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Frequency Modulation Spectroscopy of Coherent Dark Resonances of Multi-Level Atoms in a Magnetic Field

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Abstract—The resonances of coherent population trapping (CPT) excited by a frequency-modulated (FM) field at the Zeeman sublevels during the transition $F_g = 2 \longrightarrow F_e = 1$ of the ^{87}Rb D_1-absorption line were studied theoretically. The influence of the nonlinear Zeeman effect on the structure of the observed resonances was considered. The spectra of CPT resonances were calculated for different values of magnetic field induction and compared with the experimental data.

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INTRODUCTION

The effect of coherent population trapping (CPT) is one of most intriguing nonlinear phenomena that appear in multilevel systems under interaction with radiation. The CPT resonances, due to their small width, are widely used in different precision spectroscopy applications, such as metrology [1], magnetometry [2], and high-sensitivity laser interferometers for detecting gravitational waves [3]. The phenomena of quantum coherence and interference are the basis for lasers without inversion [4]. The possibility of using the CPT effect for atomic cooling and optical bistability, as well as in designing the element base of quantum computers and quantum communication systems (see, for example, [5, 6]) is of interest.

Of specific interest are the study of CPT resonances spectra in alkali metal vapors and the possibility of developing sensitive and high-precision alkali magnetometers that use the CPT effect in cells filled with atomic vapors. Alkali magnetometers are widely applied for precision measurement of magnetic fields in a wide dynamic range (up to μT) and have sensitivity at the level of nT fractions (see, for example, [7]). They are used in measurements of the magnetic fields of the Earth, laboratory fields, in medical magnetocardiography [8], and in some important fundamental research (for example, in experiments on the search for the neutron electric dipole moment [9]).

There are two experimental techniques for observing the spectra of CPT resonances. The first one uses two resonant laser fields interacting with the studied sample. The second, a simpler one, uses a single laser field modulated by frequency. Optical frequency-modulation (FM) spectroscopy, whose methods started to be developed in the beginning of the 1980s, at present play a key role in precision measurements in such branches of physics as detecting gravitational waves, frequency standards, measuring weak magnetic fields, and other branches (see the review [4]). In our previous papers [11–13] we developed the theoretical methods for calculating the dynamics of CPT resonances formation and absorption spectra in the case of a three-level Λ-system interacting with an FM field. In [13] the theoretical calculations are compared with the experimental results from the FM spectroscopy of Rb dark resonances in magnetic fields obtained at the Lebedev Physical Institute of the Russian Academy of Sciences. We studied the working transition $5^2S_{1/2}(F = 2) \longrightarrow 5^2P_{1/2}(F = 1)$ of the ^{87}Rb atom, whose levels were split in a magnetic field into three and five Zeeman sublevels, respectively, and formed three Λ-systems. The calculations were performed in the approximation of the three-level model when all the three Λ-systems are equivalent (the splittings of the lower sublevels are virtually equidistant) and can be approximated by a single three-level system. However, this approximation is true only in the case of weak magnetic fields of not more than 10 G (the so-called linear Zeeman effect). When the magnetic intensity is higher, the splitting of the lower sublevels is not equidistant and each of the three Λ-systems makes its contribution to the CPT resonance.

In this paper, the spectra of CPT resonances are calculated for multilevel atoms at different values of magnetic field.

1. THE EFFECT OF COHERENT POPULATION TRAPPING

The CPT effect is most pronounced in a three-level system consisting of two closely spaced long-living levels $|1\rangle$ and $|2\rangle$ with frequency splitting ω_{12} and the third level $|3\rangle$ distant from them (the Λ-system). The

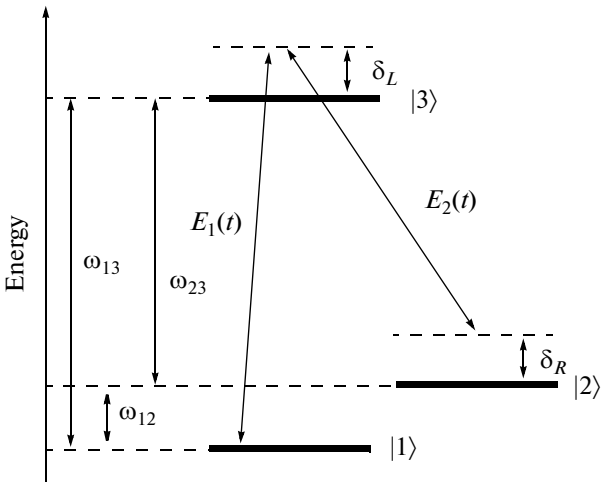

Fig. 1. Scheme of the three-level Λ-system interacting with two laser fields $E_1(t)$ and $E_2(t)$. δL is the one-photon laser detuning at the transition $|1\rangle \rightarrow |3\rangle$, δR is the two-photon Raman detuning.

system is excited by two continuous laser fields, so the distant level is optically connected with two other levels by electric dipole transitions (Fig. 1).

