

ON THE THEORY OF FREQUENCY DEPENDENCE OF THE CONDUCTIVITY OF A STRONGLY DOPED COMPENSATED SEMICONDUCTOR

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It is established that in a broad frequency band the real part of the conductivity of a strongly doped compensated semiconductor is described by the law $\text{Re } \sigma(\omega) \sim \omega^s$, where $s < 1$. The frequency and temperature dependence of the exponent s is discussed.

1. For many of the disordered semiconductors the real part of the conductivity turns out to be a power function of frequency: $\text{Re } \sigma(\omega) = A\omega^s$, where A is a constant and the exponent s is usually somewhat less than unity. This kind of frequency dependence appears when the system is characterized by a broad spread of relaxation times. In the theory of jumping conduction this spread is a consequence of the exponential dependence of the rate of transition of the charge carrier on a spatial position and/or on the energies of localized states [1].

To interpret some experimental data on amorphous and compensated semiconductors it was assumed that a substantial contribution to the alternating-current conductivity could come from band carriers located in the wells of a large-scale potential relief [2, 3]. In the present paper the frequency dependence of the ohmic conductivity is studied for a strongly doped compensated semiconductor (SDCS) with a large-scale potential, with the specific structure of the potential relief (for definiteness, we shall speak of an n -type semiconductor) taken into account.

As is known, the chaotic character of the impurity distribution in the SDCS results in potential and electron density fluctuations whose maximum size is determined by the electron screening and is equal to $r_s = N^{1/3}/n^{2/3}$ in its order of magnitude, where N and n are the impurity and electron concentrations, respectively [4]. On the large-scale fluctuations smaller-scale ones are superimposed. At low temperatures the electrons form drops located in the deepest wells of the potential relief where the potential energy is lowered by fluctuations of all scales. In conditions of strong compensation the drops occupy a small part of the crystal volume, and no leakage over them is possible.

2. The electron density can be redistributed in the external field by means of electron transitions across the potential barriers between the drops. The transitions can be due to both the above-barrier activation and the underbarrier tunneling. The tunneling effect manifests itself in the reduction of the activation energy, namely the electron tunnels under the apex of the potential barrier so that the activation energy is reduced by [5]

$$\Delta\varepsilon(T) = E_0^{5/3} (Na^3)^{2/3} T^{-2/3},$$

where $a = \hbar^2 \kappa / (me^2)$ is the Bohr radius of the impurity center, m is the effective mass, κ is the permittivity, and $E_0 = e^2 / (\kappa a)$ is the bonding energy of a hydrogen-like impurity atom.

The correction $\Delta\varepsilon$ to the activation energy decreases with temperature. For

$$T > T_{\text{act}} \equiv E_0 (Na^3)^{2/5} \quad (1)$$

it becomes smaller than T , and therefore the transitions can be regarded as being purely activation ones. In the present paper we consider the case when inequality (1) is fulfilled.

The probability of an activation transition across a barrier of height W per unit time is

$$w(W) = w_0 \exp\left\{-\frac{W}{T}\right\}, \quad (2)$$

where w_0 depends weakly on W . The quantity $w^{-1}(W)$ is the characteristic transition time.

The root-mean-square fluctuation of the potential energy is equal to

$$\gamma(r_s) = \frac{e^2 N^{2/3}}{\kappa n^{1/3}} \quad (3)$$

in its order of magnitude [4]. The barrier height can vary in the interval of the order of $\sim \gamma(r_s)$. We consider the case $\gamma(r_s) \gg T$ so that the range of the transition probability (2) turns out to be broad.

We consider the electron reaction to the switching of an external field $\tilde{\mathcal{E}}(t) = \tilde{\mathcal{E}}\theta(t)$, where $\theta(t)$ is a step function. By time t there can occur transitions whose time is less than t :

$$w^{-1}(W) \lesssim t. \quad (4)$$

Using the explicit expression (2) for the function $w(W)$ we rewrite condition (4) as

$$W \lesssim W_t \equiv T \ln(w_0 t), \quad (5)$$

which means that transitions across the barriers of the height less than W_t occur.

The drops with the height of the barrier between them less than W_t will be said to be bound. A group of drops each of which is bound with at least a single drop will be called a cluster.

