# CONCERNING ONE PROBLEM OF THE NEW NONLINEAR ELECTRODYNAMICS

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The main accomplishment of the Einstein-Cartan theory [1] is the establishment of the fundamental fact that a gravitational torsion field is connected with the nonlinearity of wave fields with spin [2]. Such a connection is established by eliminating the torsion from the field equations with the aid of the Kibble-Frolov equations [3,4] and the ensuing effective self-action terms in the equations for physical fields. These terms are responsible for the spin-spin contact interaction of the wave fields. It turns out here that in the case of fermions the torsion can both generate and cancel out a minimal nonlinearity of the $\psi^3$ type [5,6]; for bosons, at the same time, the induced nonlinearity causes violation of their gauge invariance [7]. The last fact can in the future be of fundamental significance in the construction of a theory of a unified nonlinear vacuum.

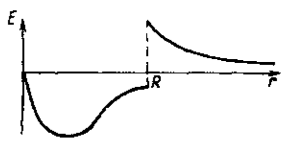


Fig. 1

This paper presents a solution of the electrodynamics equations whose nonlinearity is generated by torsion [7]

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\alpha}\left(\sqrt{-g}\,F^{\alpha\beta}\right) = -bA^\gamma F^{\alpha\beta}F_{\alpha\gamma} + \frac{4\pi}{c}j^\beta \qquad (1)$$

(where $A_\alpha$ is the 4-potential of the electromagnetic field, $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha + bA_\gamma(A_\alpha F_\beta{}^\gamma - A_\beta F_\alpha{}^\gamma)$ is the electromagnetic field tensor, $\alpha, \beta, \gamma = 0, 1, 2, 3$, $\sqrt{-g} = \det g_{\alpha\beta}$, $g_{\alpha\beta}$ is the Minkowski metric

$$b = \frac{k}{c^4} = 0{,}82\cdot 10^{-49}\ \mathrm{sec^3/g\cdot cm},$$

$k = 6{,}67\cdot 10^{-5}\ \mathrm{cm^3/g\cdot sec^2}$ is the Newton gravitational constant, $j^\beta$ is an external source) for a static uniformly charged sphere.

In this case Eq. (1) reduces to

$$\mathrm{div}\,\mathbf{E} = bE^2\varphi + 4\pi\sigma\delta(R-r) \qquad (2)$$

with boundary conditions

$$\varphi_+|_R = \varphi_-|_R;\ (E_+ - E_-)|_R = \frac{q}{R^2};\ \varphi_+ = \frac{q}{r}\ \text{for}\ r\to\infty. \qquad (3)$$

Here $A_\mu = (\varphi, 0, 0, 0)$, $\varphi = \begin{cases}\varphi_+ & r \geqslant R\\ \varphi_- & r \leqslant R\end{cases}$.

$$\mathbf{E} = (E, 0, 0),\ E = F_{01} = -\frac{\partial\varphi}{\partial r}\Big/(1 + b\varphi^2),\ E = \begin{cases}E_+ & r\geqslant R\\ E_- & r\leqslant R,\end{cases}$$

$q = 4\pi\sigma R^2$ is the charge of the sphere, $\sigma$ is the surface charge density; r is the radial coordinate, R is the radius of the sphere.

The solution of Eq. (2) with boundary conditions (3) is

$$
\mathbf{E} = \begin{cases} \mathbf{E}_- = \dfrac{q\left(1 - \operatorname{ch}\dfrac{q\sqrt{b}}{R}\right)}{\operatorname{ch}\left[\dfrac{q\sqrt{b}}{r}\left(1 - \operatorname{ch}\dfrac{q\sqrt{b}}{R}\right) + \dfrac{q\sqrt{b}}{R}\operatorname{ch}\dfrac{q\sqrt{b}}{R}\right]}\dfrac{\mathbf{r}}{r^3} & r \leqslant R, \\ \mathbf{E}_+ = \dfrac{q}{\operatorname{ch}\dfrac{q\sqrt{b}}{r}}\dfrac{\mathbf{r}}{r^3} & r \geqslant R. \end{cases}
$$

A qualitative plot of $\mathbf{E}(r)$ is shown in the figure. At $\varphi = \frac{q\sqrt{b}}{r} = \mathrm{const}$, $r_{max}$ shifts towards longer radii with increasing R (in which case the charge q increases).

It is seen from the figure that a charged sphere induces charges in the outer and inner regions, and calculation shows that these charges are equal, $|q_+| = |q_-|$.

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