THE EQUATIONS OF SCALE AND CONFORMAL INVARIANCE IN BOGOLYUBOV'S AXIOMATIC APPROACH

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Vol. 30, No. 6, pp. 752-753, 1975

UDC 538.3:530.145

Interest in scale and conformal transformations has increased in recent years since the discovery of the pointlike behavior of the form factors in deep inelastic scattering The possibility of using scale and conformal symmetry in elementary-particle theory has recently been under wide discussion. In [1-4], conformal and scale symmetry in quantum field theory were discussed within the framework of Wightman's axiomatic approach. It was found that the theory of scale and conformal invariance yields many interesting corollaries, for example, it permits unique determination of propagators and vertex functions accurate to a constant factor [1-6].

In Wightman's axiomatics, however, the basic quantities of the theory cannot be observed directly in an experiment, and this makes it difficult to obtain nontrivial, dynamic predictions as to the elements of the S-matrix.

On the other hand, Bogolyubov's axiomatic approach [7], in which the S-matrix is the principal quantity, is more directly related to the observed quantities and it is no accident that the dispersion relations were first established by following precisely this path. Scale and conformal invariance has not yet been examined in Bogolyubov's axiomatic approach. It therefore seems highly important to use to investigate the properties of scale and conformal invariance in that approach. We shall derive scale and conformal invariance equations for the functions $S_n(x_1,\dots,x_n)$ and show that the current operator has canonical dimensions in scale-invariant theory.

Bogolyubov's method [7] is based on the formalism of the variational S-matrix derivatives with respect to asymptotically free fields and on a microcausality condition that is simply formulated in terms of these variational derivatives. The scattering operator S is a functional with respect to the in-fields [7]:

$$S=\sum_n \frac{1}{n!}\int S_n(x_1\dots x_n):\varphi_1(x_1)\dots\varphi_n(x_n):d^n_x. \tag{1}$$

We shall consider massless in-fields, since only they are scale and conformally invariant [4]. The massless in-field $\varphi(x)$ satisfies the following commutation relations with the generators of the scale and conformal transformations, D and K_n [5]:

$$[\varphi(x),\ D]=i(d+x\partial)\varphi(x), \tag{2}$$

$$[\varphi(x),\ K_\mu]=i(dx_\mu+2x_\mu x_\nu\partial^\nu-x^2\partial_\mu-2ix^\nu\Sigma_{\nu\mu})\varphi(x), \tag{3}$$

where d is the dimension of the field $\varphi(x)$, which equals 1 for scalar and vector fields and 3/2 for a spinor field, and $\Sigma_{\mu\nu}=0$ for the scalar field and $\Sigma_{\mu\nu}=\frac{i}{4}[j_\mu, j_\nu]$ for the spinor field [5].

Scale invariance of the theory means that

$$[D,\ S]=0. \tag{4}$$

Expressions (2) and (4) yield an equation of scale invariance for $S_n(x_1,\dots,x_n)$:

$$\sum_{k=1}^{n}(-4+d_k-x_k\partial_k)S_n(x_1\dots x_n)=0. \tag{5}$$

Conformal invariance of the theory means that

$$[S, K_\mu] = 0. \tag{6}$$

Relations (2) and (6) yield a conformal-invariance equation for $S_n(x_1,\ldots,x_n)$:

$$\sum_{k=1}^{n} (2(4-d_k)x_{k\mu} + 2x_{k\mu}x_{k\nu}\partial_k^\nu - x_k^2\partial_{k\mu} + \tag{7}$$

$$+ 2ix_k^\nu \Sigma_{\mu\nu}) S_n(x_1\ldots x_n) = 0.$$

In Bogolyubov's axiomatics, the current operator is defined as

$$I(x) = iS^+ \frac{\delta S}{\delta\varphi(x)}. \tag{8}$$

It follows from the definition of the operator I(x) that the current operator has canonical dimensions in the scale-invariant theory, i.e., the dimension of I(x) equals 3 if $\varphi(x)$ is a scalar or vector. Hence it follows, among other things, that the dimension of the electromagnetic-current operator $I_\mu(x) = iS^+ \frac{\delta S}{\delta A^\mu(x)}$ equals 3, in agreement with the experimentally observable pointlike behavior of the form factors in deep inelastic scattering of electrons on nucleons [5].

We stress in conclusion that in contrast to Wightman's axiomatics, where the current operator I_μ can, in principle, be of any dimension, it can have only the canonical dimension in Bogolyubov's axiomatics. This indicates that the electromagnetic-current operator should have the canonical dimension.

The author thanks V. A. Matveev, A. N. Tavkhelidze, and K. G. Chetyrkin for valuable comments and amplifications.

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26 November 1974 Department of Quantum Statistics