamma, 2n)$, MeV mb
Natural mixture ([3])	13.7	582	4.3	3770		
^{200}Hg	13.5	552	4.5	3518	2586	697
^{202}Hg	13.5	562	4.6	3555	2430	855

Since cross sections for individual isotopes are absent in [3], it is impossible to perform a direct comparison with the theoretical calculation data. However, cross sections of photoneutron reactions on the two most widespread isotopes ^{200}Hg and ^{202}Hg are in agreement with the data of [3].

In the nuclei where the number of neutrons N does not coincide with that of protons Z, two isospin GDR branches are excited in the process of gamma quanta absorption, namely $T_< = T_0$ and $T_< = T_0 + 1$, where $T_0 = |N - Z|/2$ is the isospin of the nucleus ground state. The ratio of the probabilities of excitation of $T_>$ and $T_<$ states is determined by equation [12]

$$\frac{C^2(T_>)}{C^2(T_<)} = \frac{1}{T_0}\left(\frac{1 - 1.5\,T_0 A^{-2/3}}{1 + 1.5A^{-2/3}}\right).$$

The $T_>$ cross section is positioned at higher energies. The energy shift of the $T_>$ and $T_<$ states is determined by the equation [12]

$$E(T_>) - E(T_<) = \frac{60}{A}(T_0 + 1)\ \text{MeV}.$$

The components $T_>\sigma(T_>)(\sigma, abs)$ and $T_<\sigma(T_<)(\gamma, abs)$ of the total cross section of photoabsorption $\sigma(\gamma, abs)$ were calculated. The results of these calculations are summarized in Table 3. Figure 2 shows the $T_>$-component of the photoabsorption cross section (since it is more than 100 times smaller than the GDR cross section in the maximum, it is magnified 50 times for clarity).

The position of the peak of the cross section $E_m(T_<)(\gamma, abs)$ for the isospin component $T_<$ coincides with that of the GDR maximum $E_m(\gamma, abs)$. The isospin component $T_>$ is shifted by 9.8 MeV with respect to $T_<$ toward higher energies for a ^{196}Hg isotope. This energy shift increases with an increase in the mass number A and reaches 13.5 MeV for ^{204}Hg.

The integral cross section $\sigma_{int}(T_>)(\gamma, abs)$ of the resonance $T_>$ components almost coincides with the integral cross sections of the $(\gamma, 1p)$ reaction, which confirms the previously-revealed pattern [12] that the $T_>$ state in a heavy nuclei decays with the predominant ejection of protons. Moreover, the forbidenness of disintegration of the $T_>$ states with neutron ejection on the low-excited states of product nucleus makes it possible to explain the value of the $T_>$-component of the cross section in a photoproton reaction. The calculations performed based on a statistic model that does not take specific isospin peculiarities of decay of $T_>$ and $T_<$ states into account predict cross sections of photoproton reactions that are 2–4 orders smaller than those observed experimentally.

In Fig. 3, the yields of the photonuclear reactions listed in Table 2 are compared with the results of the theoretical calculations of Table 3. The obtained theoretical cross sections of the reaction $(\gamma, 1n)$, $(\gamma, 2n)$, and $(\gamma, 1p)$ on $^{196,198-202,202}$Hg isotopes were combined with the bremsstrahlung spectrum $W(E, E^{max})$, where $E^{max} = 19.5$ MeV, and the yields of the corresponding reactions were obtained. We used the bremsstrahlung spectrum that was calculated using the GEANT4 library [13]. The figure also shows the yield of the ^{201}Hg$(\gamma, 1p)$ reaction calculated based on the data of [3] (normalization to the theoretically calculated yield of the ^{196}Hg$(\gamma, 1n)$ reaction).

On the whole, sufficiently good agreement between the experimental results and the theoretically calculated yields of the $(\gamma, 1n)$ reaction was observed. The yields of the isomeric state $E = 298.93$ keV, $J^P = 13/2^+$ in a ^{197}Hg isotope and the isomeric state $E = 532.48$ keV, $J^P = 13/2^+$ in a ^{199}Hg isotope were experimentally measured for the first time. In both cases, the isomeric state yield is several percentage points from the yield of the reaction with the formation of ^{197}Hg and ^{199}Hg isotopes in the ground state. Such a large difference in yields is explained by a large difference in the values of the spins of the ground and isomeric states (see Table 2).