Since the barriers between the drops in a cluster do not exceed W_t , the electron transitions in the cluster manage to occur by time t . And different clusters are separated by barriers of height greater than W_t , and no transitions between them can occur during time t . Hence, inside a cluster a quasi-equilibrium distribution (in the external field) of the electron density sets in. The characteristic size of a cluster is determined from the condition that the fluctuation of the potential energy in its volume does not exceed W_t . In a region of a size r the root-mean-square fluctuation of the number of impurity atoms is of the order of $\sqrt{Nr^3}$. The corresponding fluctuation of the potential energy is $\gamma(r) = e^2 \sqrt{Nr} / \kappa$. Setting $\gamma(r) = W_t$ we determine the size of the cluster:

$$r_t = \frac{\kappa^2 W_t^2}{e^4 N} = \frac{\kappa^2 T^2}{e^4 N} \ln^2(w_0 t). \quad (6)$$

The excessive donor charge must be compensated for by electrons. If the cluster volume contains $\sqrt{Nr_t^3}$ excessive donors, then the number of clusters in unit volume must be of the order of

$$\nu_t = \frac{n}{\sqrt{Nr_t^3}}. \quad (7)$$

By definition, the size of a cluster cannot be smaller than that of the drop. We shall consider the case when the electron gas is nondegenerate. Then the size of the drop is determined by the expression [6]

$$r_T = \frac{\kappa^2 T^2}{e^4 N}.$$

The condition $r_t > r_T$ now means that

$$\begin{aligned} \ln(w_0 t) &\gtrsim 1, \\ t &\gg w_0^{-1}. \end{aligned}$$

The electron gas degeneration temperature can be represented in the form [2]

$$T_{\text{deg}} = E_0 (Na^3)^{4/9}. \quad (8)$$

We note that T_{deg} (8) exceeds T_{act} (1) so that for $T > T_{\text{deg}}$ condition (1) is fulfilled automatically.

On the other hand, as the clusters grow, the electron screening starts manifesting itself, and the dependence of r_t on W_t will differ from (6). The maximum size of fluctuations that are not yet screened by electrons is of the order of $r_s = N^{1/3} / n^{2/3}$ [4]. Therefore expression (6) is applicable for $r_t < r_s$, whence

$$\begin{aligned} \ln(w_0 t) &\lesssim \frac{\gamma(r_s)}{T}, \\ t &\ll w_0^{-1} \exp\left\{\frac{\gamma(r_s)}{T}\right\}, \end{aligned}$$

where $\gamma(r_s)$ is given by expression (3).

Thus, the cluster approximation is applicable in the time interval

$$w_0^{-1} \ll t \ll w_0^{-1} \exp\left\{\frac{\gamma(r_s)}{T}\right\}. \quad (9)$$

3. We shall calculate the change in the cluster dipole moment in the external field. Denote by $n(U(\mathbf{r}))$ the equilibrium electron density at the point \mathbf{r} . Here $U(\mathbf{r})$ is the sum of the potential energies of the electrons in the impurity field and in the external field:

$$U(\mathbf{r}) = U_0(\mathbf{r}) + e\vec{\mathcal{E}}\mathbf{r}.$$

For nondegenerate electron gas the function $n(U)$ is determined by Boltzmann's exponential function

$$n(U(\mathbf{r})) = n_0 \exp\left\{-\frac{U(\mathbf{r})}{T}\right\},$$

where n_0 does not depend on \mathbf{r} .

The radius vector \mathbf{r} varies within the limits of a cluster of size r_t (6). In weak external fields $e\mathcal{E}r_t/T \ll 1$ the electron density can be expanded with respect to the field:

$$n(\mathbf{r}) = n_0(\mathbf{r}) + \delta n(\mathbf{r}),$$

where $n_0(\mathbf{r}) = n_0 \exp\{-U_0(\mathbf{r})/T\}$ is the electron density in the absence of the external field,

$$\delta n(\mathbf{r}) = -n_0(\mathbf{r}) \frac{e\vec{\mathcal{E}}\mathbf{r}}{T},$$

and the terms of higher order are dropped.

The dipole moment is

$$\mathbf{D}(\mathbf{r}) = -e \int_{V_t} d^3\mathbf{r} \mathbf{r} \delta n(\mathbf{r}) = \frac{e^2}{T} \int_{V_t} d^3\mathbf{r} \mathbf{r} (\vec{\mathcal{E}}\mathbf{r}) n_0(\mathbf{r}),$$

Integration extends over the cluster volume V_t .

ely, we are interested in the average macroscopic dipole moment. By virtue of the macroscopic problem, the average moment is directed along the external field. Let us direct the axis Z for \mathcal{E} . Then for the z component of the dipole moment we obtain

$$\mathcal{D} = \frac{e^2 \mathcal{E}}{T} \int_{V_t} d^3\mathbf{r} z^2 n_0(\mathbf{r}).$$

z^2 outside the integral sign at an intermediate point. Up to within a numerical factor of the this will be the cluster size r_t (6):

$$\mathcal{D} = \frac{e^2 \mathcal{E} r_t^2}{T} \int_{V_t} d^3\mathbf{r} n_0(\mathbf{r}). \quad (10)$$

gral with respect to the coordinates is now equal to the number of the electrons in the cluster. charge must be compensated for by the charge of excessive donors in the cluster volume, and ave

$$\int_{V_t} d^3\mathbf{r} n_0(\mathbf{r}) \sim \sqrt{N r_t^3}. \quad (11)$$

To determine the averaged dipole moment over the crystal the cluster dipole moment (10) must be multiplied by the fraction of the volume occupied by the clusters, i. e., ν_t in (7). Then, in view of relation (11), we obtain

$$\langle \mathcal{D} \rangle \equiv \nu_t \mathcal{D} = \frac{1}{T} e^2 n r_t^2 \mathcal{E}.$$

