Moscow University Physics Bulletin
Vol. 47, No. 2, pp. 58–62, 1992

Vestnik Moskovskogo Universiteta. Fizika
UDC 532.526.4

ON THE RELATION BETWEEN THE KINETIC ENERGY OF PULSATORY MOTION AND THE DEPTH DISTRIBUTION OF AVERAGED VELOCITIES IN A PLANE FLOW

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Studied was the depth distribution of terms of the balance equation of the kinetic turbulent energy in a plane flow. The measurements were performed by the thermohydrometer (hot-wire) method in a measuring flume of a rectangular cross section. The experimental data were computer-processed with the use of statistic programs which enabled computing the Reynolds stress tensor components. Based on the hypothesis that, in the external layer of plane flows, a reduction of the momentum depends on the kinetic turbulent energy, an approximate equality of the von Karman parameter and the coefficient of mutual correlation between the longitudinal and vertical velocity components was obtained.

The data of many studies on the near-wall flow demonstrate that there is a relation between the mean velocity field and tangential stresses. All the attempts made to represent this dependence by a universal formula that would be valid over the entire flow thickness have failed. It follows then that new expressions must be found that would describe the interconnection between the parameters of the averaged and the pulsatory motions with allowance for the specific features of their distributions over different turbulent flow layers. This approach was employed to simulate the turbulent motion in some works [1], where the mean velocity field was represented by means of terms of the energy balance equation for pulsatory motion. However, in this approach, the initial model relationships include some parameters that cannot be physically interpreted. In order to find the actual relationships between the terms of energy balance equation and the mean velocity field, it is necessary to study the distribution of these terms over different flow layers and also their interconnection with the mean velocities. The present work is concerned with these aspects of the problem.

The experiments were performed in the $0.2\times7.0\times0.4$ m measuring flume of the Hydrophysics Laboratory of Moscow State University, using different flow characteristics (Table 1). The effects of the input and output boundary conditions on the flow structure were eliminated by arranging the measurement section at a distance of 50 H (where H is the depth) from the flume input. The mean velocities were measured by the Pitot tube at 15–20 points. Three components of the pulsation velocity were recorded by the thermohydrometer method [2] with the help of two sensors placed alternately at a specified point. One sensor recorded the longitudinal and vertical components of the velocity, the other its longitudinal and transverse components. This was attained by arranging the platinum wire holders in different planes. Use was made of the measurement circuit and the methods of initial data processing described in [3]. The flume input was 4.0 m distant from the vertical measurement section, which made it possible to eliminate the effect of input and output boundary conditions on the flow structure.

In accordance with the data obtained in measuring the mean velocities v, in all the experiments carried out in the axial region of the flow the motion was a very nearly plane-parallel flow. The logarithmic law was obeyed in a flow layer thinner than 0.2 H (where H is the flow depth) and the law of velocity deficiency in a layer (0.2–0.8) H thick. In the flow layers situated above the 0.8 H boundary, the experimental points deviated from the logarithmic curve by more than 5–7%.

The logarithmic curve in the region where the velocity deficiency law was valid was described by the

Table 1

Basic Characteristics of the Flows

No.	v, m/s	v_*, m/s	H, m	I	u, m/s
1	0.34	0.017	0.072	0.0004	0.35
2	0.22	0.013	0.041	0.0001	0.22
3	0.40	0.031	0.064	0.0016	0.49

equation

$$\frac{u_{\max} - \overline{u}(y)}{u_*} = -2.5 \ln \frac{y}{H} + 0.6, \tag{1}$$

where $u_{\max}$, $\overline{u}(y)$ are the maximum and the mean velocities along the vertical; v_* is the local friction rate; g is the acceleration due to gravity; H is the flow depth; and I is the slope of the flume bottom ($v_* = \sqrt{gHI}$).

The rate $\overline{u'v'}\, du/dy$ of turbulent energy production was computed from measured values of $\overline{u'v'}$ and du/dy. The second term in the energy balance equation for averaged motion—the dissipation ε of pulsation energy—was determined as

$$\varepsilon = 0.002\ \mathrm{Re}_* v_*^3 / y, \tag{2}$$

where $\mathrm{Re}_* = v_* H/\nu$ is the Reynolds dynamic number and ν is the kinematic viscosity factor.

Relation (2) was derived with the help of experimental data [4–6].