In the case when the transition $|1\rangle \leftrightarrow |2\rangle$ in the dipole approximation is forbidden and the two monochromatic fields $E_1\exp(i\omega_{L1}t - i\varphi_1)$, $E_2\exp(-i\omega_{L2}t - i\varphi_2)$ are in resonance with the corresponding transitions $|1\rangle \leftrightarrow |3\rangle$, $|2\rangle \leftrightarrow |3\rangle$, a CPT resonance is formed as a result of quantum interference under zero detuning $\delta_R = \omega_{L1} - \omega_{L2} - \omega_{12}$. It shows itself in the absorption spectrum as a narrow dip and can be observed, for example, when one of the acting fields, for example, ω_{L1} is scanned (Fig. 1). Thus, light-induced transparency of the medium occurs, which is also called electromagnetically induced transparency [14, 15]. CPT resonances have small spectral width and the widths of the CPT resonance in experiments on the saturation/fluorescence spectroscopy of Cs atom vapors with buffer gases at room temperature can be of tens Hz [16, 17].

The CPT phenomenon was theoretically studied in detail within the three-level model in the classical work by Arimondo [18]. The model permits one to perform basic calculations in the analytical form. A specific case, the study of alkali atoms in the approximation of high buffer gas pressure, which permits one to neglect the contribution of the Doppler effect, was considered in [19, 20]. In the general case of an N-level atom, we proposed [21] a universal theory for calculation of CPT resonances in multilevel atoms interacting with two fields. The theory permits one to calculate excitation of arbitrary schemes of atom levels with allowance for the effects of relaxation, magnetic field induction, and the Doppler effect. A two-frequency model for calculating CPT resonances in multilevel atoms interacting with a frequency-modulated (FM) field was proposed in [11, 12]. In this paper, our calculation methods will be extended to the case of a multilevel (eight-level) system describing the transition $5S_{1/2}(F_g = 2) \longrightarrow 5P_{1/2}(F_e = 1)$ of the ^{87}Rb D_1 line.

2. MODULATION LASER SPECTROSCOPY OF CPT RESONANCES IN A RB ATOM

The interaction of a three-level Λ-system with a modulated field considerably complicates the structure of absorption spectra. In contrast to the case of interaction with two monochromatic fields, where a single CPT resonance is observed, if the Λ-system interacts with an FM field, one can observe additional resonances in the absorption spectrum [11]. In addition, the presence of the Zeeman structure complicates the picture of CPT resonance formation and has a significant effect upon the observed parameters of the resonances [21].

There are two possible types of experiments on the observation of CPT resonances in a constant magnetic field based on the use of a single frequency-modulated laser:

—*Low-frequency modulation spectroscopy* provides observation of CPT resonances in the case where an FM field excites transitions from only one hyperfine sublevel, for example, $F_g = 2$, where F is the quantum number of the full moment of an atom.

—*High-frequency modulation spectroscopy* provides observation of CPT resonances under simultaneous excitation of two hyperfine sublevels $F = 1$ and $F = 2$.

Thus, there are two possible configurations for creating CPT (see, for example, [7, 13, 22]): at the Zeeman sublevels of the ground state and at the levels of a hyperfine structure of different ground states.

In this paper, we study the interaction of an FM field with a system of Zeeman sublevels of a ^{87}Rb atom at the transition $5S_{1/2}(F_g = 2) \longrightarrow 5P_{1/2}(F_e = 1)$ (Fig. 2). The choice of this transition is caused by the fact that the hyperfine components of the excited level of this transition are permitted on the background of Doppler broadening which makes it possible to study the interaction of radiation separately with this transition in a cell filled with atom vapors at room temperature. In addition, the above-mentioned transition provides the highest contrast of CPT resonances at the Zeeman sublevels [23, 24].

In the experiment, radiation is modulated by modulation of a laser diode current at frequency Ω_{mod} [13]. This is accompanied both by radiation amplitude modulation (AM), which is characterized by the amplitude modulation index R, and by frequency modulation with the frequency modulation index M. In the case of harmonic modulation, the laser field is written as follows:

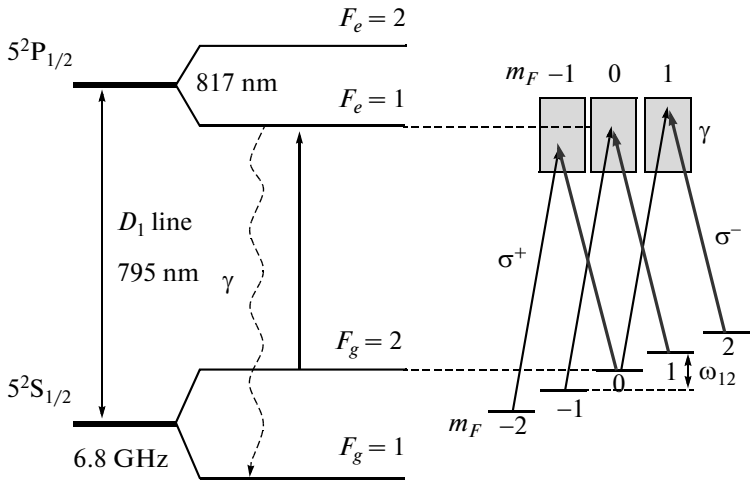

Fig. 2. Structure of the D_1 line of a ^{87}Rb atom. The working transition is $F_g = 2 \longrightarrow F_e = 1$. In a magnetic field each of the levels $F_g = 2$ and $F_e = 1$ is split into $2F + 1$ sublevels. The pairs of the Zeeman sublevels of the ground state $\Delta m_F = 2$ and the Zeeman sublevels of the excited state form a chain of three Λ-systems.

$$\begin{aligned} E(t) &= E_0[1 + R\sin(\Omega_{\text{mod}}t + \Psi)] \\ &\times \exp[i(\omega_0 t + M\sin\Omega_{\text{mod}})] + \text{c.c.} \\ &= \frac{E_0}{2}\sum_{n=-\infty}^{+\infty} a_n \exp(i(\omega_0 + n\Omega_{\text{mod}})t) + \text{c.c.}, \end{aligned} \quad (1)$$

where Ψ describes the relative phase between the frequency and amplitude modulation and the coefficients a_n in the Fourier expansion are expressed by the Bessel functions $J_n(M)$:

$$\begin{aligned} a_n &= J_n(M) - i\frac{R}{2}J_{n-1}(M)e^{i\Psi} \\ &+ i\frac{R}{2}J_{n+1}(M)e^{-i\Psi}. \end{aligned} \quad (2)$$

The field spectrum varies with variation of the index M and frequency Ω_{mod}. When Ω_{mod} is fixed, an increase of the modulation index leads to an increase of the number of spectrum bands, the distance between which is Ω_{mod}.

When a Λ-system interacts with an FM field, each pair of frequency components of the spectrum makes a contribution to the CPT resonance that is formed when the frequency difference coincides with the Zeeman splitting ω_{12} of the ground state. Thus, additional CPT resonances in the spectrum arise. They are called highest order resonances. The condition for their appearance has the form $\omega_{12} = k\Omega_{\text{mod}}$, where the integer k is called the order of resonance. Experimentally, CPT resonances are observed either when Ω_{mod} is varied at a fixed external magnetic field or due to variation of the magnetic field at a fixed Ω_{mod} [11, 12].

The amplitude modulation index R in the experiment that is described in [13] does not exceed 0.025 and so the allowance for amplitude modulation does not change the structure of the CPT resonance spectra qualitatively but has only an insignificant effect on the resonance amplitudes. Below in this paper, when we talk about an FM field we mean that the amplitude modulation is also taken into account in the calculations.

In [13], to increase the signal-noise ratio during detection of CPT resonances, the modulation frequency was additionally varied with frequency $f = 10$ kHz in the range dev = (5–500 kHz). Then the method of synchronous detection at the frequency f was used. The additional modulation is also taken into account in the calculations as $\Omega_{\text{mod}}(t) = \text{dev}\sin(ft)$.

3. ZEEMAN SPLITTING OF THE HYPERFINE STRUCTURE OF THE GROUND STATE OF A ^{87}Rb ATOM

The levels $F_g = 2$ and $F_e = 1$ of a ^{87}Rb atom are each split into $2F + 1$ sublevels in a magnetic field. The pairs of Zeeman sublevels of the ground state $F_g = 2$ with $\Delta m_F = 2$ (m_F is the magnetic quantum number) and the Zeeman sublevels of the excited state $F_e = 1$ form a chain of three Λ-systems. In a weak homogeneous magnetic field (the linear Zeeman effect) the splitting frequencies are equal and the three Λ-systems are equivalent to each other (Fig. 3a), so one can observe a single CPT resonance in the absorption spectrum. With the increase of magnetic intensity due to the nonlinear Zeeman effect the splitting of lower sublevels is no longer equidistant and each of the three Λ-systems makes a contribution to the CPT resonance. We denote the values of splitting of the lower levels by ν_1, ν_2, and ν_3.

