ON THE THEORY OF THE GENERATOR WITH OPTICAL DELAY LINE

V. N. Parygin and Kh. Gassab

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The paper presents an experimental study of a generator with an optical delay line in the feedback circuit. The amplitude and frequency characteristics of the generator are studied at various lengths of the optical delay line. The possibility of using such a generator for distance measurement is pointed out.

Occasional papers, e.g., [1], have been devoted to investigation of the generator with optical delay line in the feedback circuit. However, such a generator, a schematic diagram of which appears in Fig. 1, would be of great interest as a device whose frequency depends on the optical path length and the group velocity of the light. The generator can be used to measure distances and the velocities of transparent media and for other purposes.

In theoretical analysis of the optical delay line generator, it is necessary first of all to consider the features of this type of delay as compared to the conventional delay lines used by other authors [2-10]. First of all, the signal in the optical delay line is very broad-banded, since the modulation frequency is 9-10 orders lower than the carrier. Therefore the optical delay line can be regarded as an ideal line with fully linear dispersion. A second property of the optical line consists in the fact that the radio-circuit elements connected to it (light modulator and photodetector) are ideal unidirectional elements. In any real system, of course, there is, in addition to the optical delay τ_{opt}, a delay τ_{el} with disperse properties in the radio-circuit elements. However, the dispersion in the system can be neglected if $\tau_{opt} \gg \tau_{el}$. It should therefore be quite interesting to consider the theory of a generator with an ideal delay line in its feedback circuit

1. EQUATIONS FOR THE AMPLITUDES AND PHASES OF THE STATIONARY REGIME

We shall assume that the only nonlinear circuit element is the light modulator, whose modulation characteristic is described by the relation [11]

$$I(t) = I_0 \sin^2 \frac{\pi}{2} \frac{U(t)}{U_{\lambda/2}}, \tag{1}$$

where I(t) is the intensity of the light at the modulator output, I_0 is the intensity of the incident light, U(t) is the voltage applied to the modulator, and $U_{\lambda/2}$ is the half-wave voltage.

The nature of the nonlinearity can be controlled by applying a constant bias from an external source to the modulator. An interesting feature is the possibility of changing from in-phase oscillations (I(t) and U(t)) on one slope of the modulation characteristic to antiphase oscillations on the other slope. Thus, we can shift the phase of the system by π without changing the optical path length.

The optical delay line brings about an ideal shift in time that is independent of frequency:

$$\tau(\omega) = L/c, \tag{2}$$

where L is the length of the optical line and c is the velocity of light. As concerns the amplifier gain, it can, generally speaking, be made different at different frequencies.

The no-dispersion hypothesis enables us to assume that periodic oscillations will be excited in the system if the gain is high enough and can be represented by a Fourier series with independent amplitudes and phases:

$$U(t) = U_0 + \sum_k \sin a_k (k\omega t + \varphi_k). \tag{3}$$

The presence of multiple frequencies in the system will cause the oscillations to interact strongly with one another.

The fundamental frequency ω of the oscillations and the amplitudes a_k and phase shifts φ_k in the stationary regime can be found from the balance condition for the phases and amplitudes. Substituting (3) into (1) and introducing the dimensionless amplitudes $Z_k = \pi a_k / U_{\lambda/2}$, we obtain the following expression for the intensity of the light at the modulator output:

$$I(t) = \frac{I_0}{2} - \frac{I_0}{2}\left\{\sum_{n_1} J_{n_1}(Z_1) \sum_{n_2} J_{n_2}(Z_2) \ldots \cos[a_0 + (n_1 + 2n_2 + \ldots)\omega t + \right.$$
$$\left. + n_1\varphi_1 + n_2\varphi_2 + \ldots]\right\} = \frac{I_0}{2} + \sum_m I_m \sin(m\omega t + \psi_m). \tag{4}$$

As it propagates through the optical line, the light signal acquires a time delay τ and is then converted in the photodetector and amplified in the amplifier by a factor $\beta_{m\omega}$. The voltage $u'(t - \tau)$ obtained at the amplifier output therefore has the form

$$U'(t-\tau) = U_0 + \sum_m \beta_{m\omega} I_m \sin[m\omega(t-\tau) + \psi_m]. \tag{5}$$