Having computed ε and $\overline{u'v'}\, du/dy$, we can find, from the turbulent energy balance equation, the total diffusion $\overline{q'^2 v'}$ of kinetic energy and the pressure energy $\overline{p'v'}$. Figure 1 illustrates the energy balance components.

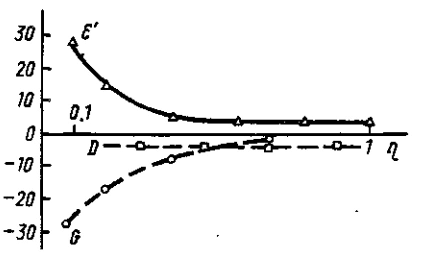

Fig. 1

Depth distribution of terms of the balance equation for the kinetic turbulent energy. Circles: production of turbulent energy $G = u'v'(du/dy)H/v_*^3$; triangles: dissipation $\varepsilon' = H\varepsilon/v_*^3$; squares: diffusion $D = (\overline{p'v'} + \overline{q'^2 v'})/v_*^3$.

Analysis of the $G(y)$, $\varepsilon'(y)$, and $D(y)$ curves, which describe plane turbulent flows, supports the existence of universal relationships, first discovered by Schubauer [7], in the depth distribution of normalized components of the pulsatory energy balance. This conclusion enables one to use formula (2) for the computation of the turbulence pulsatory energy dissipation, which can hardly be found by measurements due to the limitations of the currently available equipment.

Let us analyze the mean velocity profile taking into account the data obtained on the distribution of terms of the energy balance equation for pulsatory motion. It is common knowledge that the presence of velocity gradient in the near-wall current close to the rigid boundary is related to the flow-wall friction. At relative distances from the bottom, $\eta = 0.10$–0.15, where $\eta = y/H$, the velocity gradient will be determined by the kinetic energy transferred by convection and gradient-type diffusion. This fact manifests itself in the validity of the velocity deficiency law in the $0.3 < \eta < 0.8$ range, where the viscosity effect on the velocity distribution can be ignored. Suppose that the momentum drop, characterized by the dimensionless velocity deficiency $(u_{\max} - \overline{u}(y))/v_*$, must be a function of the kinetic energy of turbulence and diffusion:

$$\frac{u_{\max} - \overline{u}(y)}{v_*} = \frac{\overline{q'^2}}{v_*^2} + \frac{D}{v_*^3}. \tag{3}$$

As follows from the analysis of the data summarized in Table 2, expression (3) is fulfilled to a high accuracy for various types of near-wall flow, including flows in hydrodynamic flumes [8], in pipes [4], and over smooth plates [6].

Table 2

Data on Kinetic Energy $\overline{q'^2}/v_*^2$, Diffusion D/v_*^3, and Velocity Deficiency $(u_{\max} - \overline{u}(y))/v_*$ by Different Authors

η	$\frac{u_{\max}-\overline{u}(y)}{v_*}$				$\frac{\overline{q'^2}}{v_*^2}$				$\frac{D}{v_*^3}$			
	Present work	[4]	[6]	[9]	Present work	[4]	[6]	[9]	Present work	[4]	[6]	[9]
0.2	4.9	4.6	6.0	4.3	5.3	5.9	6	7.2	−1.0	−1.5		−2.9
0.3	3.0	3.5	5.1	3.8	4.8	5.0	5.2	5.8	−1.8	−1.7		−2.0
0.4	2.2	2.6	3.9	3.2	3.7	4.1	4.3	4.3	−1.5	−1.4		−1.1
0.5	1.6	1.8	2.2	2.9	3.5	3.5	3.5	3.9	−1.9	−1.6		−1.0
0.6	1.1	1.6	1.8	3.2	2.9	2.6	2.8	2.8	−2.1	−1.8	−0.8	−1.0
0.7	0.6	0.7	1.1	1.4	2.9	2.5	1.8	2.4	−2.3	−1.9	−1.0	−1.0
0.8	0.1	0.3	0.4	0.2	2.7	2.1	1.1	1.2	−2.6	−1.9	−1.0	−1.0
0.9	0.0	0.0	0.0	0.0	2.7	1.9	0.6	1.0	−2.7	−2.0	−0.9	−1.0

Equation (3) implies that at a constant cross-flow diffusion, the relationship

$$-\frac{du}{dy} = -\frac{1}{v_*}\frac{d\overline{q'^2}}{dy}. \tag{4}$$

will hold.