The value of the Zeeman splitting is described by the Breit–Rabi formula, which yields a more exact expression that takes the corrections of the second approximation of the perturbation theory into

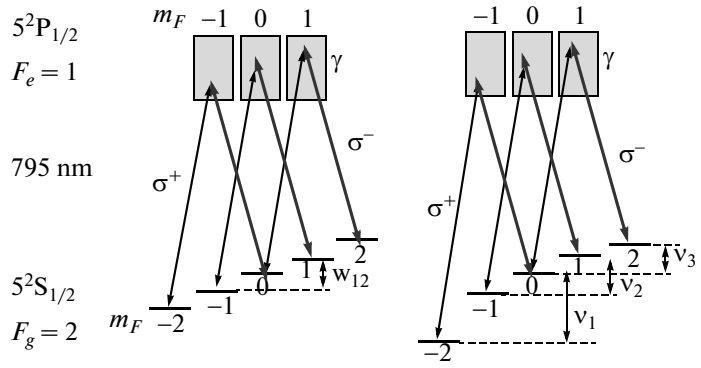

Fig. 3. Structure of the D_1 line of a ^{87}Rb atom in a magnetic field. The case of (a) linear Zeeman effect, (b) nonlinear Zeeman effect. ν_1, ν_2, and ν_3 are splittings of the lower levels with $\Delta m_F = 2$; the width of the upper level is γ.

account, as compared with the simple linear Zeeman effect,

$$\nu_{m_F} = -\frac{\nu_{\rm hf}}{2(2I+1)} + m_F g_I B\frac{\mu_B}{h} \pm \frac{\nu_{\rm hf}}{2}\sqrt{1+\frac{4m_F}{2I+1}x+x^2}, \quad (3)$$

where B is the magnetic field induction and x is defined by the expression

$$x = (g_J - g_I)B\frac{\mu_B}{h\nu_{\rm hf}},$$

where I, J are the quantum numbers of the nuclear momentum and momentum of the electron shell, $F = I \pm J$, g_I, g_J are the nuclear and electron g-factors, μ_B is the Bohr magneton, μ_F is the magnetic quantum number, and $\nu_{\rm hf}$ is the value of hyperfine splitting. The values of the parameters are presented in the table in [28]:

In the case of weak magnetic fields ($x \ll 1$), the value of the splitting of the magnetic sublevels for which $\Delta_{m_F} = 2$ can be written in the form

$$\delta\nu = \delta\nu_{\rm LIN} + \delta\nu_{\rm NLIN} \approx 2A'B - A''(m_{F,i-1} + m_{F,i+1})B^2, \quad (4)$$

$$A' = \frac{\mu_B}{h}g_F, \quad A'' = \frac{(\mu_B/h)^2(g_J-g_I)^2}{8\nu_{\rm hf}},$$

Constants for ^{87}Rb

g-factor g_I	−0.0009951414(10) [29]
g-factor $g_J(5^2S_{1/2})$	2.00233113(20) [29]
g-factor $g_J(5^2P_{1/2})$	0.666 [29]
$\nu_{hf}(5^2S_{1/2})$	$h \times 3.417341305452145(45)$ GHz [31]
$\nu_{hf}(5^2P_{1/2})$	$h \times 407.24(77)$ MHz [29]
Bohr's magneton μ_B/h	1.399624604(35) MHz/G [30]

where $g_F \approx g_J/(2I+1)$, and the values for the coefficients A', A'', and B are presented in [22]. The splitting of lower levels in a weak field may be considered as equidistant; as a result, one can observe only one CPT resonance in the absorption spectrum. This is valid when the value of the magnetic field is less than 10 G. On further increase of the magnetic field one should take into account the nonlinear Zeeman effect due to which the splitting of the magnetic components becomes nonequidistant (Fig. 3b) and three Λ-systems with different splitting of the lower levels ν_1, ν_2, and ν_3 are formed. To find the values of splitting, we expand the Breit–Rabi formula (3) into a series in powers of x and obtain the following expressions,

$$\nu_1 = \nu_{m_F=0} - \nu_{m_F=-2} = (g_J + 3g_I)\frac{\mu_B}{2h}B + \Delta,$$

$$\nu_2 = \nu_{m_F=1} - \nu_{m_F=-1} = (g_J + 3g_I)\frac{\mu_B}{2h}B, \quad (5)$$

$$\nu_3 = \nu_{m_F=2} - \nu_{m_F=0} = (g_J + 3g_I)\frac{\mu_B}{2h}B - \Delta,$$

where

$$\Delta = \frac{1}{4\nu_{\rm hf}}(g_J - g_I)^2\left(\frac{\mu_B}{h}B\right)^2. \quad (6)$$

Thus, the splitting of CPT resonances is primarily determined by the term Δ which is quadratic in the field.

