SECULAR AND LONG-TERM PERTURBATIONS OF THE INNER ORBIT OF THE MULTIPLE SYSTEM ξU.Ma

A. A. Orlov* and N. A. Solovaya

Vestnik Moskovskogo Universiteta. Fizika,
Vol. 42, No. 6, pp. 61-64, 1987

UDC 521.13

Perturbations of the inner orbit of the multiple stellar system ξU.Ma are calculated using formulas derived in previously published papers. Secular changes in the elements e_1, Ω_1 and I_1 are reported. The results show that the perturbations of the inner orbit are substantial and must be taken into account.

This paper is a continuation of [1] and is devoted to an application of the theory of motion of ternary stars, developed in [2], to secular and long-term perturbations of the orbit of the close pair ξU.Ma with a period of 1.8 years. It was noted in [1] that changes in the outer orbit were detected by Heintz [3] as a result of an analysis of the observations of the ternary stellar system ξU.Ma. Changes in the inner orbit of this system were not detected, probably because the time interval over which the close spectral binary was observed was too short.

The aim of this paper is to elucidate the evolution of the orbit of this close binary and to investigate secular and long-term perturbations of the periastron and node. The results of the analysis can be subsequently used to determine the most favorable conditions for determining the perturbations of a close binary from observational data.

As in [1], we shall use the same notation for the stars in the ξU.Ma system that was employed in [3]: the components of the close binary with a period of 1.8 years will be denoted by A and a, and the stars of the distant pair with the very short period of 3.98 days will be denoted by B and b. We shall assume that the latter pair is a single star of mass equal to the sum of the masses of B and b. The orbit of a around A will be referred to as the inner orbit and the orbit of B around the center of mass of Aa will be called the outer orbit.

1. BASIC FORMULAS

We shall be interest exclusively in the elements of the inner orbit as functions of time. This orbit is an ellipse with variable longitude of the ascending node and periastron, variable eccentricity, and constant semi-major axis. In our numerical calculations, we use the formulas given in [2, §§3,7,8] retaining the same notation and order of calculation.

Let the masses of the components be m_0, m_1, m_2, and let the elements of the inner and outer orbits in the fixed Laplace plane be denoted by $a_j, e_j, i_j, l_j, g_j, h_j$ (j =

*Deceased.

= 1, 2), where a, e, i, l, g, h are, respectively the semi-major axis, the eccentricity, the inclination, the mean anomaly, the argument of the periastron, and the longitude of the ascending node.

According to [2], we can then write

$$\begin{aligned}
&a_1 = \text{const}, \qquad e_1 = \sqrt{1-\xi}, \\
&l_1 = B_1 + \varkappa_1 (t - t_0) + Q_1 I_1(u) + Q_2 I_2(u) + Q_3 I_3(u), \\
&h_1 = B_5 + Q_8 I_1(u) + Q_6 I_4(u) - Q_7 I_5(u), \\
&\sin i_1 = \overline{G}_2 \sqrt{1 - q^2/\bar{c}}, \\
&\cos i_1 = (q\overline{G}_2 + \sqrt{\xi})/\bar{c},
\end{aligned} \tag{1}$$

in which the variable ξ is related to the time by

$$\frac{1}{12}\overline{G}_2 \int_{\xi_1}^{\xi_2} \frac{d\xi}{\sqrt{\Delta}} = \frac{B_3}{A_1} + \frac{1}{16}\frac{\gamma m^2}{(1-e_2^2)} n_1 (t - t_0), \tag{2}$$

where

$$\begin{aligned}
&\overline{G}_2 = \frac{(m_0+m_1)m_2}{m_0 m_1}\sqrt{\frac{(m_0+m_1)(1-e_2^2)}{m_0+m_1+m_2}\,\frac{a_2}{a_1}}, \\
&\gamma = \frac{m_2}{m_0+m_1+m_2}, \\
&n_1 = \frac{k}{a_1}\sqrt{\frac{m_0+m_1}{a_1}}, \quad n_2 = \frac{k}{a_2}\sqrt{\frac{m_0+m_1+m_2}{a_2}}, \\
&m = \frac{n_2}{n_1}, \quad q = \frac{\bar{c}^2 - \overline{G}_2^2 - \sqrt{\xi}}{2\overline{G}_2\sqrt{\xi}},
\end{aligned}$$

k is the Gauss constant, and $\Delta = f_2(\xi)\cdot f_3(\xi)$ is a polynomial of degree five in ξ.

The formulas for Q_j, $I_j(u)$, ξ_j, $\bar{c}$, A_1, B_3 are given in [2] and will not be reproduced here.

The type of the intermediate inner orbit is determined by the sign of the parameter $\bar{h}$:

$$\begin{aligned}
&2\overline{G}_2^2 \bar{h} = 3e_{10}[e_{10} - 4\overline{G}_2 \eta_0 q_0 - 8\overline{G}_2^2 (1 - q_0^2)\sin^2 g_{10}], \\
&\eta_0 = \sqrt{1 - e_{10}^2}, \quad q_0 = \cos(i_{10} + i_{20}).
\end{aligned} \tag{3}$$

The subscript 0 in these and subsequent formulas indicates that the value of the particular variable is taken at the initial time.

For $\bar{h} < 0$, the orbit is circular, i.e., the periastron of the inner orbit exhibits a secular motion. For $\bar{h} > 0$, the periastron does not execute the secular motion, but oscillates between finite limits, and we have a libration orbit.