Using the explicit expression (6) for r_t we find

$$\langle \mathcal{D} \rangle = \left(\frac{T}{\gamma(r_s)} \right)^3 \mathcal{E} \ln^4(w_0 t).$$

The current density can be determined from the time derivative of the dipole moment:

$$j(t) = \frac{d\langle \mathcal{D} \rangle}{dt} = 4 \left(\frac{T}{\gamma(r_s)} \right)^3 \mathcal{E} \frac{\ln^3(w_0 t)}{t}. \quad (12)$$

To calculate the conductivity, we find the Fourier components of the external field and the current density:

$$\begin{aligned} \mathcal{E}_\omega &= \int_{-\infty}^{\infty} \mathcal{E} \theta(t) \exp\{i\omega t\} dt = \frac{\mathcal{E}}{\varepsilon - i\omega}, \quad \varepsilon \rightarrow +0, \\ j_\omega &= \int_0^{\infty} j(t) \exp\{i\omega t\} dt. \end{aligned}$$

The conductivity $\sigma(\omega)$ is given by the formula

$$j_\omega = \sigma(\omega) \mathcal{E}_\omega,$$

whence

$$\sigma(\omega) = -\frac{i\omega}{\mathcal{E}} \int_0^{\infty} j(t) \exp\{i\omega t\} dt. \quad (13)$$

The explicit expression for the current (12) is known only in the time interval (9) whereas the calculation of conductivity (13) requires the knowledge of the integrand function throughout the interval of integration from 0 to ∞ . However, we note that in the frequency band

$$w_0 \exp\left\{-\frac{\gamma(r_s)}{T}\right\} \ll \omega \ll w_0 \quad (14)$$

the major contribution to the real part of conductivity (13) comes from the integration over the interval (9). Indeed, for $\omega \ll w_0$ the integral from 0 to w_0^{-1} does not depend on frequency and is simply equal to a constant making a contribution only to the imaginary part of the conductivity. The integral from $w_0^{-1} \exp\{\gamma(r_s)/T\}$ to ∞ for $\omega \gg w_0 \exp\{-\gamma(r_s)/T\}$ contains a rapidly oscillating exponential function, and its contribution to the conductivity is small. Therefore in frequency band (14) function (12) can be extended throughout the interval of integration.

The real part of conductivity (13) is calculated in Supplement:

$$\text{Re}\sigma(\omega) = 4 \left(\frac{T}{\gamma(r_s)} \right)^3 \omega \ln^3\left(\frac{w_0}{\omega}\right). \quad (15)$$

Expression (15) can be represented in the standard form

$$\text{Re}\sigma(\tilde{\omega}) = A \tilde{\omega}^s,$$

where $\tilde{\omega} = \omega/w_0$ is the dimensionless frequency, $A = 4\kappa w_0 (T/\gamma(r_s))^3$, and

$$s = 1 - \frac{\ln \ln(\tilde{\omega}^{-1})}{\ln(\tilde{\omega}^{-1})}.$$

In frequency band (14) the exponent s increases with increasing frequency and varies within the limits

$$1 - \frac{T}{T_0} < s < 1,$$

where $T_0 = \gamma(r_s) \ln(\gamma(r_s)/T)$ depends weakly (logarithmically) on temperature. We note that for $T/\gamma(r_s) \ll 1$ the values of s are always close to unity.

SUPPLEMENT

Substituting the function $j(t)$ (12) into the expression for conductivity (13) we find the real part of the conductivity:

$$\operatorname{Re}\sigma(\omega) = 4\omega \left(\frac{T}{\gamma(\tau_s)} \right)^3 L(\omega), \quad (16)$$

where

$$L(\omega) = \int_0^{\infty} dt \frac{\sin(\omega t)}{t} \ln^3(\omega t).$$

The change of the variable $\omega t = \xi$ results in

$$L(\omega) = \int_0^{\infty} d\xi \frac{\sin \xi}{\xi} \ln^3 \left(\frac{w_0}{\omega} \xi \right).$$

The factor $\sin \xi / \xi$ truncates in fact the integrand function at the values $\xi \sim 1$, and therefore to estimate $L(\omega)$ we can set

$$L(\omega) \approx \int_0^1 d\xi \ln^3 \left(\frac{w_0}{\omega} \xi \right) = \ln^3 \frac{w_0}{\omega} - 3 \ln^2 \frac{w_0}{\omega} + 6 \ln \frac{w_0}{\omega} - 6.$$

Since $w_0/\omega \gg 1$ (see (14)) and setting $\ln(w_0/\omega) \gg 1$, we obtain

$$L(\omega) \approx \ln^3 \frac{w_0}{\omega}. \quad (17)$$

Substituting (17) into (16) we arrive at formula (15).

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