In the generator's stationary regime, it is obvious that the amplifier output voltage (5) must equal the voltage (3) across the light modulator. From this we obtain the amplitude and phase balance equations:

$$Z_m = K_m \frac{I_m}{I_0}, \tag{6}$$

$$\varphi_m = \psi_m - m\omega\tau, \tag{7}$$

where

$$K_m = \frac{\pi\beta_{m\omega} I_0}{U_{\lambda/2}} \tag{8}$$

is the generalized system gain at the frequency $m\omega$. The amplitude (6) and phase (7) balance equations can, in principle, be used to determine the amplitudes and phases of all harmonic components of the voltage on the modulator. In fact, this can be done only with certain simplifying assumptions because of the complexity of relation (4).

We consider below certain elementary cases in which complete solution of the problem is possible.

2. SINGLE-FREQUENCY GENERATOR

Let us assume that the gain K_m equals zero (or is quite small) at all values of m except one (for example, m = 1). In this case, even though the light at the modulator exit contains harmonics of the frequency ω, only two of all of the amplitude and phase balance equations remain:

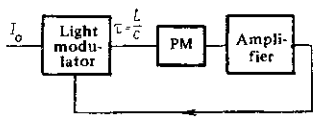

Fig. 1. Schematic diagram of generator.

$$Z_1 = K_1 |\sin a_0| J_1(Z_1), \tag{9}$$

$$\omega\tau = 2n\pi \quad \text{for } \sin a_0 > 0 \quad \text{and} \quad \omega\tau = (2n+1)\pi \quad \text{for } \sin a_0 < 0. \tag{10}$$

We see from relations (10) that generation occurs at one of the frequencies that are multiples of $1/\tau$ in work on the rising slope of the modulation characteristic. Frequencies $1/2\tau$, $3/2\tau$, etc. can be generated in work on the descending slope. Which of these frequencies is excited in the system will depend on the frequency-response curve of the amplifier.

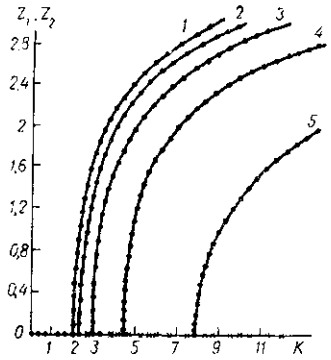

Fig. 2. Amplitude as a function of gain in the single-frequency regime with various biases:

1) $\frac{U_{\lambda/2}}{2}$, 2) $\frac{U_{\lambda/2}}{3}$, 3) $\frac{U_{\lambda/2}}{4}$, 4) $\frac{U_{\lambda/2}}{6}$ and 5) $\frac{U_{\lambda/2}}{12}$

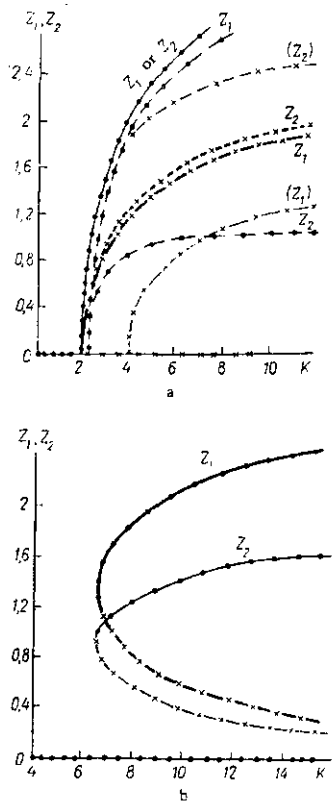

Fig. 3. Amplitudes in two-frequency regimes as functions of K: a) $U_0=\frac{1}{2}U_{\lambda/2}$, solid curves; $U_0=\frac{1}{3}U_{\lambda/2}$, dashed; b) $U_0 = 0$. The stable branches of the curves are dotted and the unstable branches are marked with crosses.