4. MATHEMATICAL TECHNIQUE OF CALCULATIONS

The dynamics of a quantum system are described by the density matrix, and the temporal evolution is determined by the kinetic equation of the form

$$\frac{\partial\hat{\rho}}{\partial t} = \mathcal{L}_t\hat{\rho}. \quad (7)$$

By the use of the superoperator technique (described in detail in [21]), the Liouvillian of an N-

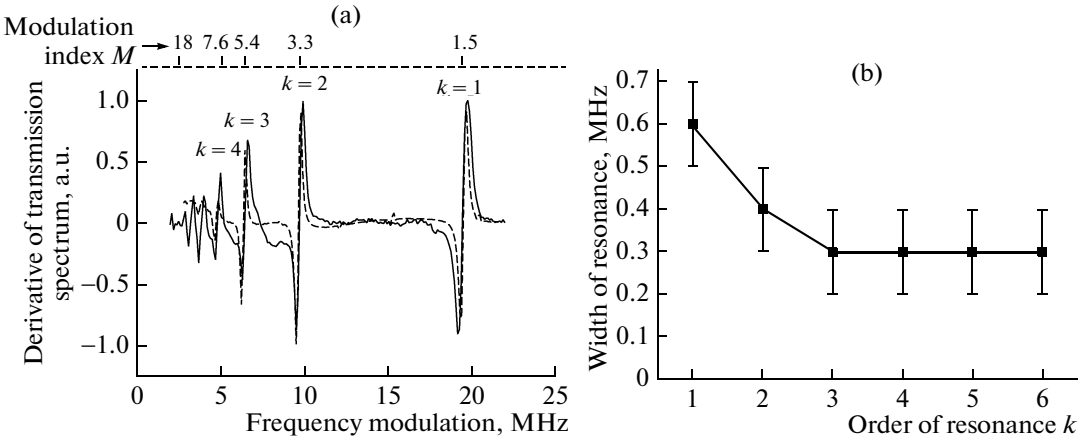

Fig. 4. (a) Experimentally detected spectrum of CPT resonances (solid line) in a cell with ^{87}Rb vapor at a fixed magnetic field $B = 14$ G, deviation dev = 200 kHz, power of exciting FM light $P \approx 500$ μW, and the constant of synchronous detector time $\tau =$ 3 ms. The upper scale presents the frequency modulation indices M for several resonant frequencies. The result of the theoretical calculation for the derivative of the transmission spectrum of a cell for the above-mentioned parameters of the experiment is plotted as dashed line. The resonance orders k are written over the resonances. For the presented parameters, $k_{\max} = 4$. (b) Resonance width as a function of their order k.

level atom in a laser wave in the rotating field approximation, as in the two-level case, can be represented as a sum of contributions

$$\mathcal{L}_t = \mathcal{L}_r + \mathcal{L}_e + \mathcal{L}_\delta + \mathcal{L}_i, \tag{8}$$

where $\mathcal{L}_r$ is the superoperator of radiative decay, $\mathcal{L}_e$ is the superoperator of elastic dephasing, $\mathcal{L}_i$ is the superoperator of interaction with the laser field, and $\mathcal{L}_\delta$ is the superoperator of laser detuning, which complements the chosen superoperator of nonperturbative evolution to the superoperator of free dynamics of an atom in the zero laser field. The latter includes the corresponding detunings of all acting laser fields with allowance for the fact that free precession with frequencies of these fields is included in the superoperator of nonperturbative dynamics.

The radiative decay superoperator has the form:

$$\mathcal{L}_r = \sum_{kl} \gamma_{kl} \left(\hat{P}_{lk} \odot \hat{P}_{kl} - \frac{1}{2} [\hat{P}_{kk}, \odot]_+ \right), \tag{9}$$

where the two-dimensional array γ_{kl} describes the rates of spontaneous decay for $k > l$ and pumping rates for $k < l$. The superoperator of elastic dephasing is described by the sum

$$\mathcal{L}_e = \sum_{k<l} (\mathcal{L}_{\text{in}}^{kl} + \mathcal{L}_{\text{ex}}^{kl}) = -\Gamma_{\text{in}}^{kl} (\hat{P}_{kk} \odot \hat{P}_{ll} + \hat{P}_{ll} \odot \hat{P}_{kk}) - \Gamma_{\text{ex}}^{kl} [\hat{I}_{kl}, \odot]^2, \tag{10}$$

where Γ_{in}^{kl}, Γ_{ex}^{kl} are the dephasing rates.

The superoperator of laser detuning depends on the type of the considered resonance and usually can be written in the form of an antisymmetric superoperator represented as a commutator with the population operators

$$\mathcal{L}_\delta = i \sum_k \delta_k (\hat{P}_{kk} \odot - \odot \hat{P}_{kk}), \tag{11}$$

where δ_k is the array of frequency detunings.