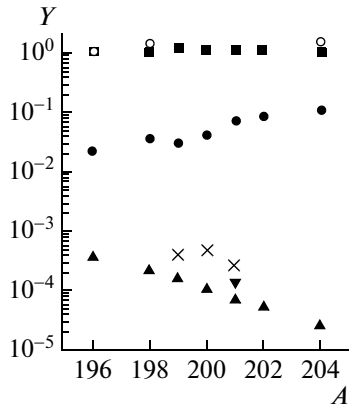

Fig. 3. Yields of photonuclear reactions on Hg isotopes normalized to the total yield of the ^{196}Hg$(\gamma, 1n)^{195}$Hg reaction. Theoretical calculations: (■) $\sigma(\gamma, 1n)$, (●) $\sigma(\gamma, 2n)$, (▲) $\sigma(\gamma, 1p)$. (▼) the data on $\sigma(\gamma, 1p)$ from [4]. Our result: (○) $\sigma(\gamma, 1n)$, (×) $\sigma(\gamma, 1p)$.

The authors of the present study were the first to measure the yield of photoproton reactions of ^{199,200,201}Hg isotopes (Table 2). The yields of photoproton reactions on ^{199,200,201}Hg isotopes are 3.9×10^{-4}, 4.6×10^{-4}, and 2.3×10^{-4}, respectively, and make $\sim 10^{-4}$ from the yield of photoneutron reactions. The decrease in the theoretically calculated yield of a photoproton reaction with an increase in the mass number A is explained by an increase in the energy of proton bonds with an increase in the mass number A (Table 1).

Within the limited accuracy of the experiment and the modes the calculations were based on [5, 6], the yields of photoproton reactions we measured are in agreement with the calculations of [5, 6] and the data of [4].

The peaks corresponding to isomer yields in the ^{201}Hg$(\gamma, p)^{200m}$Au and ^{199}Hg$(\gamma, p)^{198m}$Au reactions were not observed in the measured spectra. Based on this, the yields of these isomers were estimated. The isomeric ratios did not exceed 0.01. This result, as well as the yield of isomers in photoneutron reactions, is related to the large difference in the spins of the ground states and isomers. The s spin parity of ^{200m}Au and ^{198m}Au is $J^P = 12^-$, being $J^P = 1/2^-$ and $J^P = 3/2^-$ for ^{199}Hg and ^{201}Hg, respectively.

The $(\gamma, 2n)$ reaction were not observed in the present experiment, since in the majority of cases this reaction results in the formation of stable isotopes or the isotopes that are also formed in the $(\gamma, 1n)$ reaction. The yields of the $(\gamma, 1p1n)$ and $(\gamma, 2p)$ reactions are strongly suppressed as compared to the yield of the $(\gamma, 1p)$ reaction owing to the lower average energy of the formed photons and the suppression of their passage through a nucleus potential barrier.

CONCLUSIONS

The yields of photonuclear reactions on stable isotopes of Hg were measured on a bremsstrahlung beam with an energy of $E^{max} = 19.5$ MeV.

The yields of photoproton reactions on ^{199,200,201}Hg isotopes and photoneutron reactions on ^{196,198,200,204}Hg isotopes were measured for the first time. The yield of isomeric state formed in photonuclear reactions was measured for ^{195,197,199}Hg isotopes.

Cross sections of partial channels $(\gamma, 1n)$, $(\gamma, 2n)$, and $(\gamma, 1p)$ on stable Hg isotopes were calculated based on a collective model of a nucleus taking isospin splitting of the GDR into account. On the whole, the results of the theoretical calculations are in good agreement with the experimental results.

These results point to the manifestation of isospin peculiarities during the decay of $T_>$ states along proton and neutron channels. The analysis of the dependences of yields and cross sections of photoneutron and photoproton reactions on the mass number A reveals a tendency towards an increase of photoproton reaction cross sections when moving off the region of beta stability to that of neutron-deficit nuclei and an increase in cross sections of photoneutron reactions during a shift toward the region of nuclei that are overladen with neutrons.

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