In the case of single-frequency generation, the amplitude of the steady-state oscillation is determined by Eq. (9). Since $J(Z_1) = Z_1/2$ at small Z_1, the condition for single-frequency generation has the form

$$K_1|\sin a_0| > 2. \qquad (11)$$

At $U_0 = 0$ or $U_0 = n\pi U_{\lambda/2}$, i.e., in operation on the square-law segment of the modulation characteristic, single-frequency generation is totally impossible. Oscillations are most easily excited at $U_0 = \frac{1}{2}U_{\lambda/2}$, i.e., at the middle of the linear segment of the characteristic. Figure 2 presents computed results for the stationary amplitude at various values of U_0 and K_1. The stability condition of these oscillations is the same as (11), i.e., if a single-frequency oscillation is possible in the system, it is stable.

3. GENERATION OF TWO MULTIPLE FREQUENCIES ω AND 2ω

Let us consider the case in which the amplifier gain has a frequency dependence such that simultaneous excitation of oscillations at the frequencies ω and 2ω is possible, and that the gains are the same for both frequencies and equal to K.

The instantaneous intensity of the light at the modulator output then has the form

$$I(t) = \frac{I_0}{2} - \frac{I_0}{2}\sum_{n_1=-\infty}^{+\infty} J_{n_1}(Z_1) \sum_{n_2=-\infty}^{+\infty} J_{n_2}(Z_2)\cos[a_0 + (n_1 + 2n_2)\omega t + n_2\varphi], \qquad (12)$$

where φ is the phase difference at t = 0 between the oscillations at the frequencies 2ω and ω.

Since the amplifier passes only the frequencies 1ω and 2ω, only those among all of the possible values of n_1 and n_2 in (12) for which $n_1+2n_2=\pm 1$ and $n_1+2n_2=\pm 2$ are of interest. However, there are infinitely many of even these values of n_1 and n_2. In calculations, the number can be limited from physical considerations. It is natural to assume when the gain is not too large that the maximum amplitude of the oscillations will not exceed $U_{\lambda/2}$. In this case, all Z_k are smaller than π. We know from the theory of Bessel functions that these functions are small when their order exceeds their arguments. In (12), therefore, we may omit all of the functions for which $|n_1| > 5$ or $|n_2| > 5$.

In this case, after the appropriate bookkeeping, we find $\varphi=\pi/2$ from the phase equations. With this condition, the amplitude equations assume the form

$$
\begin{aligned}
Z_1 &= K\{[J_1(Z_1)J_0(Z_2) + J_3(Z_1)J_2(Z_2) - J_5(Z_1)J_1(Z_2)]\sin a_0 + \\
&\quad + [J_1(Z_1)J_2(Z_2) - J_3(Z_1)J_1(Z_2) - J_5(Z_1)J_3(Z_2)]\cos a_0\}, \\
Z_2 &= K\{[J_0(Z_1)J_1(Z_2) + J_4(Z_1)J_1(Z_2) - J_4(Z_1)J_3(Z_2)]\sin a_0 + \\
&\quad + [J_2(Z_1)J_0(Z_2) - J_2(Z_1)J_2(Z_2)]\cos a_0\}.
\end{aligned}
\tag{13}
$$

Equation system (13) is quite complex, and for this reason it was solved with a BESM-6 computer. The computation yielded Z_1 and Z_2 as functions of K for each U_0. Usually, several values of Z_1 and Z_2 that satisfy Eqs. (13) correspond to a given value of K.

All of the solutions obtained can be classified into the following four types.

$Z_1 = 0$, $Z_2 = 0$—system at equilibrium.

$Z_1 \neq 0$, $Z_2 = 0$—an oscillation at frequency ω arises in the system. This solution can exist only when $\cos a_0 = 0$.

$Z_1 = 0$, $Z_2 \neq 0$—an oscillation at frequency 2ω occurs in the system.

$Z_1 \neq 0$, $Z_2 \neq 0$—the oscillations are of biharmonic nature.

The results of analysis of specific operating modes of this generator with the operating point at various positions on the modulation characteristic appear in Fig. 3.

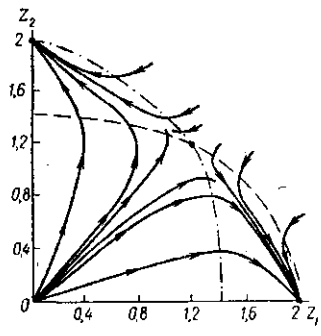

Fig. 4. Behavior of integral curves on the Z_1, Z_2-plane.