Interaction with a laser field can be represented in the form of an antisymmetric commutator with the polarization operators

$$\mathcal{L}_i = -\frac{i}{2} \sum_{k<l} \Omega_{kl} [(\hat{P}_{kl} + \hat{P}_{lk}), \odot], \tag{12}$$

where Ω_{kl} is the two-dimensional array of Rabi frequencies for the kl-transitions.

The presented technique is efficient for use in analytical and numerical calculations connected with solving both the problem of the stationary state and the full spectral problem for the evolution superoperator $\mathcal{L}_t$ [25, 26].

When solving the system of equations (7) for a specific problem, one can calculate, for example, the absorption spectrum for a Λ-system. For the considered eight-level system, the number of solved equations is 64.

5. CALCULATION OF THE FM ABSORPTION SPECTRA OF THE ^{87}Rb D_1 LINE IN A MAGNETIC FIELD

To compare the theoretical calculations with the experimental data, we chose a corresponding set of parameters starting from the experimental data for the specific experiments with buffer gas and vacuum cells [13]. The dipole moments d of the transitions were taken from [27], the Rabi frequencies g and decay rates γ were calculated by the formulas

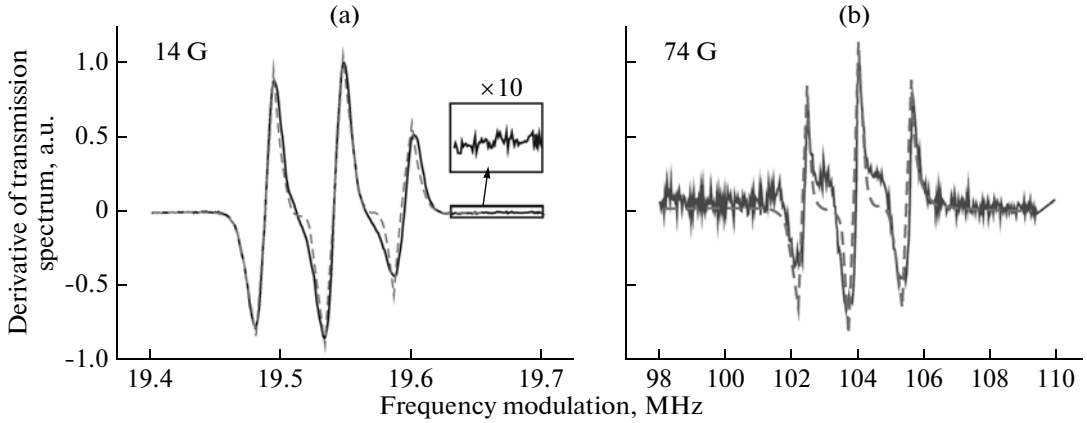

Fig. 5. Experimental spectra of CPT resonances (solid line) obtained in the buffer cell and theoretical calculations (dashed line) for (a) $B \approx 14$ G, (b) $B \approx 74$ G.