A constant diffusion [7–9] is observed in the external layer of the near-wall flow, where the velocity profiles are usually slightly curved. Then we can change over from the differential form of equation (4) to the finite-difference equation

$$\frac{u_{\max} - \overline{u}(y)}{\Delta y} = \frac{1}{v_*}\frac{\overline{q'^2_{\mathrm{sur}}} - \overline{q'^2}(y)\big|_{y=y_{\mathrm{av}}}}{\Delta y}, \tag{5}$$

where $\overline{q'^2}(y)\big|_{y=y_{\mathrm{av}}}$ is the kinetic energy at the point of application of mean velocity on the vertical; Δy is the distance between the points of application of the maximum and mean velocities.

When the condition $\overline{q'^2_{\mathrm{sur}}} \ll \overline{q'^2}(y)\big|_{y=y_{\mathrm{av}}}$ and the known equation

$$\varkappa = \mathrm{Def}^{-1} = \frac{v_*}{u_{\max} - \overline{u}(y)} \tag{6}$$

are fulfilled, we obtain the equality

$$\frac{v_*^2}{\varkappa \Delta y} = -\frac{\overline{q'^2}(y)\big|_{y=y_{av}}}{\Delta y}. \tag{7}$$

In [6] a relationship has been established between the tangential stresses $\overline{u'v'}$, the correlation coefficient r_{uv}, and the doubled kinetic turbulent energy $k = 2\overline{q'^2}$:

$$\frac{1}{3} k r_{uv} = \overline{u'v'}. \tag{8}$$

In formula (7) we substitute r_{uv} and $\overline{u'v'}$ for $\overline{q'^2}(y)$ subject to (8). After transformations performed in (7) we get

$$\frac{2v_*^2}{\varkappa} = -\frac{3\overline{u'v'}}{r_{uv}}. \tag{9}$$

In plane flows, the tangential stresses vary linearly along the depth:

$$\overline{u'v'} = -v_*^2(1-\eta), \tag{10}$$

and the mean velocity coordinate is at a distance of 0.37 H from the bottom [10].

Hence, at the point of application of $u(y)$,

$$\overline{u'v'} \approx -0.63 v_*^2, \tag{11}$$

which leads to the fulfillment of the approximate equality

$$|\varkappa| \approx |r_{uv}|. \tag{12}$$

Table 3 lists the values of $\varkappa$ and r_{uv} obtained by different authors in flumes of a rectangular cross section when the flows travel in the resistance mode under conditions of hydraulically smooth walls. Analysis of the data of Table 3 points to the validity of equality (12) for the range of Re = 1400–64 400 irrespective of the correlation layer size, and this is also corroborated by the data of [11, 12].

Table 3

Data on the Flow Parameters by Different Authors

Flow parameter	Present work		[8] Re = 2 150	[14] Re = 14 000	[13] Re = 19 500	[15] Re = 21 000	[17] Re = 25 000	[13] Re = 35 000	[9] Re = 64 400
	Exp. 1	Exp. 2							
Width, cm	20	20	2.5	34	27	15	20.8	27	10
H, cm	7.2	4.1	0.65	10.3	6.3	8.3	5.1	8.8	2.1
r_{uv}	0.36	0.40	0.40	0.38	0.40	0.32	0.40	0.38	0.46
$\varkappa$	0.36	0.40	0.40	0.38	0.40	0.32	0.40	0.38	0.46

For example, in the experiments performed by Sugrei in the developing boundary layers [13], the size of the region where $|r_{uv}| = |\varkappa|$ varied along the flume by (0.1–0.2) H; in the experiments by Nikitin [9] the layer depth was 0.3 H and Fidman [14] got 0.4 H. It should also be noted that Minskii [15] and Fidman [14] conducted measurements in flows with secondary effects over the entire flow depth.

The equality sign in [12] can be explained in terms of the general view that there is a relation between the geometric dimensions of the highest-energy vortices and the linear dimensions of the hydraulic units. Actually, the vortex dimensions determine the values of r_{uv}. Alternatively, according to the data from [8] for plane flows in flumes of a rectangular cross section, the following equation holds:

$$\varkappa = R/H, \tag{13}$$

where R is the total hydraulic radius.

According to [16], r_{uv} is a function of the relative value of L/H, where L is the linear dimensions of the vortices corresponding to the value of the total hydraulic radius. Hence, relation (12) holds for plane flows.

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