In the case $U_0 = \frac{1}{2}U_{\lambda/2}$, the system is self-excited at K > 2. Solutions belonging to all four types then appear.

It is convenient to use the Z_1, Z_2-plane for stability analysis of the various solutions for given K and U_0. The general pattern of the integral curves on this plane gives a clear presentation of the stable and unstable oscillation regimes. Complete construction of the integral-curve pattern on the Z_1, Z_2-plane is, of course, extremely difficult, but to investigate stability it is sufficient to consider a qualitative picture of the integral-curve behavior. It can be constructed by the isocline method [12]. Two of these isoclines on the Z_1, Z_2-plane can be indicated at once. In fact, the first equation in (13) defines the curve of stationary Z_1 on this plane. This is the isocline of the vertical tangents to the integral curves. Accordingly, the second equation of (13) defines the isocline of the horizontal tangents on the Z_1 and Z_2 plane. Figure 4 shows the Z_1, Z_2-plane and both isoclines for the case $U = \frac{1}{2}U_{\lambda/2}$ and K = 3.5. In this case, oscillations with the amplitudes $Z_1 = 2$, $Z_2 = 0$, $Z_1 = 0$, and $Z_2 = 2$ are stable. The equilibrium position $Z_1 = Z_2 = 0$ and the biharmonic regime are unstable. The regions of initial conditions from which the oscillations enter one of the stationary regimes are also easily seen on inspection.

In operation at the middle of the linear modulation-characteristic segment, therefore, the generator produces harmonic oscillations with the frequency ω or 2ω, depending on the initial excitation conditions, when the self-excitation conditions are exceeded. The nature of the resulting solutions changes on a change in the operating-point position on the modulation characteristic. For example, with $U_0 = \frac{1}{3}U_{\lambda/2}$ and K = 5 we have a stable biharmonic regime for which $Z_1 > Z_2$. The other biharmonic regime ($Z_2 > Z_1$) is unstable.

When $U = \frac{1}{6}U_{\lambda/2}$, the nature of the regimes in the system changes again: two biharmonic regimes and a stable equilibrium position exist at gains 3 < K < 3.5. This is the range of hard excitation, in which oscillations can arise only after a rather strong initial jolt. The equilibrium position is highly stable in operation on the square-law segment of

the modulation characteristic ($U_0 = 0$). At $K > 6.5$, however, hard excitation of biharmonic oscillations having $Z_1 > Z_2$ is possible.

Thus, two different types of oscillations can be excited in the system, depending on initial conditions, in operation on the linear modulation-characteristic segment. However, with the approach to the nonlinear range, excitation of one of these types becomes difficult and then even impossible.

4. OTHER TYPES OF TWO-FREQUENCY GENERATION

The investigations of the preceding section showed that the oscillations at frequencies ω and 2ω do not interact at the center of the linear modulation-characteristic segment (at $U_0 = \frac{1}{2} U_{h\omega}$). This is because of the absence of even terms of the characteristic. Strong coupling between the oscillations at frequencies ω and 3ω is observed. We therefore considered the problem of combined generation of these frequencies. It was shown by analysis that two stable oscillation modes are possible in this case. In the first case, with $Z_1 = 0$ but $Z_3 \neq 0$, an oscillation appears at frequency 3ω. The second stable oscillation is of biharmonic nature, with $Z_1 > Z_3$. This is in fact an oscillation at frequency ω with a small third harmonic.

We also solved the problem of generation when two nonmultiple frequencies fall in the amplification band. Such conditions may arise from the use of an amplifier with a band filter whose passband includes two frequencies with neighboring values of n, neither of which equals unity.

In this case, the equations determining the stationary amplitudes of the oscillations take the form

$$Z_1 = K|\sin a_0| J_1(Z_1) J_0(Z_2),$$
$$Z_2 = K|\sin a_0| J_0(Z_1) J_0(Z_2). \qquad (14)$$

Stability analysis of the possible oscillations showed that the biharmonic regime with nonmultiple frequencies is always unstable. Only harmonic oscillations with frequency ω_1 or ω_2 can be excited in this system. In this case the amplitude characteristics are the same as those of the single-frequency regime (see Fig. 2).

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Department of the Physics of Oscillations