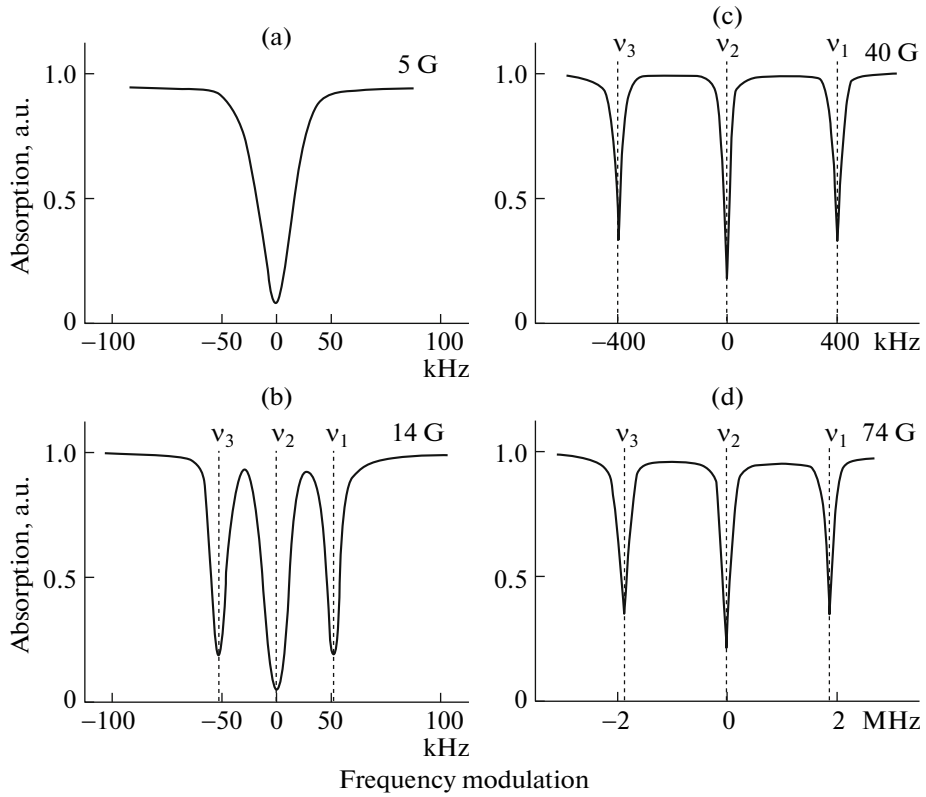

Fig. 6. Calculated spectra of CPT resonances for four values of magnetic field induction $B \approx 5$, 14, 40, 74 G for a cell with pure rubidium. The position of the resonance at frequency ν_2 is taken as zero.

$$g = dE(t)/\hbar, \quad \gamma = 4d^2\omega^3/3\hbar c^3,$$

where m and e are the mass and charge of an electron, respectively, c is the velocity of light, ω is the frequency of a given transition. The field $E(t)$ is represented in the form (1) with allowance for the additional modulation. The Zeeman splitting was calculated by formulas (4) and (5) in the cases of the linear and nonlinear Zeeman effect, respectively.

On the basis of the mathematical technique described in section 4, we calculated the absorption spectra of the D_1 line of ^{87}Rb atoms within the eight-level model. The spectra correspond to the experimental parameters, both in cells with pure rubidium and in cells with the buffer gas Kr [13].

Let us consider the case when the magnetic field is fixed, i.e., the value of Zeeman splitting does not vary but the modulation frequency Ω_{mod} does. Figure 4a

presents the experimental and theoretical spectra of CPT resonances (calculated within the eight-level model) in a vacuum cell. The separate scale in the experimental plot shows the values of the frequency modulation index M [13] for several frequencies. One can see that the experimental and theoretical plots are in good agreement.

The plot of resonance width against their order k (Fig. 4b) shows that for small k one can observe a perceptible decrease of the resonance width with increase of k, which is saturated afterwards. The presence of saturation seems to be connected with the modulation technique of detection because the width of the observed resonance during frequency modulation cannot be less than the modulation amplitude dev.

Figure 5 presents the fundamental ($k = 1$) CPT resonance detected in a buffer cell in a constant magnetic field $B \approx 14$ G and comparison with the theoretical calculation. The resonance at frequency $\nu \approx 19.55$ MHz is split into three components due to the quadratic Zeeman effect [32]. The differences of frequencies between the components are $\nu_3 - \nu_2 = \nu_2 - \nu_1 = \Delta \simeq 50$ kHz. The width of CPT resonances in a cell with a buffer is determined mainly by inhomogeneity of the magnetic field in the cell and by the contribution of the quadratic Zeeman effect. When the magnetic field induction is greater than 10 G, the resonance components can be resolved; their width is approximately 15 kHz for $B = 14$ G. For a cell without buffer gas, the typical value of the width is about 200 kHz.

Figure 6 presents the calculated spectra of CPT resonances in pure rubidium for four values of magnetic field induction. For $B = 5$ G (Fig. 6a), a single resonance is observed at frequency ν_2 because the contribution of the quadratic Zeeman effect to splitting of the magnetic sublevels of the ground state is less than the resonance width. In the other three cases (Figs. 6b–6d), the resonance is split into three components at frequencies ν_1, ν_2, and ν_3, and the greater the difference of splitting frequencies is, the less the distortion of the resonance shape is and the less the resonance width is. The resonance widths in Fig. 6a–6d are (a) ~17 kHz, (b) ~20 kHz, (c) ~50 kHz, and (d) ~0.25 MHz, respectively.

CONCLUSIONS

In this paper, the effect of a magnetic field on FM spectra of coherent dark resonances at the D_1 line of ^{87}Rb was studied with allowance for all the transition levels (the eight-level model). In a magnetic field the upper and lower sublevels are split into three and five Zeeman sublevels, respectively, and form three Λ-systems. On the base of methods for theoretical calculation of dynamics of CPT resonance formation, which were developed earlier in the works [11, 12, 13], the calculations have been performed for the cases of weak magnetic fields <10 G (the regime of the linear Zeeman effect) and magnetic fields with induction in the range from 14 to 74 G (the regime of the nonlinear Zeeman effect). The experimental and theoretical results have been compared for two values of magnetic field induction, 14 and 74 G. It has been shown that the results of numerical calculations completely reproduce the experimental data for the position and width of the resonances, and the shapes of the spectra.

