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AN EXACT SOLUTION FOR THE LIGHT SHELL IN THE RGT TYPE THEORY

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An exact continuous solution is found for the light shell in the RGT type theory. It is shown that the solution is clearly nonstatic, which is what must be the case for any RGT theory.

The nonstatic character of the external spherically symmetric gravitational field in the relativistic gravitation theory (RGT) can be regarded as an established fact [1]. Nevertheless, it is always interesting to demonstrate the way the nonstatic behavior appears for an exact continuous solution to the equations. In the present paper we give such an exact solution to the equations of the RGT type theory (we remind the reader that RGT type theories differ in the choice of an additional covariant vector equation relating the metrics γ_{ij} and g_{ij} of the Minkowski and effective curved spaces).

Consider a singular spherically symmetric light shell contracting from infinity in an originally planar space-time. Since no information can propagate at a rate exceeding the light speed, there is no gravitational field inside that shell until it attains the center $\rho = 0$, and the metric in this region coincides with that of the planar space-time:

$$ds_-^2 = dt^2 - d\rho^2 - \rho^2 d\Omega^2 = \gamma_{ij}(\xi) d\xi^i d\xi^j.$$

In the coordinates t and ρ of the Minkowski space the equation of light shell motion is

$$d\rho/dt = -1,$$

whence

$$\rho = t_0 - t. \tag{1}$$

Outside the light shell the space-time metric corresponds to the curved space and can be reduced to the Schwarzschild form in the variables τ and r:

$$ds_+^2 = d\tau^2 A - dr^2/A - r^2 d\Omega^2 = g_{ij+}(x) dx^i dx^j,$$

where $A = 1 - 2m/r$ and m is the total mass-energy of the light shell.

In the variables τ and r the equation of shell motion is written

$$dr/d\tau = -A,$$

which yields

$$r + 2m \ln |r - 2m| = \tau_0 - \tau.$$

On the boundary of the light shell the interior and exterior metrics must coincide (this is the joining condition; for a light shell it is, for instance, discussed in paper [2] and in the works cited there):

$$\gamma_{ij}(\xi) = \frac{\partial x^m}{\partial \xi^i} \frac{\partial x^n}{\partial \xi^j} g_{mn+}(x). \tag{2}$$

Relation (2) immediately implies that on the boundary the equations

$$r = \rho, \qquad \tau - \tau_0 = t - t_0 - 2m \ln |t_0 - t - 2m|,$$
$$1 = \dot{\tau}^2 A - \dot{r}^2/A; \quad -1 = \tau'^2 A - r'^2/A; \quad 0 = \tau' \dot{\tau} A - r' \dot{r}/A$$

are fulfilled, from whose solution it follows that on the boundary we have

$$r = \rho, \qquad \tau - \tau_0 = t - t_0 - 2m\ln|t_0 - t - 2m|,$$
$$\dot{\tau} = \frac{t_0 - t - m}{t_0 - t - 2m}, \qquad \tau' = -\frac{m}{t_0 - t - 2m},$$
$$\dot{r} = -\frac{m}{t_0 - t}, \qquad r' = 1 - \frac{m}{t_0 - t}. \tag{3}$$

Relations (3) imply that any exterior metric in the variables ξ that is joined with the interior metric (2) is clearly nonstatic with respect to the variable t because the derivatives $\dot{r}$ and τ' do not vanish.

There are an infinite number of functions $\tau(t, \rho)$ and $r(t, \rho)$ satisfying boundary conditions (3) and the asymptotic conditions $g_{ij+}(\xi) \to \gamma_{ij}(\xi)$ for $\rho \gg t_0 - t$. These functions are selected in RGT type theories by imposing on γ_{ij} and g_{ij} a certain covariant vector differential equation.

If, for instance, the relation

$$D_i(g/\gamma) = 0$$

is taken as the additional equation (it was first suggested in [3]), then for the problem under study it reduces to the equation

$$r'\dot{\tau} - \tau'\dot{r} = \rho^2/r^2. \tag{4}$$

The solutions to Eq. (4) satisfying boundary conditions (3) are the following functions:

$$r = \rho + m\left(\frac{t_0 - t}{\rho} - 1\right)$$

and $\tau = \tau(t, \rho)$, the latter being expressed in terms of the parameters λ and σ:

$$\rho = (2m\lambda + (t_0 - \sigma)^2)^{1/2};$$
$$t - t_0 = (\lambda + (t_0 - \sigma)^2/(2m))^{1/2}[-(2m)^{1/2} + (2/m)^{1/2}(\sigma - t_0) + 2(\lambda + (t_0 - \sigma)^2/(2m))^{1/2}],$$
$$\tau = m\lambda^2(t_0 - \sigma)^{-2} + \lambda + \sigma - 2m\ln|t_0 - \sigma - 2m| + \text{const}. \tag{5}$$

Expression (5) is an exact continuous solution to the original equations of the theory and satisfies the asymptotic condition $g_{ij+}(\xi) \to \gamma_{ij}(\xi)$ for $\rho \gg t_0 - t$. By virtue of the joining condition (2), the exterior metric in the variables of the Minkowski space coincides with the metric of the Minkowski space on the boundary of the light shell and, consequently, is finite when the shell attains the Schwarzschild sphere.

Some other RGT type theories, e. g., those with an additional equation of the form of $D_i[(-g)^{1/2}g^{ij}] = 0$, result in a different relationship between the variables x and ξ for the exterior solution, but all such solutions are nonstatic by virtue of the boundary conditions (3). In this paper the equation $D_i(g/\gamma) = 0$ was taken for the problem in question because it yields an exterior solution in the compact analytic form (5) whereas an equation of the form of $D_i[(-g)^{1/2}g^{ij}] = 0$ has no compact solution in an analytic form for the problem under consideration.

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