ON A QUANTUM THEORY OF STIMULATED SYNCHROTRON RADIATION BY ELECTRONS WITH RELATIVISTIC LONGITUDINAL VELOCITY

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We examine the possibility of stimulated synchrotron radiation being generated by an electron moving at a relativistic velocity along the magnetic field direction. The limiting condition for the highest harmonic that can be generated is obtained, and it is shown that stimulated synchrotron radiation is possible for high harmonics as well as low. The treatment is based on quantum theory.

Relativistic beams are currently in wide use for generating powerful electromagnetic radiation in the millimeter, submillimeter, and infrared regions of the spectrum. The theory of the interaction of a relativistic beam with an electromagnetic wave has been developed along classical lines and usually involves numerical integration on a computer (see, for example, Refs. [1,2]).

In our view, the quantum approach based on the fundamental papers of Sokolov and Ternov on synchrotron radiation [3,4] is extremely interesting, since it enables one to treat in a unified way the problems of both stimulated and spontaneous synchrotron radiation over a wide spectral region and in a large interval of electron energies.

Suppose that electrons are moving on a helical trajectory in a constant and uniform magnetic field $\mathbf{H}=\{0, 0, H\}$. Their energy spectrum will be given by the formula [3,4]:

$$W_n = c\hbar K_n = c\hbar \sqrt{k_0^2 + 4\gamma n + k_z^2}, \tag{1}$$

where $k_0 = \frac{m_0 c}{\hbar}$, $\gamma = \frac{eH}{2c\hbar}$, $\hbar k_z$ is the momentum of an electron along the field, e and m_0 are the charge and rest mass of the electron, and n is the principal quantum number (n = 0, 1, 2, ...).

The radiative eigenfrequencies of the electron will be of the form

$$\omega_{n,n-\nu} = \frac{W_n - W_{n'}}{\hbar} = \frac{\nu\omega_c}{1-\beta_\parallel \cos\theta}\left(1 + \frac{\nu}{4n}\beta_\perp^2 \frac{\sin^2\theta}{(1-\beta_\parallel \cos\theta)^2}\right), \tag{2}$$

where W_n and $W_{n'}$ are the energies of the initial and final states of the electron, $\omega_c = \frac{eHc}{W}$, $c\beta_\parallel$, and $c\beta_\perp$ are the parallel and perpendicular components of the velocity of the electron, ν is the number of the harmonic ($\nu \ll n$, $n \gg 1$), and θ is the angle between the wave vector of the radiated photon and the magnetic field direction.

Suppose that the electron interacts with a linearly polarized external electromagnetic wave propagating at an angle θ to the direction of **H**. Then, using the expressions obtained in [3] for the probability of stimulated transitions of the electron and for its wave functions, we will have the following formula for the power of stimulated synchrotron radiation:

$$P_n = \frac{2e^2E^2c^2\tau}{W}\,\frac{\nu\omega_c}{\omega}\,S, \tag{3}$$

where

$$S = \frac{J_\nu'^2(z)}{1+x^2}\left\{\frac{\nu^2-z^2}{z}\,\frac{J_\nu(z)}{J_\nu'(z)} - \frac{\beta_\perp^2}{1-\beta_\parallel\cos\theta} + \frac{\beta_\perp^2\,xQ\tau}{1+x^2}\right\},$$
$$x = \tau(\omega_n-\omega),\ Q = \frac{\nu\omega_c\sin^2\theta}{(1-\beta_\parallel\cos\theta)^2},\ z = \frac{\omega_c}{\omega_c}\,\beta_\perp\sin\theta, \tag{4}$$

$I_\nu(z)$ is a Bessel function of order ν, τ/2 is the average lifetime of an electron in the initial state, and E is the amplitude of the electric field of the wave.

It can be seen from formulas (3) and (4) that for x < 0 one has $P_n < 0$ and the electrons emit stimulated radiation; this is analogous to the weakly relativistic case [5]. However, in the relativistic case stimulated emission is also possible when the frequency of the external wave ω coincides exactly with a radiative eigenfrequency of the electron (x = 0), i.e., in the case of resonance. In this case expressions (3) and (4) assume the form

$$P_n = \frac{2e^2E^2c^2\tau}{W}(1-\beta_\parallel\cos\theta)\,S, \tag{5}$$

$$S = J_\nu'^2(z)\left\{\frac{\nu^2-z^2}{z}\,\frac{J_\nu(z)}{J_\nu'(z)} - \frac{\beta_\perp^2}{1-\beta_\parallel\cos\theta}\right\}, \tag{6}$$

where z is defined as

$$z = \frac{\nu\beta_\perp\sin\theta}{1-\beta_\parallel\cos\theta}.$$

In the ultrarelativistic case β → 1, one has $\beta_\parallel \to \cos\alpha$ (α is the angle between the directions of the instantaneous velocity of the electron and the magnetic field). For small values of α, $\beta_\parallel \gg \beta_\perp$, $\theta \sim \alpha$ and $\frac{z}{\nu} \sim 1$, one can use the following approximate expressions for the Bessel functions [6]:

$$J_\nu(z) = \frac{\Gamma(1/3)}{\pi 2^{2/3}\,3^{1/6}}\,\nu^{-1/3} \simeq 0{,}45\nu^{-1/3},$$
$$J_\nu'(z) = \frac{\Gamma(2/3)\,3^{1/6}}{\pi 2^{1/3}}\,\nu^{-2/3} \simeq 0{,}41\nu^{-2/3}. \tag{7}$$

With these expressions taken into account, Eqs. (5) and (6) assume the form

$$P_n = 0{,}336\nu^{-1/3}\,\frac{e^2E^2c^2\tau}{W}(1-\beta_\parallel\cos\theta)\,S, \tag{8}$$

$$S = 1{,}1\nu^{4/3}\left(1 - \frac{\beta_\perp^2\sin^2\theta}{(1-\beta_\parallel\cos\theta)^2}\right) - 1. \tag{9}$$

Formula (9) implies that stimulated emission will dominate over absorption for harmonics

$$\nu < \sqrt{\nu_{max}}, \tag{10}$$

where

$$\nu_{max} \simeq \frac{1}{\varepsilon^{3/2}}, \quad \varepsilon = 1 - \frac{\beta_\perp^2 \sin^2\theta}{(1-\beta_\parallel \cos\theta)^2},$$

ν_{max} gives the maximum for the intensity of spontaneous synchrotron radiation [7]. An analogous result for the case $\beta_\parallel = 0$, $\theta = \pi/2$ was obtained in Ref. [8].

Thus, in the case studied here it is possible to observe stimulated synchrotron radiation for high harmonics as well.

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