REFERENCES

1. S. Knappe, V. Velichansky, H. G. Robinson, et al., Rev. Sci. Instrum. **74**, 3142 (2003).
2. M. Fleischhauer and M. O. Scully, Phys. Rev. A **49**, 1973 (1994).
3. C. M. Caves, Phys. Rev. D: Part. Fields **23**, 1693 (1981).
4. O. A. Kocharovskaya and Ya. I. Khanin, Pis'ma Zh. Eksp. Teor. Fiz. **48**, 581 (1988) [JETR Lett. **48**, 630 (1988)].
5. B. Julsgaard, J. Sherson, J. I. Cirac, et al., Nature **432**, 482 (2004).
6. C. H. van der Wal, M. D. Eisaman, A. Andre, et al., Science **301** (5630), 196 (2003).
7. P. D. D. Schwindt, S. Knappe, V. Shah, et al., Appl. Phys. Lett. **85**, 6409 (2004).
8. G. Bison, R. Wynands, and A. Weis, Appl. Phys. B, No. 3, 325 (2003).
9. M. Kuzniak, K. Bodek, T. Brys, et al., J. Res. Nat. Inst. Stand. Technol. **110**, 461 (2005).
10. G. E. Hall and S. W. North, Ann. Rev. Phys. Chem. **51**, 243 (2000).
11. Yu. V. Vladimirova, B. A. Grishanin, V. N. Zadkov, et al., Zh. Eksp. Teor. Fiz. **130**, 130 (2006) [J. Exp. Theor. Phys. **103**, 528 (2006)].
12. Yu. Vladimirova, B. Grishanin, V. Zadkov, et al., Laser Phys. Lett. **3**, 427 (2006).
13. Yu. V. Vladimirova, V. N. Zadkov, A. V. Akimov, A. Yu. Samokotin, et al., Appl. Phys. B**97**, 35 (2009).
14. S. E. Harris, J. E. Field, and A. Imamoglu, Phys. Rev. Lett. **64**, 1107 (1990).
15. M. D. Lukin, M. Fleischhauer, M. O. Scully, and V. L. Velichansky, Opt. Lett. **23**, 295 (1998).
16. S. Brandt, A. Nagel, R. Wynands, and D. Meschede, Phys. Rev. A **56**, 1063 (1997).
17. M. Erhard and H. Helm, Phys. Rev. A **63**, 043813 (2001).
18. E. Arimondo, in *Progress in Optics,* vol. 35, Ed. by E. Wolf (Amsterdam, 1999), p. 257.
19. A. V. Tachenachev, A. M. Tumaikin, V. I. Yudin, et al., Phys. Rev. A **67**, 033810 (2003).
20. A. V. Tachenachev, A. M. Tumaikin, V. I. Yudin, et al., Phys. Rev. A **69**, 024501 (2004).
21. Yu. V. Vladimirova, B. A. Grishanin, V. N. Zadkov, et al., Zh. Eksp. Teor. Fiz. **123**, 710 (2003) [J. Exp. Theor. Phys. **96**, 629 (2003)].
22. Ch. Andreeva, G. Bevilacqua, V. Biancalana, et al., Appl. Phys. B **76**, 667 (2003).
23. A. V. Tachenachev, A. M. Tumaikin, V. I. Yudin, and G. Nienhuis, Phys. Rev. A **69**, 033410 (2004).

24. S. A. Zibrov, V. L. Velichanskii, A. S. Zibrov, et al., Pis'ma Zh. Eksp. Teor. Fiz. **82**, 534 (2005) [JETP Lett. **82**, 477 (2005)].

25. B. A. Grishanin, V. N. Zadkov, and D. Meschede, Phys. Rev. A **58**, 4235 (1998).

26. I. V. Bargatin, B. A. Grishanin, and V. N. Zadkov, Proc. SPIE–Int. Soc. Opt. Eng. **3736**, 246 (1998).

27. D. A. Steck, http://steck.us/alkalidata/index.html (2009).

28. C. W. White, W. M. Hughes, G. S. Hayne, and H. G. Robinson, Phys. Rev. A **7**, 1178 (1973).

29. E. Arimondo, M. Inguscio, and P. Violino, Rev. Mod. Phys. **49**, 31 (1977).

30. P. J. Mohr and B. Taylor, Rev. Mod. Phys. **72**, 351 (2000).

31. S. Bize, Y. Sortais, M. S. Santos, et al., Europhys. Lett. **45**, 558 (1999).

32. A. V. Taichenachev, V. I. Yudin, A. Yu. Samokotin, et al., Pis'ma Zh. Eksp. Teor. Fiz. **88**, 409 (2008) [JETP Lett. **88**, 355 (2008)].