In the case of ξU.Ma, we have $\bar{h}$ = 1.8727 and a libration inner orbit.

The quantities sin g_1 and cos g_1 are given by

$$\sin^2 g_1 = \frac{\xi f_2(\xi)}{5(1-\xi)[\xi - (\bar{c} - \overline{G}_2)^2][(\bar{c} - \overline{G}_2)^2 - \xi]}, \tag{4}$$

	a	B
a_j	1,56 a. u.	19,46 a. u.
e_j	0,56	0,414
i_j	86,3°	122,65°
T	1935,410	1935,170
ω_j	146°	127,53°
Ω_j	326°	101,59°

$$\cos^2 g_1 = \frac{f_3(\xi)}{5(1-\xi)[\xi-(\bar{c}-\bar{G}_2)^2][(\bar{c}-\bar{G}_2)^2-\xi]}. \qquad (4)$$

If ε_1, ε_2 are the roots of the quadratic equation $f_2(\xi) = 0$, ε_3, ε_4, ε_5 are the roots of the cubic $f_3(\xi) = 0$, and the functions $f_2(\xi)$ and $f_3(\xi)$ are products of linear factors

$$f_2(\xi) = (\varepsilon_1-\xi)(\varepsilon_2-\xi),$$
$$f_3(\xi) = (\xi-\varepsilon_3)(\varepsilon_4-\xi)(\varepsilon_5-\xi),$$

then, as shown in [2], the roots ε_3 and ε_4 are less than unity, whereas ε_1, ε_2, ε_5 are greater than unity. The quantity ε_1 in the numerator of $\sin^2 g_1$ is always less than ξ, so that $\sin g_1$ can never be zero in this case. The quantity g_1 can vary either within the range $0° < g_1 < 180°$ or $180° < g_1 < 360°$.

When ξ takes on the limiting values ε_3 and ε_4, we have $\cos g_1 = 0$ and, consequently, the value of ξ corresponding to the extremum of the deviation of g_1 from 90° or 270° must lie strictly inside the interval $\varepsilon_3 < \xi < \varepsilon_4$.

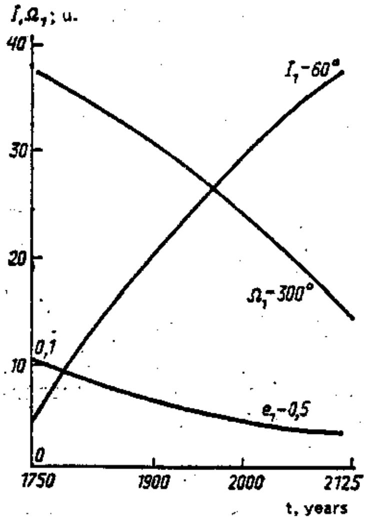
We shall now determine the extremum of the function $\cos g_1$ in order to find the limits of variation of g_1 during the librational motion. Equating to zero the derivative of the function $\cos g_1$, we obtain an algebraic equation of degree four for ξ. The equation can be solved by the method of successive approximations and the resulting value of ξ can be substituted in (4) to obtain the range of values of g_1.

2. INITIAL DATA

We have used the elements of the system ξU.Ma obtained in [3]. Before calculations based on the above formulas were carried out, the orbital elements were transformed in accordance with [1, §2].

The elements of the inner and outer orbits in the system ξU.Ma in the projected plane are listed in the table (the semi-major axis of a is referred to the center of mass of A and the semi-major axis of B is referred to the center of mass of Aa) and T_1 and T_2 are the times of crossing of the periastron. The elements are given for the epoch 1900. The masses m_0, m_1, m_2 of A, a, and B in units of the solar mass are as follows: $m_0 = 0.83$, $m_1 = 0.31$, $m_2 = 0.92$.

The elements listed in the table must be transformed to the coordinate frame in which the constant Laplace plane is the principal plane. The position of the Laplace plane and the elements deduced from spherical geometry are obtained in [1, §2].



3. RESULTS

The formulas given in §2, above, were used to calculate the values of e_1, Ω_1 and I_1 for component *a* in the projected plane in the range 1750-2125. The results are plotted in the figure. It is clear that the elements of the inner orbit are nonlinear functions of time, as are the elements of the outer orbit in [1]. We cannot compare the results obtained here for the inner orbit with those reported in [3] because the inner-orbit perturbations were neglected by Heintz, although he considered that they were present but could not be detected because the size of the orbit was too small.

The average changes in the elements e_1, Ω_1 and I_1 per 100 years are as follows: $\delta e_1 = 0.024$, $\delta\Omega_1 = -6.1°$, $\delta I_1 = 9.9°$. The periastron of the inner orbit oscillates around 270° in the range $268.93° < g_1 < 277.07°$.

Our calculations suggest that the perturbations of the orbit are considerable and cannot be neglected. We plan to analyze observations of ξU.Ma with allowance of the perturbations of both the inner and outer orbit of the ternary stellar system.

REFERENCES

1. A. A. Orlov and N. A. Solovaya, Vestn. Mosk. Univ. Ser. 3, Fiz., Astron. [Moscow University Physics Bulletin], no. 6, pp. 74-77, 1986.
2. A. A. Orlov and N. A. Solovaya, Tr. GAISh, vol. 45, -p. 119-136, 1974.
3. W. D. Heintz, Astron. Nachrichten, vol. 289, pp. 269-277, 1967.

25 June 1986

Shternberg State Astronomical Institute